BEAM TRANSFER FUNCTION AND TRANSVERSE IMPEDANCE MEASUREMENTS IN THE FERMILAB MAIN RING

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ABSTRACT

Knowledge of the Main Ring(MR) impedance is crucial for the understanding of beam instabilities. Beam transfer function measurements provide a direct measurement of beam impedance[1, 2]. With proper calibration procedures, values for the transverse impedances can be extracted from beam transfer function measurements. The principle of measurements is reviewed and results are presented.

I. REVIEW OF MEASUREMENT PRINCIPLES

A. *Beam transfer function*

Beam transfer function(BTF) measurement is performed by driving the beam with an external RF excitation, then one measures the induced signal from a transverse pickup. We use the vertical feedback system of Fermilab Main Ring to drive the beam. The measurement setup is depicted in Figure 1.

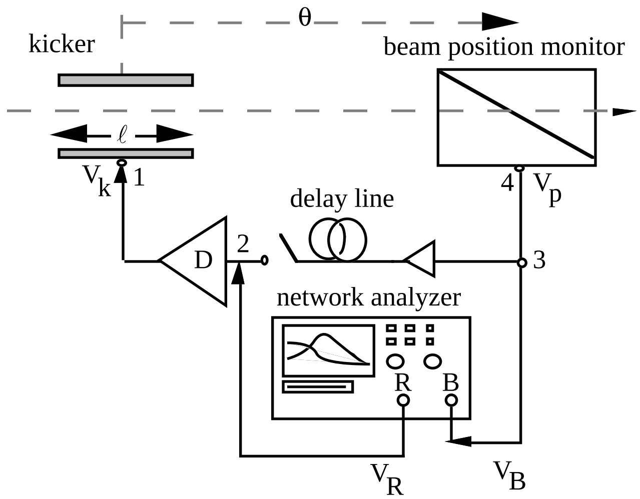

Figure 1: The setup for open-loop beam transfer function measurement. θ is the betatron phase advance from kicker to pickup.

The equation of motion for a single particle driven by external excitation at the location of kicker is given by:

$$\frac{d^2y}{dt^2} + Q^2\Omega^2 y = G\,e^{-i\omega t} + i\frac{eI_b}{\gamma m_0}\frac{Z_\perp(\omega)}{L}\langle y\rangle$$

where e=charge of proton, I_b=beam current, m_0=rest mass of particle, γ =Lorentz relativistic factor, <y>=averaged beam displacement, L=accelerator circumference, Q=betatron tune of individual particle, Ω=angular revolution frequency of individual particle, ω=angular frequency of external RF excitation and $Z_\perp$ is the transverse impedance of accelerator.

The beam transfer function[3] is defined as:

$$B(\omega) = -iQ_0\Omega_0\left\langle\frac{\tilde{y}}{G}\right\rangle$$

For zero accelerator impedance the BTF is given by[1]:

$$B_0(\omega) = \frac{(-/+)}{2}\left[\pi\rho(\omega) + i\cdot pv\int\frac{\rho(\omega_\beta)}{\omega_\beta-\omega}d\omega_\beta\right]$$

where $\rho(\omega_\beta)$ is the normalized betatron frequency distribution of particle beam, ω_β is the betatron frequency of individual particle. When ω_β is near the Ω(n+Q) sideband, the minus sign is used. The plus sign is used for the Ω(n-Q) sideband.

If one inverts the BTF and plots the imaginary part vs. real part, one will get the stability diagram. The equation of the stability diagram is given by:

$$\frac{1}{B(\omega)} = \frac{1}{B_0(\omega)} + \frac{eI_b(\omega)}{\gamma m_0 Q_0\Omega}\frac{Z_\perp(\omega)}{L}$$

From the equation of the stability diagram one can determine the accelerator impedance simply by measuring the displacement of the contour from the origin of the complex plane.

B. *Calibration*

The driven beam response can be measured by the induced voltage on beam position monitor(BPM) at point 4:

$$V_p = I_b\langle y\rangle S_\perp$$

where $S_\perp$ is the detector sensitivity[4]. The actual signal measured by the network analyzer is V_B. From the definition of the beam transfer function, the induced voltage V_p can be rewritten as:

$$V_p = \frac{I_b(\omega)S_\perp(\omega)B(\omega)}{-iQ_0\Omega_0}G$$

If h denotes the cable length from port 4 of BPM to port B of the network analyzer, then the S_{21} parameter can be written as:

$$S_{21} = \frac{eI_b(\omega)S_\perp K_\perp D}{-i\gamma m_0\ell Q_0\Omega_0}B(\omega)e^{-i\omega h/c}$$

where ℓ is the length of kicker plate, $K_\perp$ is the kicker constant[4] which characterizes the response of the kicker and D is the voltage gain factor of the amplifier.

* Operated by Universities Research Association Inc., under contract with the U.S. Department of Energy.

One can compensate the cable delay during data analysis. Since the frequency span for a BTF measurement is much smaller than the 3 dB frequency bandwidth of feedback system, S_{21} parameter and B(ω) is only different by a proportionality constant. One can just do a linear fit to determine the proportional constant, which is given by the following:

$$\frac{eI_b(\omega)S_\perp K_\perp D}{-i\gamma m_0 \ell Q_0 \Omega_0}$$

The above arguments will not hold if the frequency span of the BTF measurement is beyond the 3 dB bandwidth of feedback system. To improve the signal to noise ratio, a time domain gating technique[5] is applied to the analysis. Because we are interested in the low frequency range of the transverse impedance, a detector optimized for low frequency signals was built as depicted in Figure 2.

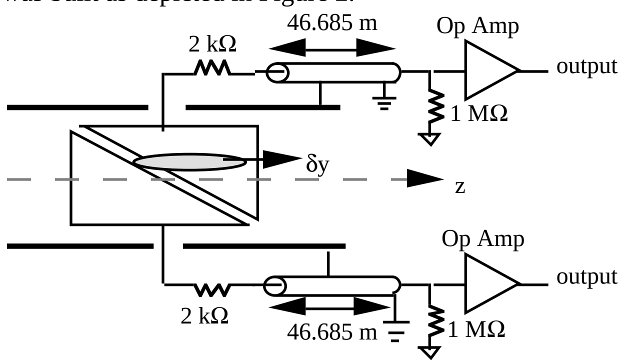

Figure 2: The configuration of beam position monitor used for BTF measurements.

II. RESULTS OF BEAM EXPERIMENTS

Vertcial BTF measurements were done at the injection energy of the MR, 8 GeV, with debunched beam. Figure 4 and 5 are examples of measured raw data. Figure 6 is the stability diagram after the time domain gating technique is applied. The signal is still quite noisy. The fitting results are shown as dashed lines in Figures 6 -9. Only the central portion of resonance response is used for fitting because of the noise. The fitting result for vertical impedance is shown in Figure 10.

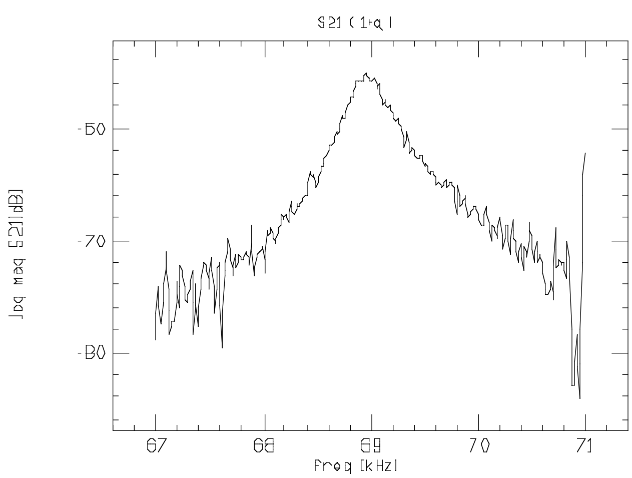

Figure 3: Amplitude of S21(raw data) for the n=1+q sideband.

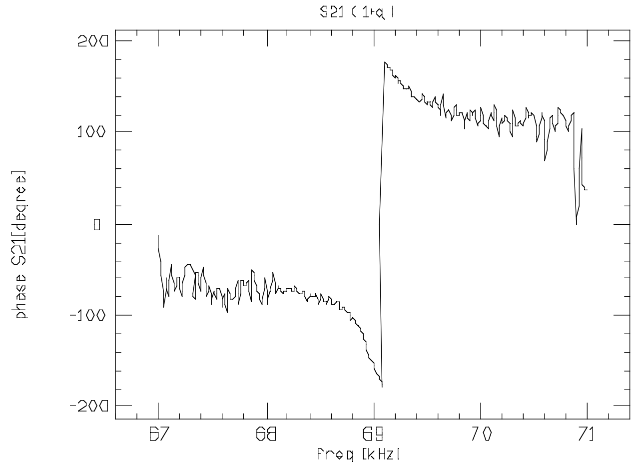

Figure 4: Phase of S21(raw data) for the n=1+q sideband.

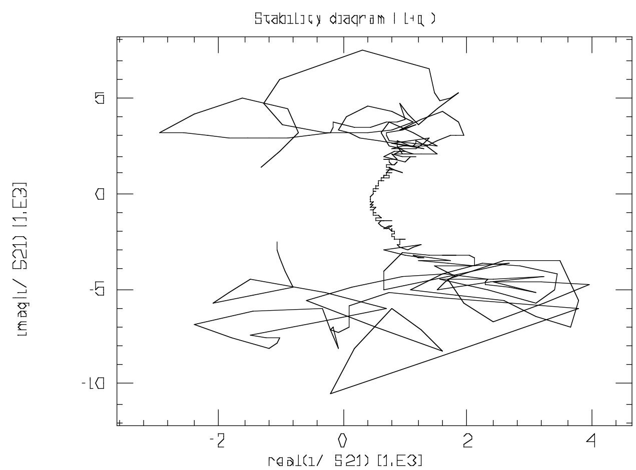

Figure 5: Stability diagram for the n=1+q betatron sideband processed by applying the time domain gating technique.

The revolution frequency of Main Ring is 47.4 kHz and the fractional tune is 0.4 . The beam pipe is made of stainless steel and the thickness is 0.065”. If we assume only beam pipe of circular cross section is used around the accelerator, we can calculate the vertical impedance. The result is depicted in Figure 11.

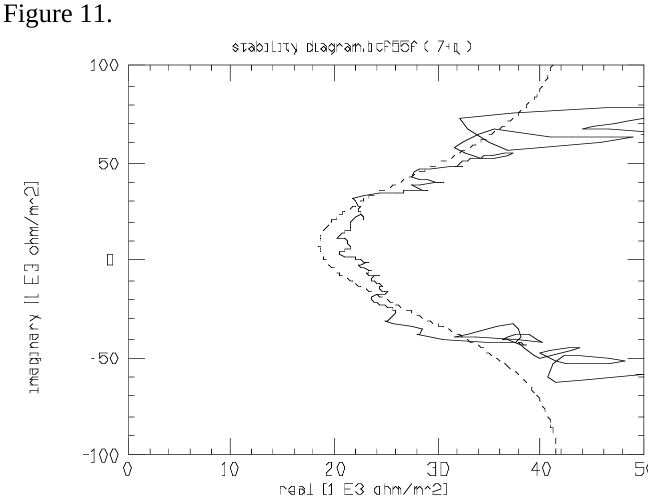

Figure 6: Stability diagram for the n=7+q betatron sideband.

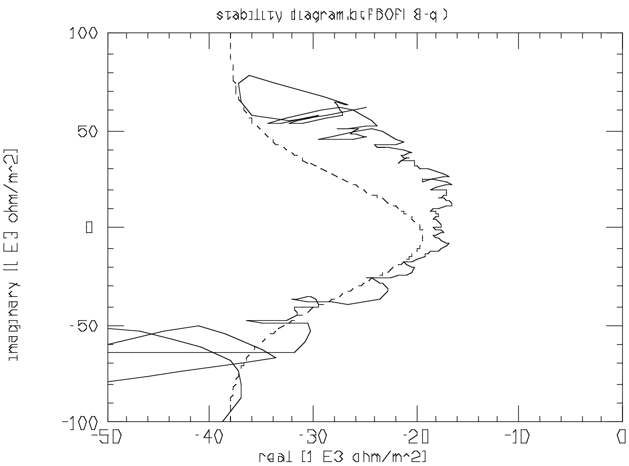

Figure 7: Stability diagram for the n=8-q betatron sideband.

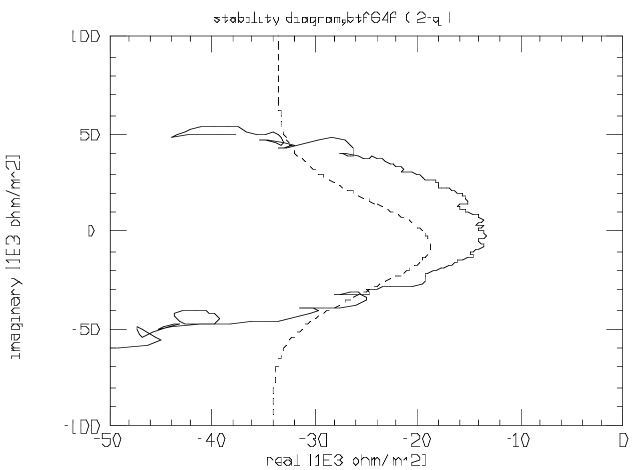

Figure 8: Stability diagram for the n=2-q betatron sideband.

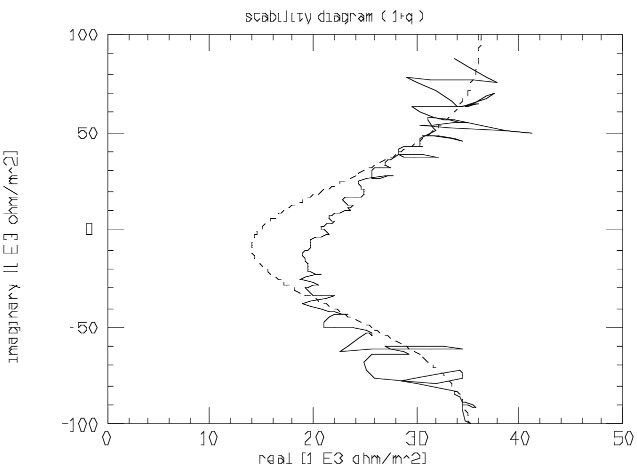

Figure 9: Stability diagram for the n=1+q betatron sideband.

We are particularly interested in the vertical impedance of the first few betatron sidebands because of evidence showing the signs of collective beam instabilities due to the resistive wall impedance[6]. The results from BTF measurements do not show the characteristic $\omega^{-1/2}$ dependence of resistive wall impedance. Other sources besides wall resistance of the beam pipe may contribute to the observed phenomena. Work is still underway to improve the accuracy of measurements.

III. ACKNOWLEDGEMENT

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IV. REFERENCES

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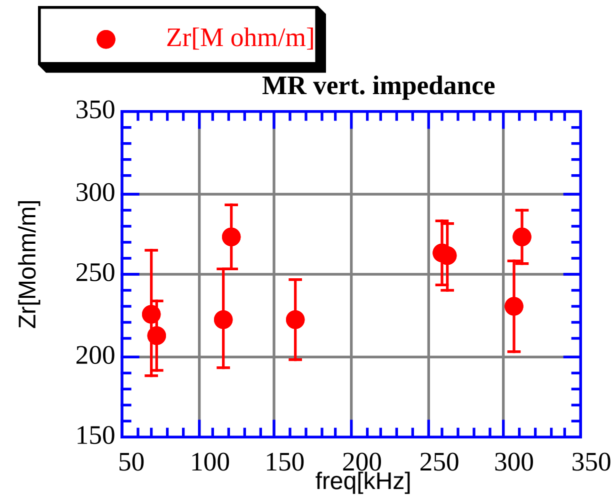

Figure 10: The vertical impedance of the Fermilab Main Ring from BTF measurements. The unit of y axis is in MΩ/m.

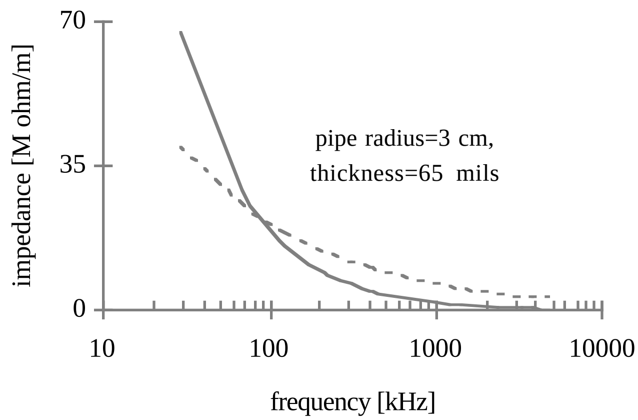

Figure 11: Theoretical calculation of vertical impedance for Main Ring. Only the real part is plotted. The dashed line is the value for thick wall model and the solid line is the thin wall model.