Experimental Observations of Nonlinear Coupling of Longitudinal Modes in Unbunched Beams

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Abstract

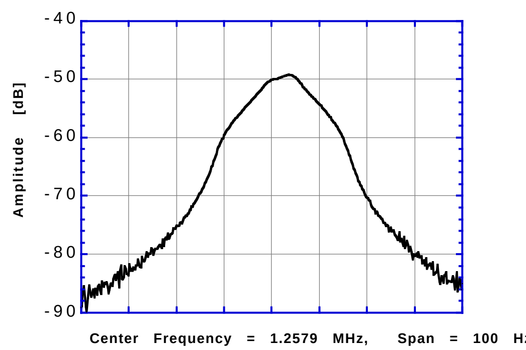
In an unbunched beam in a synchrotron, under conditions where the beam distribution is marginally stable, coherent fluctuations can cause a nonlinear coupling of longitudinal modes. Experimental observations of this parametric resonance have been reported previously.[1] Frequency domain measurements in the Fermilab Main Ring show pathological beam distributions associated with regions of marginal stability in longitudinal phase space; these are believed to be a requirement for the manifestation of the observed coupling. We have extended this investigation to the case of the beam response to an impulse excitation. The temporal development of the response indicates the formation of soliton-like structures within the beam distribution whose characteristics depend on beam conditions and the machine impedance.

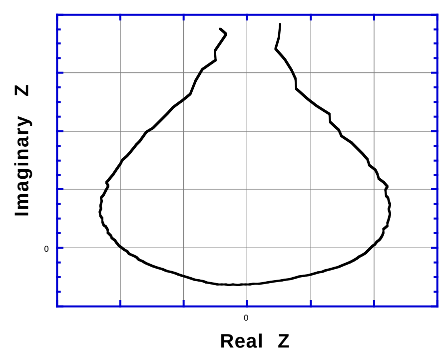
I. INTRODUCTION

A certain type of nonlinear resonance, known as a parametric resonance[2], is a process whereby energy present in oscillatory modes of a system can be transferred into another mode through frequency mixing, provided the difference frequency mode is also a normal mode of the system. In our case, these modes are the longitudinal modes of an unbunched coasting beam in a synchrotron, and the observation of this type of coupling in the Fermilab Tevatron has been documented.[1] The purpose here is to suggest conditions under which parametric coupling is likely to arise, and to present new time domain observations which provide further information concerning the phenomena. Besides clarifying the temporal sequence of energy transfer, these observations show an interesting non-linear self-bunching effect.[3] Aspects of this self-bunching are favorably compared to results from a particle tracking simulation which includes wakefields in the dynamics in order to provide a mechanism for the nonlinear resonance and self-bunching of the beam.

II. FREQUENCY DOMAIN MEASUREMENTS

Parametric coupling in the Tevatron and Main Ring was stimulated by applying an excitation to the beam at a single revolution harmonic. Once the initial oscillation was excited, it spontaneously transferred its energy into other revolution harmonics, and became damped as other modes grew. It was not possible to achieve parametric coupling in the Fermilab Accumulator using the same technique. The coupling coefficient for this specific kind of nonlinear coupling is dependent on the machine impedance at the frequencies of the excited modes, and is also extremely sensitive to the form of the beam distribution. Beam transfer function measurements were used to examine the beam distribution and to investigate the linear stability boundary. Results from a measurement in the Accumulator are shown in Figures 1a and 1b. These are, respectively, the magnitude response of the beam at h=2, and the corresponding stability boundary plotted in the complex impedance plane.

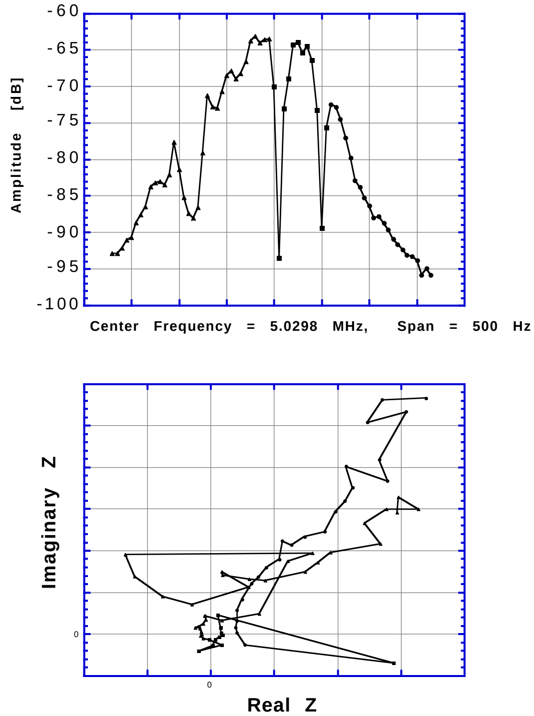




Figs. 1a and 1b Transfer function results from the Fermilab Accumulator, magnitude response and stability boundary

In contrast, the results of beam transfer function measurements in the Main Ring are shown in Figures 2a. and 2b. These are, respectively, the magnitude response of the beam at h=106, and an attempted construction of the linear stability boundary using the magnitude and phase information from the transfer function measurement.

* Operated by the Universities Research Association under contract with the U.S. Department of Energy.



Figs 2a and 2b Transfer function results from the Fermilab Main Ring, magnitude response and stability boundary

The magnitude response in Main Ring, unlike the Accumulator, shows deep notches in the beam distribution. These notches indicate that there are unstable regions in the phase space that would otherwise be occupied by the beam. Further investigation determined that the chromaticity and horizontal tune affected the position and depth of the notches, implying that certain momenta particles were unstable due to their mapping into tune space. The stability boundary in Figure 2b. displays large loops which correspond exactly to the notches in the beam distribution shown in Figure 2a.

III. TIME DOMAIN MEASUREMENTS

A series of measurements was performed the in the Main Ring where an external voltage was applied at a single frequency for .5 ms, approximating an impulse excitation. The drive frequency was at h=106, since this is the center frequency of the broadband cavity (Q~42) which was being used as the longitudinal kicker. The beam was not bunched, and the energy of the beam was constant at 8.9 Gev.

One result of these experiments was the demonstration that the parametric coupling was occurring in such a way that harmonics on the lower side of h=106 were excited sequentially. The initial impulse first caused the power in mode h=106 to grow, followed by modes h=105 to h=93 in sequential order. There was no significant excitation of harmonics at frequencies higher than h=106. This pattern of sequential excitation is shown in Figure 3, which is an overlay of six spectrum analyzer traces. Only the odd harmonics are shown for clarity.

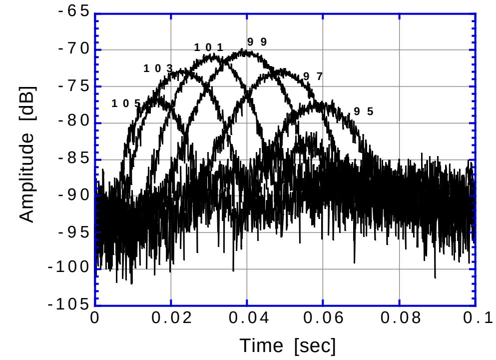


Fig 3 Power vs. Time for odd Main Ring harmonics 105-95. Growth of these harmonics occurs in sequential order.

For each trace the spectrum analyzer was set to monitor power at a particular harmonic as a function of time. This can be done by choosing the center frequency of the rotation harmonic as the center frequency of the sweep and setting the frequency span to zero. Since rotation harmonics in the Main Ring are 47 kHz apart, the resolution bandwidth was set to 30 kHz. The trigger occurred at the same time relative to the applied impulse, with the sweep time set to 100 ms. So, the relative timing shown by the overlaid traces is also the actual timing.

It is likely that the magnitude of the response at successive harmonics is in part due to the impedance of the 5 MHz cavity used to apply the initial kick. The resonant response of this cavity is shown in Figure 4. Nine harmonics away from the center frequency of the cavity the response is already 26 dB down.

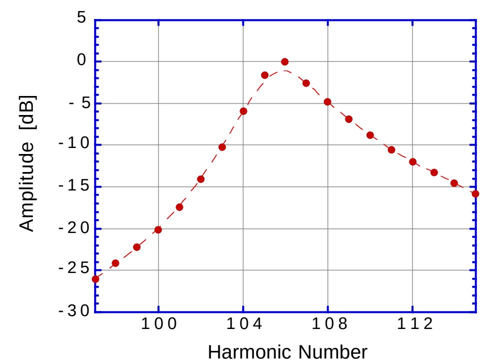


Fig. 4 Response of the Main Ring 5 MHz cavity

There are other interesting features of the beam response which are more evident upon examination of the power development at a single harmonic. The growth and decay of oscillation at h=105 is shown in Figure 5. The impulse excitation (at h=106) was applied at .0055 s with respect to the zero of the time axis, and lasted for .001 s. As energy is transferred from h=106, the coherent response at h=105 grows, but it does not continue to grow as there is a finite amount of energy available. The oscillation eventually decays due to power transfer to lower harmonics and dissipation. Note that there are several bounces in the signal level while it is losing power. This is a manifestation of a phase space rotation in which there is an exchange between the energy and phase coordinates of particles which have been bunched at h=105.

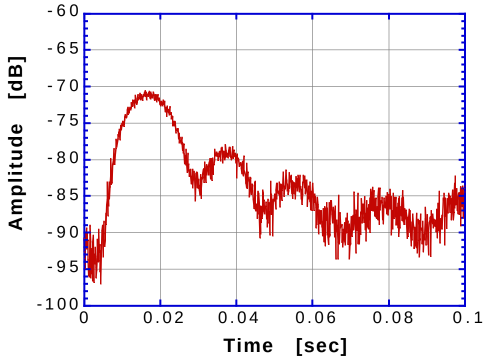


Fig. 5 Power vs. Time for mode h=105

The oscillation of the power level of a beam signal such as that shown in Figure 5 has been successfully reproduced with a particle tracking simulation. Besides generating output showing power level versus time for selected harmonics, the simulation also produces a phase space diagram which develops in time. An example of the phase space output is shown in Figure 6.

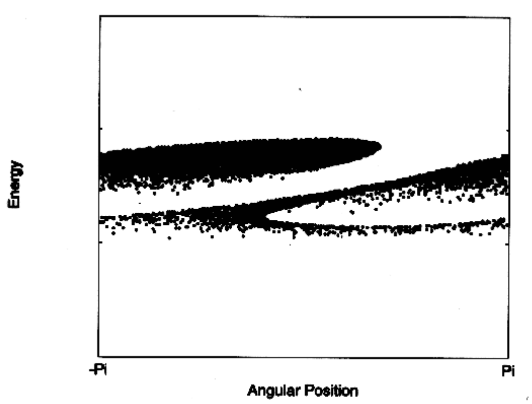


Fig. 6 Phase space output of particle tracking simulation

Inspection of Figure 6 reveals a tight clumping of particles on the higher energy side of the beam distribution. Guided by the simulation, a search for these high frequency bunchlets, and for this asymmetry in the phase space, was successfully undertaken. While much of the energy flow from the initial perturbation is governed by parametric coupling, if the initial pulse length is long enough, there is also significant higher harmonic generation. For example, with a 45 ms applied pulse, it was possible to see up to 40 higher harmonics of h=105; that is, h=2x105, h=3x105, up to h=40x105. Examination of these higher harmonics on either side of the center frequency yielded the projected asymmetry and characteristic signature of bunch rotation. These results are shown in Figure 7.

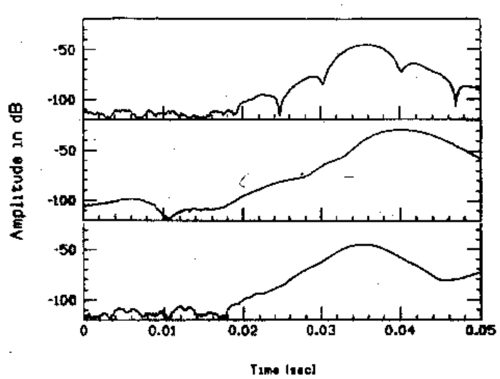


Fig. 7 From top to bottom respectively; the upper, middle, and lower portions of the frequency distribution of the beam at higher harmonic 5x105

The center trace of Figure 7 shows the time development of power at the center frequency of the 5x105 harmonic. For all three traces the spectrum analyzer had a narrow resolution bandwidth (300 Hz) for the purpose of monitoring only a fraction of the beam distribution. The upper and lower traces were taken in a manner similar to the center trace, except they were taken at the center frequency plus .5 kHz and at the center frequency minus .5 kHz, respectively. The power signal at the upper frequency has structure which indicates bunchlets of particles rotating in phase space. An exchange between energy and phase as a bunchlet rotates can cause this faster modulation of the current envelope.

IV. CONCLUSION

In machines with moderate impedances as well as regions of marginal stability in the phase space distribution of the beam, nonlinear effects may be observed. In particular, nonlinear frequency mixing otherwise known as parametric decay, higher harmonic generation, and self-bunching effects at high frequencies have been observed. Since these phenomenon are in part dependent on wakefields, perhaps these nonlinear effects may be used to quantify impedances.

V. ACKNOWLEDGEMENTS

The authors would like to thank Leo Michelotti, J. Holt, J. Beda, and A. Gerasimov for help with the simulations; and D. Wildman, G. Jackson and S. Assadi for their assistance with experimental techniques.

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