Transverse Multibunch Instabilities for Non-Rigid Bunches*

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Abstract

In this paper, we present a method for computing growth rates and frequency shifts of a beam containing multiple non-rigid bunches. With this approach, we calculate non-rigid multibunch effects which can impact phenomena which are traditionally treated as single-bunch effects, such as the transverse mode-coupling instability. This approach is important for high current storage rings such as PEP-II at SLAC (the B-Factory) which have very strong inter-bunch forces. Typical calculations treat multibunch and single bunch effects separately, and thus eliminate important interactions between the two. To illustrate the technique, we calculate growth rates and frequency shifts using PEP-II as an example.

I. Introduction

In [1], we describe a method for computing transverse multibunch instabilities. This method allows us to include the effects of internal degrees of freedom of the bunches, including coupling between the resulting modes. Previous work by other authors has either considered coupling between internal degrees of freedom for only a single bunch, or multiple bunches where the internal degrees of freedom are not coupled (see [1] for references and more discussion).

In this paper, we briefly describe the formalism, leaving the reader to [1] for more details. We then describe in detail the effects that are seen due to transverse multibunch mode coupling. This is done by plotting the mode frequencies and growth rates versus current for previously studied cases (transverse single bunch mode coupling and transverse multibunch modes without coupling), and comparing those plots to similar plots obtained by finding the transverse multibunch modes including coupling between internal degrees of freedom.

II. The eigenvalue equation

We can write a Vlasov equation describing the time evolution of the distribution for each bunch in terms of all the bunch distributions. The bunch distribution is assumed to be a time-independent stable distribution which satisfies the Vlasov equation for zero current, plus a small time-dependent perturbation.

That Vlasov equation can then be turned into a nonlinear eigenvalue equation for the coherent mode frequencies Ω. If we assume that 1) the bunches are identical and equally spaced, 2) the non-wakefield forces are all linear and independent of position in the ring, and 3) the bunch distribution we're perturbing about is Gaussian and only depends on the non-wake Hamiltonian, then

our eigenvalue equation becomes [1]

$$\phi_m(\Omega) = \sum_{n=0}^{\infty} K_{m+n}(\Omega + p\omega_0)F_n(\Omega)\phi_n(\Omega) \tag{1}$$

$$F_n(\Omega) = \frac{1}{2^n n!}\sum_{k=0}^{n}\binom{n}{k}\frac{[\omega_y + (n-2k)\omega_z]^2}{\Omega^2 - [\omega_y + (n-2k)\omega_z]^2} \tag{2}$$

$$K_k(\omega) = -i\frac{r_0 c^2 \beta_y N M}{\gamma_0 L^2 \omega_y}\sum_{\alpha}\left(\frac{\sigma_l}{\beta_0 c}\right)^k (\omega + M\alpha\omega_0)^k$$
$$Z_\perp(\omega + M\alpha\omega_0)e^{-\sigma_l^2(\omega + M\alpha\omega_0)^2/\beta_0^2 c^2}. \tag{3}$$

where ω_0 is the angular revolution frequency of the ring, ω_y is the betatron frequency, ω_z is the synchrotron frequency, r_0 is the classical radius of the electron, c is the speed of light, β_y is the average beta-function, N is the number of particles in a bunch, M is the number of bunches, γ_0 is the nominal beam energy divided by the rest mass energy of the particle, L is the length around the ring, σ_l is the bunch length, $\beta_0 c$ is the nominal particle velocity, and $Z_\perp$ is the transverse impedance. Feedback can be added by adding an additional term to K_k with $Z_\perp(p\omega_0 + \Omega)$ replaced by $Z_{\text{FB}}(p\omega_0 + \Omega)e^{-2\pi i p\Delta s/L}$, where Z_{FB} is the Fourier transform of the feedback response, and Δs is the distance between the pickup and kicker.

III. Illustrative Example

We will use the PEP-II B-Factory low energy ring [2] to illustrate the effects that arise from multibunch mode coupling. We have used an estimate for the broadband impedance using parameters from [3] and higher-order mode impedances for the cavities from [2], [4]. For this example, we truncate equation (1) at $m = 1$.

Fig. 1 shows mode coupling appearing when we consider only a single bunch. For small currents, the growth rates of the modes are negligible, and the mode frequencies change with increasing current. At the current where the mode frequencies coincide,
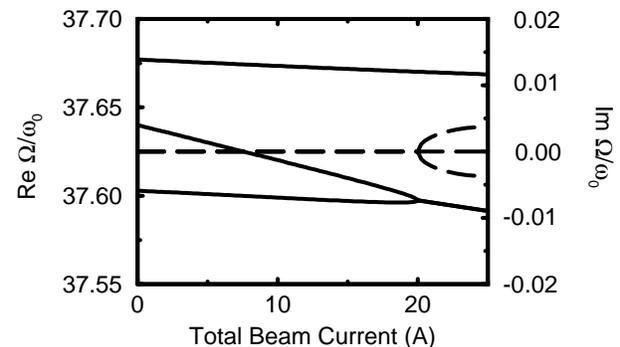
Figure. 1. Single bunch mode coupling. Solid lines are real part, dashed lines are imaginary part.
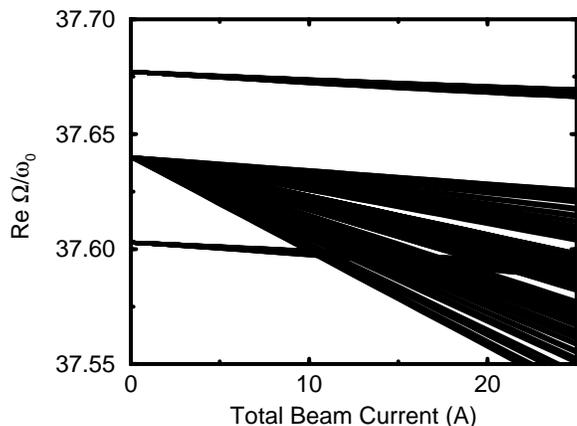
Figure. 2. Multibunch mode frequencies, no coupling.
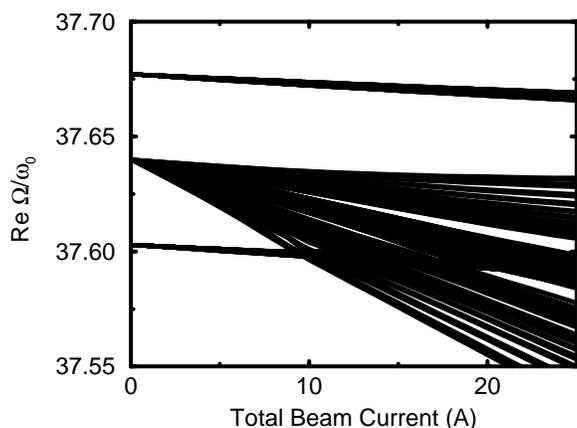
Figure. 3. Multibunch mode frequencies, with coupling.

Figure. 4. Multibunch $m = 0$ growth rates, no coupling.
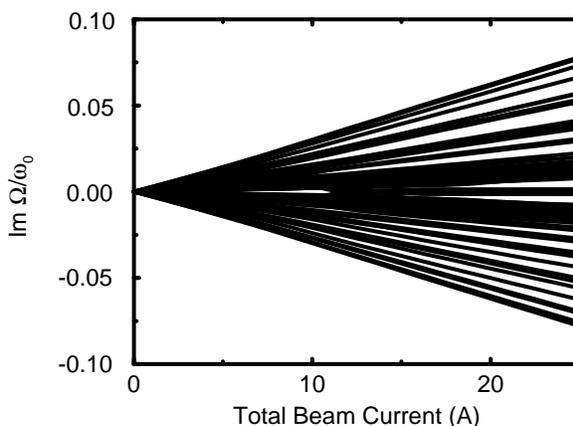
Figure. 5. Multibunch $m = 0$ growth rates, with coupling.
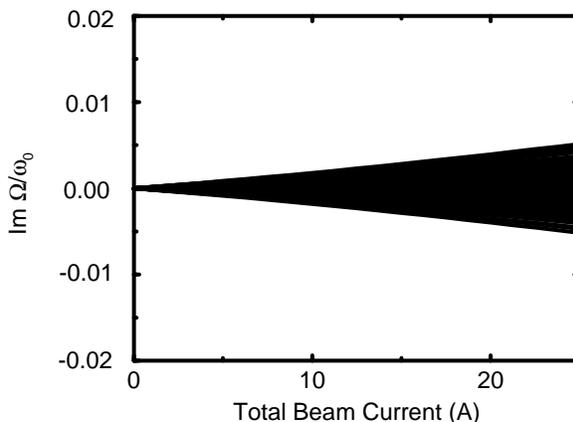
Figure. 6. Multibunch $m = 1$ growth rates, no coupling.

two of the modes resonantly drive one another, and exponential growth results.

Now consider multiple bunches. In all the following diagrams, we have only shown the modes with the largest growth rates. If we ignore coupling between the $m = 0$ and $m = 1$ multibunch modes, Fig. 2 demonstrates that for many of the multibunch modes, the frequencies of the $m = 0$ and one of the $m = 1$ modes coincide at currents much lower than where the frequencies coincide in the single bunch case of Fig. 1. When we allow the multibunch modes to couple, then Fig. 3 shows the real parts having nearly identical behavior to the uncoupled case. In the uncoupled case, the frequency shifts were nearly linear with current. Once coupling occurs, the curvature of the mode frequencies with current increases, and so we notice that in Fig. 3, the current where the mode frequencies intersect is even lower than what we see in the uncoupled case. Note that this current is still well above the intended operating current of the PEP-II low energy ring [2].

Now we examine the growth rates of the multibunch modes. First we look at the $m = 0$ modes. In the uncoupled case, Fig. 4 shows the growth rates increasing nearly linearly with current. If we allow the modes to couple, there are minimal changes to the growth rates of the $m = 0$ modes, as can be seen in Fig. 5. This is largely due to the fact that the largest growth rates are significantly larger than the growth rates due to mode coupling

(compare Fig. 1). In fact, if we were to look at modes with very small growth rates, we would see behavior almost identical to the single bunch case in Fig. 1.

Next, consider the $m = 1$ modes. In the uncoupled case, we see the growth rates increasing linearly with current in Fig. 6. Notice also that the growth rates are much smaller than they were in the $m = 0$ case. In fact, they are comparable to what one sees in single bunch mode coupling (Fig. 1). When we include coupling, we in fact see significant increases in the growth rate
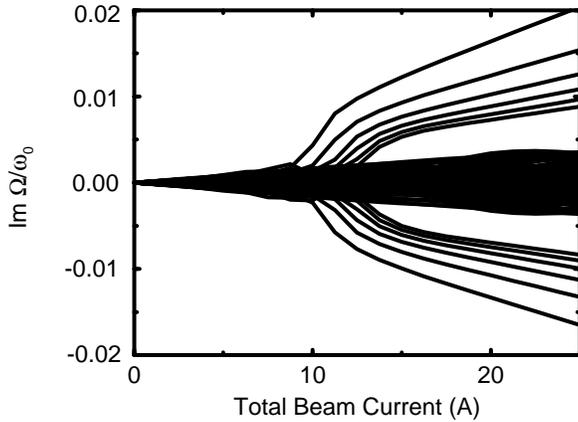
Figure. 7. Multibunch $m = 1$ growth rates, with coupling.
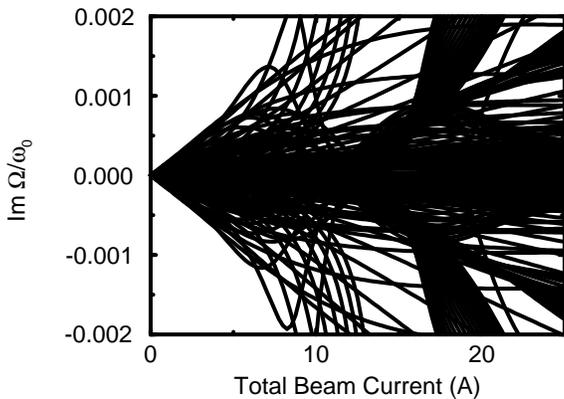
Figure. 8. Multibunch $m = 1$ growth rates, with coupling. Expanded vertical scale shows curvature of mode lines with current even at low currents.

Figure. 9. Multibunch growth rates, with feedback. Dashed lines are $m = 0$ modes, solid lines are $m = 1$.

[3] S. Heifets *et al.*, "Impedance study for the PEP-II B-factory," Tech. Rep. SLAC/AP-99, SLAC, March 1995.
[4] Y. Cai. Private communication.

of the $m = 1$ modes, as shown in Fig. 7. Notice that the growth rates increase sharply at just the point where the real part of the mode frequencies coincide (see Fig. 3). The multibunch mode coupling also causes the mode frequencies to no longer increase linearly with current, even for currents well below the current where the real part of the frequencies coincide. This is demonstrated in Fig. 8. This can cause $m = 1$ growth rates to be significantly increased over their uncoupled values, even at very small currents.

Finally, we can consider the effects of adding a feedback system. Typically, a transverse feedback system does not operate at frequencies sufficiently high to damp $m = 1$ modes. Thus, using parameters similar to those proposed for the PEP-II B-factory, Fig. 9 shows how the $m = 0$ modes are well damped, but the $m = 1$ modes still exhibit significant growth rates due to multibunch mode coupling.

References

[1] J. S. Berg and R. D. Ruth, "Transverse instabilities for multiple rigid bunches in a storage ring," Tech. Rep. SLAC-PUB-95-6829, SLAC, April 1995. Submitted to Phys. Rev. Lett.
[2] "PEP-II, an asymmetric B factory," Tech. Rep. SLAC-418, SLAC, June 1993.