SIMULATIONS OF TRANSITION CROSSING IN THE MAIN INJECTOR

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Abstract

The design goal for the Fermilab Main Injector (FMI) is to accelerate a minimum of 6×10^{10} protons per bunch through the transition. We present here the results from simulation studies of the transition crossing in the FMI using the particle tracking code ESME[1].

I. INTRODUCTION

The Fermilab Main Injector (FMI)[2] that is under construction is intended to be a high intensity 150 GeV proton injector to the Tevatron. The beam in the FMI will be accelerated from 8 GeV to 150 GeV through a transition energy of 20.48 GeV. The longitudinal emittance of the proton beam at injection is about 0.1 eVs, and the intensity will be more than 6×10^{10} protons per bunch. Maintaining the beam intensity as well as its longitudinal emittance through the acceleration cycle is very important for the FMI operation. In the past, preserving the beam emittance and the intensity through transition crossing in a proton synchrotron has been one of the major problems. A number of techniques have been suggested to cure these problems[3,4]. Two of the suggested techniques viz., a) γ_t-jump scheme[3] and b) focus free transition crossing(FFTC) [4] have been investigated in some detail for proton synchrotron along with with the normal transition phase jump (NTPJ) scheme. Here, the particle tracking code ESME[1] has been used to study the longitudinal beam dynamics of the transition crossing in the FMI for these three different schemes.

The condition of non- adiabaticity[5] exists in a proton synchrotron when,

$$\frac{\mid \gamma-\gamma_t \mid}{\gamma_t} \leq \left[\frac{\gamma_t (eV_{rf}\, sin\phi_s)^2}{4\pi h E_o eV_{rf} \mid cos\phi_s \mid}\right]^{1/3} \quad (1)$$

where γ_t is the relativistic quantity γ at transition, V_{rf} is the peak rf voltage at transition, ϕ_s is the synchronous angle of the beam with the rf wave form, h is the harmonic number of the machine and E_o is the rest mass of proton. By assuming that the γ is increasing linearly near transition at a rate $\dot{\gamma}$ this expression can be converted to a non-adiabatic time period in the vicinity of the transition time,

$$T_{na} = \pm T_s \left[\frac{f_s E_o {\gamma_t}^4}{4\pi h\dot{\gamma} eV_{rf} \mid cos\phi_s \mid}\right]^{1/3} \quad (2)$$

where f_s=1/T$_s$ is the revolution frequency of the synchronous particle. Since all the particles in a bunch do not pass through the transition at the same time, there will be a non-linear period during which some particles are above the transition energy while others are below it. The non-linear time is given by,

$$T_{nl} = \pm\gamma_t \left[\frac{\beta^2 + \alpha_1/\alpha_o + 1/2}{\dot{\gamma}}\right] \frac{\Delta p}{p} \quad (3)$$

where β is the ratio of particle velocity and the velocity of light. α_1 is the second order term in the expansion of path length in Δp/p and $\alpha_o = \gamma_t^{-2}$. During this time the rf focusing force causes increased momentum spread and a number of different instabilities come into play. Since the non-adiabatic and non-linear time decrease with increased $\dot{\gamma}$, the simulations have been carried out for two different values of $\dot{\gamma}$ for the FMI operating scenarios.

II. ESME SIMULATIONS OF TRANSITION CROSSING

In ESME, the collective behavior of the beam particles is treated using a pair of Hamilton-like difference equations describing synchrotron oscillations in the energy-angle (ΔE, ϕ) phase space, (where ΔE = E-E$_o$ and $0\leq \phi \leq 2\pi$). The particles in a bunch are assumed to have an elliptical distribution which is a good representation of the beam bunches coming from the Fermilab Booster. For a cylindrical beam pipe of radius 'b' and a co-axial beam of radius 'a', the impedance, Z_ω seen by a single Fourier component of the beam current at a frequency $\omega/2\pi$, is,

$$\frac{Z_\omega}{n} = -j\frac{Z_o g}{2\beta\gamma^2} + \frac{Z_W}{n} + \frac{Z_\parallel(\omega)}{n} \quad (4)$$

where Z_o = 377 Ohm (Impedance of free space), Z_W is total wall impedance of the beam pipe and the geometry factor g = 1+ $2ln(b/a)$. The average values of 'a' and 'b' are listed in Table I. The $Z_\parallel$ is given by,

$$Z_\parallel(\omega) = \frac{R_s}{1+jQ(\frac{\omega_c}{\omega} - \frac{\omega}{\omega_c})} \quad (5)$$

For quality factor Q=1, Equation 5 represents the broad-band impedance. R_s is the strength of the effective shunt impedance. For the FMI we have taken design value R_s = 5 Ohm which is almost surely a considerable over estimate with enough safety margin.

The effect of transverse space charge force producing horizontal betatron tune shift is proportional to the particle density distribution in a bunch at a longitudinal position ϕ. Very close to the transition, η goes to zero. Therefore even a very small correction to γ_t becomes a sensitive parameter to determine the longitudinal beam dynamics. In the present calculations the dispersion of momentum compaction factor was taken into account by expanding,

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Table I
The parameters used for ESME simulations.

Parameter	Values
Mean radius of FMI	528.3019 m
γ_t (nominal)	21.838
$\dot{\gamma}$ at transition	167(Slow Ramp) sec^{-1} 300 (Fast Ramp) sec^{-1}
α_1	0.002091
Principal rf sys.	53 MHz 4 MV (max)
Init. emittance and Bunch intensity	0.1 and 0.2 eVs 6 $\times 10^{10}$
Coup. imp. $Z_{\|\|}$/n	5 Ω 2.17 GHz cutoff
Transverse Beam size(a)	0.0022 (m)
Beam pipe Radius (b)	0.03 (m)
FFTC : Shaping rf for FFTC Type of Tran. Crossing	 159 MHz 280 kV (max) Non-symmetric
γ_t - jump : $\Delta\gamma$ Type of Tran. Crossing	 1.0 Non-symmetric

$$\alpha_p \approx \alpha_o + (\alpha_o + 2\alpha_1 - \alpha_o^2)\frac{\Delta p}{p} \qquad (6)$$

For the Main Injector we take α_1 to be 0.002091. This corresponds to a Johnsen parameter[3] of 0.8. Thus, each particle has its characteristic γ_t depending on the deviation of its momentum from that of the synchronous particle. Table I lists the parameters used in the present simulation studies. The results of ESME simulations have been displayed in Table II. The FFTC and γ_t-jump scheme prefer symmetric settings for beam emittance larger than 0.2 eVs. For smaller emittance beam, where the space charge forces play important role in emittance blow up, the non-symmetric transition crossing is essential. Figure 1 shows a comparison of evolution of ϵ_l for NTPJ ,FFTC and γ_t-jump schemes in the Main injector for initial longitudinal emittance of 0.1 eVs. All these calculations have been performed by incorporating both space charge effects and the broad band Z/n. Since the $\Delta p/p$ increases as a bunch approaches transition energy, it is necessary to take into account the momentum acceptance of the FMI. From these simulations we find that the γ_t-jump scheme is preferable compared to FFTC. However, for emittance $\leq$ 0.1 eVs, and with the fast ramps the benefits are limited. With the FFTC scheme the emittance growth will be in between those for NTPJ and the γ_t-jump scheme. For emittance $\geq$ 0.2 eVs we find that the FFTC and γ_t-jump schemes give almost no emittance growth, while, with the NTPJ there is a maximum of about 10% emittance growth. Thus, with $\dot{\gamma}\ |_{transition}$ = 300 /sec and with $\epsilon_l \geq$ 0.2 eVs we may not need any of the schemes like FFTC or the γ_t-jump for transition crossing in the FMI.

Table II
The results of the longitudinal beam dynamics simulations for transition crossing using ESME. The fractional growth $\Delta\epsilon$ / ϵ for different schemes is listed.

$\dot{\gamma}\ \|_{trans.}$ (sec^{-1})	Init. Long. Emittance (eVs)	NTPJ	FFTC	γ_t-jump
167	0.1	3.0	0.6[a]	0.15
	0.2	0.09	0.04[a]	0.02
300	0.1	1.6	-	0.25
	0.2	0.06	-	0.02

[a] In these cases the ESME simulations have been carried out for $\dot{\gamma}\ |_{transition}$ = 169 /sec.

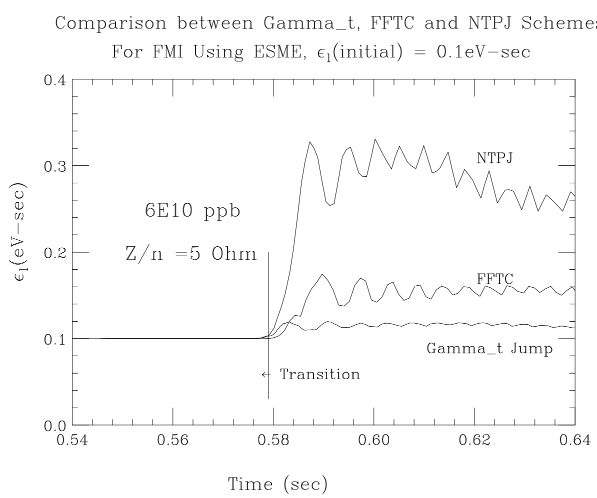

Figure 1. A comparison between γ_t-jump, FFTC and NTPJ schemes for the FMI. The initial emittance is 0.1 eVs, number of protons per bunch = 6$\times 10^{10}$. The $\dot{\gamma}\ |_{transition}$ = 167 /sec.

In a separate set of calculations we have estimated the negative mass instability using ESME. Our results confirm the calculations of Ng[6], who employed the analysis of Hardt[7]. We find for 6$\times 10^{10}$ protons/bunch a limit of $\epsilon_l\ \leq$ 0.16 eVs for $\dot{\gamma}\ |_{transition}$ = 167 /sec and $\epsilon_l\ \leq$ 0.12 eVs for $\dot{\gamma}\ |_{transition}$ = 300 /sec.

III. SUMMARY AND CONCLUSIONS

We have simulated the transition crossing for the proton beam with 6$\times 10^{10}$ particle /bunch. Three different schemes of transition crossing in the FMI have been investigated. We find that for an operating scenario of $\dot{\gamma}\ |_{transition}$=300 /sec and $\epsilon_l \geq 0.2$ eVs we do not need any special schemes like γ_t-jump or FFTC.

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