

# BUNCH LENGTHENING THRESHOLDS ON THE DARESBUURY SRS

J. A. Clarke, CCL Daresbury Laboratory, Warrington, WA4 4AD, UK.

The bunch length of the SRS has been studied over a large range of beam currents at energies between 0.6 GeV and 2.0 GeV. Longitudinal microwave instability thresholds have been observed and used to estimate the broadband impedance of the SRS. The results are compared with measurements based on other effects.

## I. INTRODUCTION

The behaviour of single bunch beams in the SRS have been studied in some detail [1, 2]. This behaviour is largely dependent upon the basic impedances of the vacuum chamber experienced by the electron beam. This paper presents recent results which provide further information on these impedances and their effects. In particular, the data clearly demonstrates microwave instability thresholds and potential well distortion.

The bunch length data has been collected with a photodiode based system [3] which measures a significantly shorter value than the previously used stroboscopic image dissector [4]. Earlier data has been revised and re-evaluated in the light of these discrepancies.

## II. BUNCH LENGTHENING THEORY

The length of an electron bunch in a storage ring is dependent upon the peak current of the bunch. The two effects which alter the length are potential well distortion and microwave instability. For potential well distortion the bunch length varies due to the electro-magnetic fields induced by the electrons altering the RF voltage seen by the bunch. This effect is present even at very low currents. The second effect, microwave instability, is only observed after a certain threshold current has been reached. Above this threshold the energy spread of the beam increases until the peak current of the bunch reduces to equal the threshold current again.

### A. Potential Well Distortion

At very small currents the electron beam is described by the natural bunch length,  $\sigma_{l_0}$ :

$$\sigma_{l_0} = \frac{c\alpha}{\omega_{s_0}} \sigma_{p_0} \quad (1)$$

where  $c$  is the speed of light,  $\alpha$  is the momentum compaction,  $\sigma_{p_0}$  is the natural relative energy spread of the beam and  $\omega_{s_0}$  is the associated synchrotron oscillation frequency. The bunch length,  $\sigma_l$ , modified by the potential well distortion is given by [5]:

$$\left(\frac{\sigma_l}{\sigma_{l_0}}\right)^3 - \left(\frac{\sigma_l}{\sigma_{l_0}}\right) + I_b \frac{e\alpha \text{Im}\{[Z/n]_{eff}\}}{\sqrt{2\pi}E v_{s_0}^2} \left(\frac{R}{\sigma_{l_0}}\right)^3 = 0 \quad (2)$$

where  $I_b$  is the average beam current,  $e$  the electron charge,  $R$  the ring average radius,  $E$  the beam energy and  $v_{s_0}$  is the synchrotron tune. The size of the effect is dependent upon the reactive part of the effective longitudinal coupling impedance  $[Z/n]_{eff}$ .

### B. Microwave Instability

If the peak current of the beam exceeds the so-called microwave instability threshold then the energy spread of the beam,  $\sigma_p$ , increases thus increasing the bunch length (from Eqn (1)). The bunch length continues to grow until the peak current reduces to equal the threshold current. The threshold current, written in terms of the average beam current, is given by [6]:

$$I_b = \frac{\sqrt{2\pi}\alpha E \sigma_p^2 \sigma_l}{eR[Z/n]_{BB}} \quad (3)$$

where  $[Z/n]_{BB}$  is the longitudinal broadband impedance. Combining Eqns (1) (for  $\sigma_p$ ) and (3) leads to the dependence of the bunch length purely on the microwave instability:

$$\sigma_l^3 = \frac{ec^2\alpha R[Z/n]_{BB}}{\sqrt{2\pi}E\omega_{s_0}^2} I_b \quad (4)$$

Since the instability increases the energy spread of the beam the horizontal beam size,  $\sigma_x$ , is also increased. The dependence of  $\sigma_x$  on  $\sigma_p$  is given by:

$$\sigma_x^2 = \epsilon_x \beta_x + \eta_x^2 \sigma_p^2 \quad (5)$$

where  $\epsilon_x$  is the horizontal emittance,  $\beta_x$  is the horizontal betatron function and  $\eta_x$  is the horizontal dispersion function. Note that since in general the bunch length depends upon two effects at high current, the horizontal beam size gives a more direct observation of the broadband impedance.

### C. Combination of Both Effects

Above the microwave threshold the bunch length is modified by a combination of both effects. The potential well distortion alters the synchrotron frequency and the microwave instability increases the energy spread. When both effects are present Eqn (1) can be rewritten as:

$$\sigma_l = \frac{c\alpha}{\omega_s} \sigma_p \quad (6)$$

By deriving  $\omega_s$  from Eqns (1) and (2) and  $\sigma_p$  from Eqns (1) and (3) it is possible to show that the bunch length dependence upon the two effects is given by:

$$\sigma_l^3 = \frac{ec^2 \alpha R}{\sqrt{2\pi E \omega_{s_0}^2}} \left\{ [Z/n]_{BB} - \text{Im}[Z/n]_{eff} \right\} I_b \quad (7).$$

### III. BUNCH LENGTHENING BELOW THRESHOLD

In order to determine the contribution that potential well distortion has on the bunch length of the SRS an experiment was carried out below the threshold current. The length of a single bunch beam was recorded as a function of beam current at 1.7 GeV. Throughout the experiment  $\sigma_x$  remained constant, confirming that the beam was indeed below the threshold of microwave instability. The variation of the bunch length with the beam current is given in figure 1. The solid line on the figure is a curve derived using Eqn (2) for  $[Z/n]_{eff} = -0.7 \Omega$ .

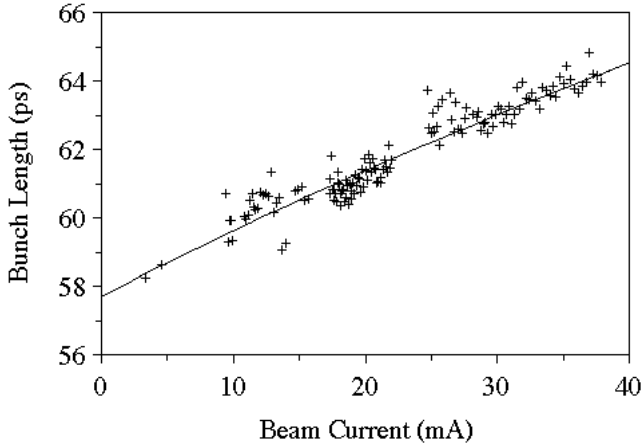


Figure 1. The bunch length as a function of beam current at 1.7 GeV.

### IV. BUNCH LENGTHENING ABOVE THRESHOLD

A similar experiment to the one above was carried out at the lower energy of 0.6 GeV. The microwave threshold in this case was expected to be  $\approx 1$  mA so all of the measurements were taken well above this value. The bunch lengthening observed is shown in figure 2. The solid line shows the best fit to be to a power of 0.295. If the fit is forced to a one-third dependence following Eqn (7) then a combined vacuum chamber impedance of  $3.1 \Omega$  is estimated. Subtracting the contribution from the effective longitudinal coupling impedance derived in section III gives a value for the longitudinal broadband impedance of  $2.4 \Omega$ .

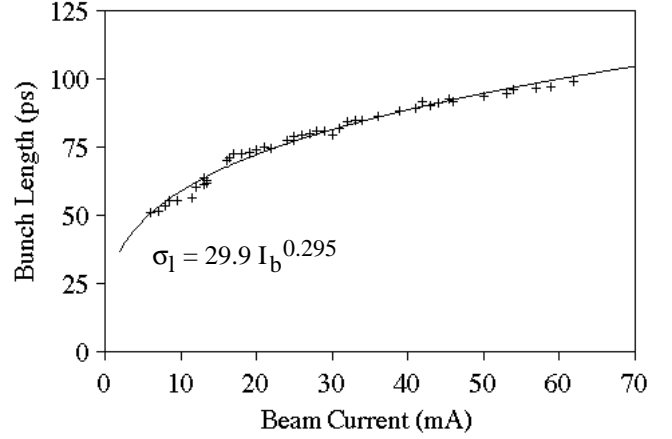


Figure 2. The bunch length as a function of beam current at 0.6 GeV.

### V. OBSERVATION OF BUNCH LENGTHENING THRESHOLD

Direct observation of the onset of microwave instability in the SRS is only possible at an intermediate energy. Measurements have been carried out between 1.0 and 1.375 GeV. In these experiments all three beam dimensions were recorded. Surprisingly the threshold was clearly visible on the vertical beam size as well as the horizontal one. A subsequent experiment confirmed the presence of finite vertical dispersion in the SRS lattice [7].

The bunch length measured as a function of current at 1.125 GeV is shown in figure 3. The microwave instability threshold appears at around 19 mA. The bunch length data above threshold has been fitted to the model described by Eqn (7). This implies a combined impedance of  $2.9 \Omega$ . Again subtracting  $0.7 \Omega$  due to the effective longitudinal coupling impedance gives a longitudinal broadband impedance of  $2.2 \Omega$ , similar to the value found earlier in section IV.

The horizontal beam size growth should be entirely due to the microwave instability (ie no potential well distortion term) so the data can be used to derive the broadband impedance directly. By substituting the value for  $\sigma_p^2$  from Eqn (3) into (5) it is clear that a plot of  $\sigma_x^2$  against  $I_b/\sigma_l$  should give a linear graph with gradient proportional to  $[Z/n]_{BB}$ . The horizontal beam size data is shown in this form in figure 4. The gradient implies a longitudinal broadband impedance of  $1.2 \Omega$ , significantly less than the values derived above.

It is also possible to derive the value for  $[Z/n]_{BB}$  from the microwave threshold current itself (see Eqn (3)). If the threshold current is taken to be 19 mA then this produces a value for the longitudinal broadband impedance of  $1.5 \Omega$ .

The data taken at 1.375 GeV was not so clear as the 1.125 GeV values. The bunch length data was not recorded correctly due to a triggering failure and the horizontal beam size was unusually noisy. However, the threshold was clearly observed

in the vertical plane, due to the unexpected finite vertical dispersion in the SRS. A threshold of 28 mA is apparent in figure 5. This leads to a value for the longitudinal broadband impedance of  $1.9 \Omega$ .

One further set of data was recorded at 1.0 GeV. The microwave instability threshold was observed to occur at 9 mA in this instance, implying a value for the longitudinal broadband impedance of  $1.8 \Omega$ .

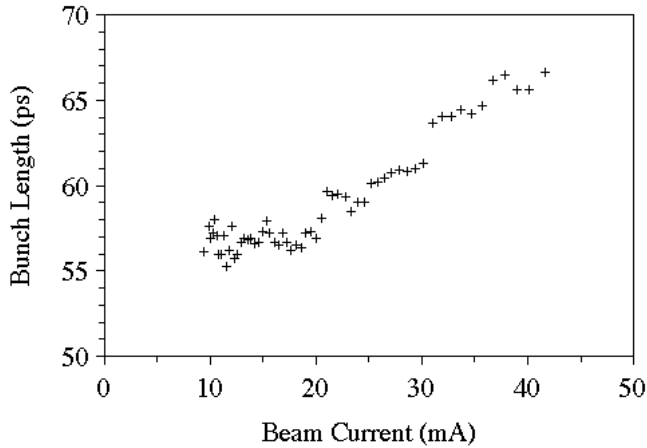


Figure 3. The bunch length as a function of beam current at 1.125 GeV.

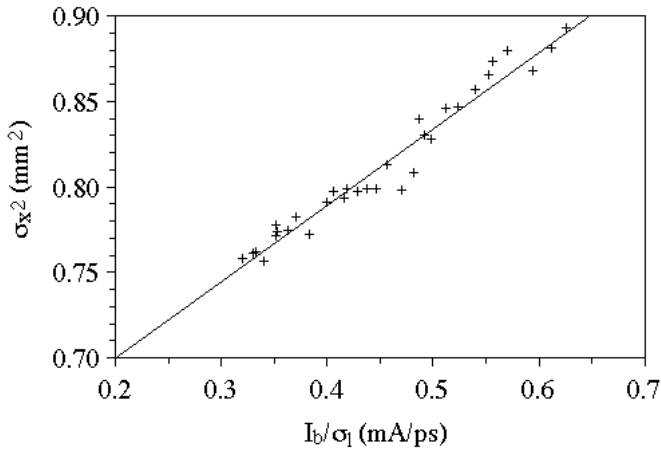


Figure 4. The horizontal beam size dependence upon the ratio of the beam current to the bunch length at 1.125 GeV.

| Energy (GeV) | Technique         | $[Z/n]_{BB}$ ( $\Omega$ ) | $[Z/n]_{BB} - \text{Im}[Z/n]_{eff}$ ( $\Omega$ ) |
|--------------|-------------------|---------------------------|--|
| 0.6          | $\sigma_l$ growth | $2.4 \pm 0.7$             | $3.1 \pm 0.5$                                    |
| 1.0          | threshold         | $1.8 \pm 0.5$             | $2.5 \pm 0.7$                                    |
| 1.125        | $\sigma_l$ growth | $2.2 \pm 1.1$             | $2.9 \pm 0.9$                                    |
| 1.125        | $\sigma_h$ growth | $1.2 \pm 0.4$             | $1.9 \pm 0.6$                                    |
| 1.125        | threshold         | $1.5 \pm 0.4$             | $2.2 \pm 0.6$                                    |
| 1.375        | threshold         | $1.9 \pm 0.5$             | $2.6 \pm 0.7$                                    |

Table 1. Summary of the impedances measured in the SRS.

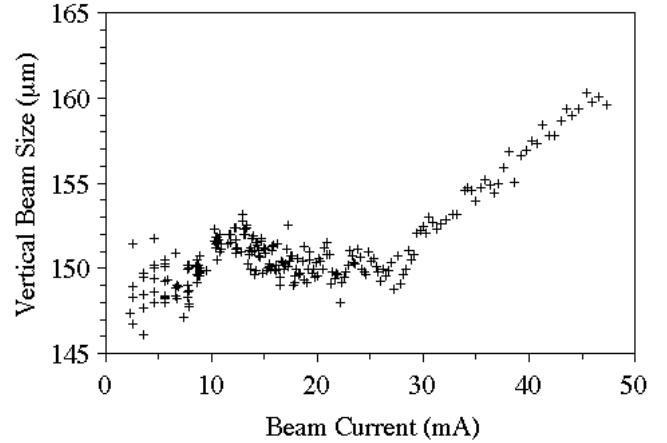


Figure 5. The vertical beam size as a function of beam current at 1.375 GeV.

## VI. SUMMARY AND CONCLUSIONS

Two types of bunch lengthening have now been evaluated on the SRS in single bunch mode. Potential well distortion has been observed at high energy. The bunch lengthening measured has been used to estimate a value for the reactive part of the longitudinal coupling impedance to be  $-0.7 \pm 0.2 \Omega$ .

The microwave instability has been observed over a wide range of energies. Bunch lengthening and beam size growth in both planes has been noted. The increases to the beam dimensions and the threshold currents have been used to derive estimates for the longitudinal broadband impedance. The values derived are summarised in Table 1. It appears that the best estimate for the broadband impedance of the SRS is  $1.8 \pm 0.6 \Omega$ . This value is significantly smaller than that previously published since the earlier work was largely based upon incorrect bunch length data [3].

## VII. REFERENCES

- [1] J. A. Clarke et al, "Revised SRS Impedance Estimates", in Proc Euro Part Accel Conf, London, 1994, p 1096.
- [2] L. A. Welbourn et al, "Single Bunch Effects in the Daresbury SRS", in Proc IEEE Part Accel Conf, Washington, 1993, p 3672.
- [3] J. A. Clarke, "A New Picosecond Bunch Length Monitor on the SRS", in Proc Euro Part Accel Conf, London, 1994, p 1682.
- [4] G. S. Brown et al, "Measurement of Bunch Length with an Image Dissector Tube", IEEE Trans. Nucl. Sci., NS-30, No. 4, p 2348, August 1983.
- [5] B. Zotter, "Potential-Well Bunch Lengthening", CERN SPS/81-14 (DI), 1981.
- [6] J. LeDuff et al, "Single Beam Collective Effects on DCI", Proc 11th Int Conf High Energy Accel, CERN, 1980, p 566.
- [7] J. A. Clarke, Internal Note SRS/APES/95/13, March 1995.