Microwave Instabilities in Electron Rings with Negative Momentum Compaction Factor

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Abstract

Bunch lengthening (or shortening) caused by the potential well distortion and the microwave instability in electron rings with negative momentum compaction factor is discussed in detail based on the resonator impedance model; further, a comparison with rings of positive momentum compaction factor is given. It was found that the bunch shape is less deformed and the current threshold of the microwave instability is higher in the rings with negative momentum compaction factor over a very wide range of the impedance parameters. The results also show that even within the range where the threshold for positive momentum compaction is higher than that for negative momentum compaction, the bunch lengthening is still less serious in most cases. Finally, an example of bunch lengthening in the case that $\alpha < 0$ with a real wake field of the SLC damping ring (old vacuum chamber) is given in contrast to the case in which $\alpha > 0$.

Introduction

In most of the existing low energy electron-positron colliders, a positive momentum compaction factor ($\alpha > 0$) is usually adopted. Bunch lengthening in such machines is one of the most important factors which limit the luminosity performance and its upgrade. Using negative momentum compaction is one of the hopeful approaches[1] to shorten the bunch length.

The method and computer program developed by K. Oide and K. Yokoya[2] were used to study the longitudinal bunch behaviour. The potential well distortion (PWD) and the threshold of microwave instabilities with a resonator impedance model for both the $\alpha > 0$ and $\alpha < 0$ cases are discussed and compared in detail in the first and second sections. Finally, as an example, bunch lengthening in a ring with a real wake field of the SLC damping ring (old vacuum chamber) for $\alpha > 0$ and $\alpha < 0$ is demonstrated. Generally speaking, bunch lengthening is less serious in rings with $\alpha < 0$.

I. Potential well distortion

When the beam intensity is low enough,the bunch behaviour is determined by its own potential well field. The bunch shape is deformed and its length changed. A self-consistent solution of the bunch shape is found by applying numerical solution of the Haissinski equation[2][3]. Using the well-known dimensionless Keil-Schnell criterion parameter, $I_{k,s} = \dfrac{I R_s}{\eta \sigma_{E0}^2 (\frac{E}{e})}$, where I is the current intensity, R_s the shunt impedance of a broad-band resonator, and $\frac{E}{e}$ the nominal beam energy in unit of Volts, we give the typical distribution of the particles in the bunch vs $I_{k,s}$ in the Figs. 1(a) and 1(b) for the resonator impedance model with $Q = 1$ and $K = 1$. Here Q is the quality factor, $K = \omega_r \frac{\sigma_{z0}}{c}$, ω_r is the resonator frequency, σ_{z0} is rms natural bunch length, c is the speed of light, respectively. One can see the cases $\alpha > 0$ and $\alpha < 0$ are quite different .

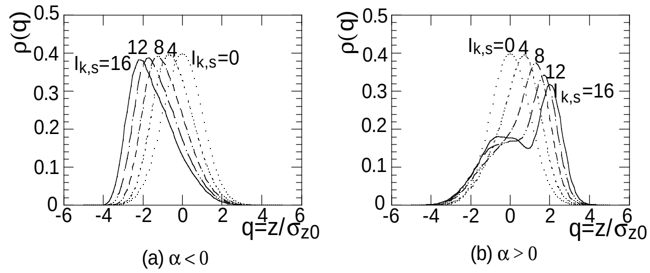

Figure. 1. Bunch shape (density distribution within the bunch) for various intensities.
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1. As usual, the bunch leans forward to compensate for the energy loss by the RF voltage in the case $\alpha > 0$, and backward at $\alpha < 0$.
2. The bunch shape is seriously distorted for $\alpha > 0$; for high intensity, sometimes two peaks appear. However, it is less distorted for $\alpha < 0$.

The reason is obvious: in the case $\alpha > 0$, most particles move to the front part of the bunch; then, the wake field produced by them disturbs the tail part of the beam, and a large deformation occurs at the tail part at high intensity. However, in the case $\alpha < 0$, most particles move to the back of the bunch; since the wake field generated by these particles is mostly located outside of the beam, only a portion of the beam is disturbed. A calculation shows that the two peaks never appear in the $\alpha < 0$ case, and that the beam always remains in good order.

Figs. 2(a) and 2(b) give the bunch length variation vs the current. Bunch lengthening is very serious in the case that $\alpha > 0$; at high intensity[1], the lengthening factor is between 1.45 to 1.65. However, it is between 1.05 to 1.25 for $\alpha < 0$. A similar phenomenon occurs for different Q. This can be explained as follows.

For a long bunch ($K > 1$), in the case $\alpha > 0$, the spectrum of the bunch is mainly in the inductive part of the impedance, which causes the bunch to lengthen and moves the spectrum even further to the inductive part of the impedance. It is like a "positive feedback", which causes the bunch length to increase exponentially with the intensity. However, in the case $\alpha < 0$,

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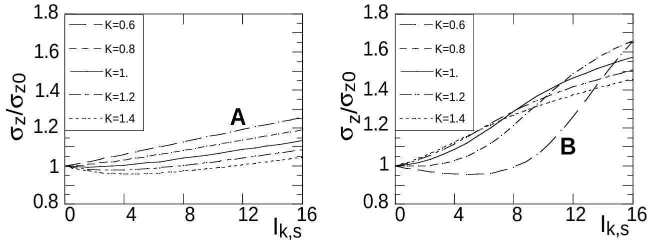
[1]For example, $I_{k,s} = 16$ corresponds to 40 mA for BEPC



(a) $\alpha < 0$ (b) $\alpha > 0$

Figure. 2. Bunch length vs current, $Q = 1$.

most of the spectrum of a long bunch is in the negative-inductive part of the impedance, which causes the bunch to shrink, thus forming a "negative feedback". As a result, the bunch length increase is nearly linear and very slightly.

For a short bunch ($K < 1$) , the situation is similar in the case $\alpha < 0$, in that most of the spectrum of the short bunch overlaps with the negative-capacitive part of the impedance, which causes bunch lengthening, The spectrum thus moves to the negative-inductive part of the impedance, which partly cancels the negative-capacitive part and a "negative feedback" mechanism is formed, so that the lengthening becomes weaker. This is why bunch lengthening of a short bunch is always mild, as shown in Fig. 2(a), curve "A". However, it is quite different in the case $\alpha > 0$; it seems there should be a "positive feedback" mechanism to push the bunch length so as to become shorter. However, it is not true when the bunch is shortened to some extent; in this case the wake field generated by the head part of the bunch is so strong that it causes a serious deformation of its tail part. The bunch shortening is then stopped and lengthening starts. Once the bunch becomes lengthened, "positive feedback" plays an important role, and the bunch length increases even more significantly (curve "B" in Fig. 2(b)).

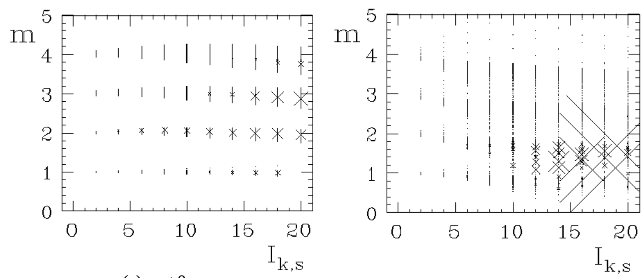
II. Microwave Instability

Above the current threshold of the microwave instability, the main results for the resonator impedance model with various parameters are described as follows:

1. In the case $\alpha < 0$, the mechanism of microwave instabilities is mainly due to radial mode coupling within a single azimuthal band; in the case $\alpha > 0$, however, besides the radial mode coupling belonging to same azimuthal band, instabilities are sometimes excited by the mixture of radial and azimuthal mode coupling . Qualitative analysis has been given in Ref. [4]. Here only the brief results are given in Figs. 3(a) and 3(b) for $Q = 1$ and $K = 1$.

One can see, the adjacent azimuthal modes remain separated until a very high current in the case $\alpha < 0$, whereas different modes merge even at a very low intensity in the case $\alpha > 0$. Usually the azimuthal mode coupling instabilities are stronger, the growth rate rapidly increases with the intensity. This is also shown in Figs. 3(a) and 3(b).

2. From a general point of view the threshold of microwave instabilities is higher over a rather wide range of K values for the case $\alpha < 0$. Even in the range where the threshold is lower than in the case $\alpha > 0$, in most cases bunch lengthening is still much



(a) $\alpha < 0$ (b) $\alpha < 0$

Figure. 3. Adjacent azimuthal modes merge with the intensity and the magnitude of the growth rate (propotional to the size of the cross).

less serious than in the case $\alpha > 0$, if the machine is operated at the same intensity.

Comparisons of the current thresholds between both $\alpha > 0$ and $\alpha < 0$ cases are given in Fig. 4. In this comparison, the growth rate 5×10^{-3} (in units of ω_{s0}) is chosen as the threshold for all instabilities modes, and radiation damping is omitted. Similar characters have been obtained for different Q, so we take

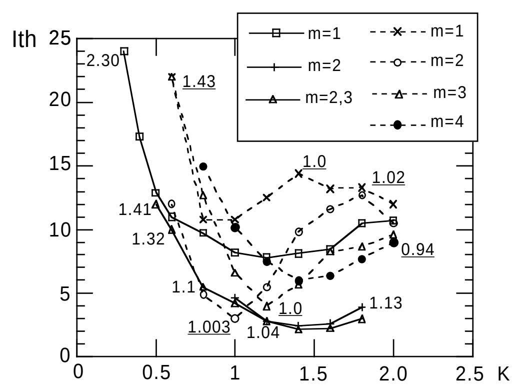


Figure. 4. Current threshold vs K for different $Q = 0.6$ (Solid lines for $\alpha > 0$ and dashed lines for $\alpha < 0$).

the more typical one, $Q = 0.6$, as an example for a detailed description. From this picture, one can find three different regions: $K < 0.75$; $0.75 < K < 1.1$; $K > 1.1$.

1). $K > 1.1$. In this region the threshold of $\alpha < 0$ is higher than the threshold of $\alpha > 0$, and the system appears to be inductive (or negative inductive) for $\alpha > 0$ (or $\alpha < 0$).

In the case where $\alpha < 0$, since all of the azimuthal modes are fully separated, a total of four pure radial mode instabilities (each occurs in its own azimuthal band), appear in this region. All of these instabilities are not strong. The lowest threshold among the four instabilities is $I_{k,s} = 4$ at $K = 1.2$.

In the case where $\alpha > 0$, two pure radial mode and one azimuthal mode instabilities appear in this region. The azimuthal mode instability is the strongest; it occurs in the merging region of mode 2 and mode 3 when the intensity is high. Nevertheless, its threshold is not the lowest one. The lowest threshold which becomes the boundary in this region is determined by one of the two pure radial modes which occur when azimuthal modes 1 and 2 are separated at low intensity. They finally both disappear at $K > 1.8$; mode 2 also disappears at $K < 1.0$. The corresponding threshold with different K is lower than that in the case $\alpha < 0$.

The bunch lengthening ($\frac{\sigma_z}{\sigma_{z0}}$) at the threshold is also shown in Fig. 4, the numbers attached to the marks with underline for $\alpha < 0$ and without that for $\alpha > 0$. Obviously, the bunch lengthening is much stronger in the case $\alpha > 0$.

2). $0.75 < K < 1.1$. The threshold of $\alpha < 0$ is lower than that of $\alpha > 0$; in this region the resistive character is dominant. In the case $\alpha < 0$, although the four pure radial mode instabilities still exist, the threshold of mode 2 decreases very rapidly with decreasing K, and arrives at its minimum at $K = 1$, giving the lowest threshold value. For $\alpha > 0$, only mode 1 still exists, and gives the boundary of the threshold.

The maximum difference in the threshold value between $\alpha < 0$ and $\alpha > 0$ is less than 50 % that at $K = 1$; the former is $I_{k,s} = 3$, and the latter is $I_{k,s} = 4.2$. If a machine with $\alpha > 0$ is operated at the same intensity as the threshold of $\alpha < 0$, i.e., $I_{k,s} = 3.0$, the bunch lengthening is 1.07, which is higher than 1.001 for $\alpha < 0$.

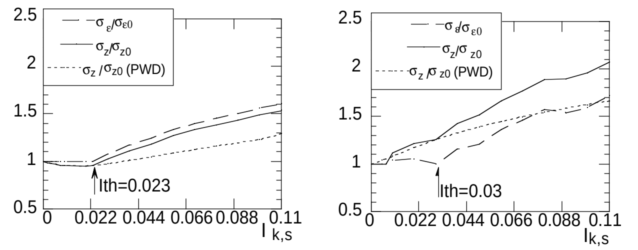
For increasing the threshold of $\alpha < 0$ in this region, the simplest way is to increase the energy spread of the beam; one can roughly estimate the bunch lengthening at the new higher threshold by the scaling. A calculation shows that it is effective in most cases; only about a 5$\sim$10% energy spread increment is needed. One can thus easily increase the threshold of the $\alpha < 0$ case to the same value as in the $\alpha > 0$ case. In the meantime, the bunch lengthening is still less than that in the case $\alpha > 0$.

3). $K < 0.75$. The threshold of $\alpha < 0$ is higher than that for $\alpha > 0$, and the capacitive character is dominant in this region. The lowest threshold boundary in the case that $\alpha < 0$ is determined by the pure radial mode 2 instability; in the case that $\alpha > 0$ it is determined mainly by the azimuthal mode instability, which is very strong, and the bunch shape is seriously deformed and two peaks appear.

It is surprising that the threshold is unbelievably high in this region. This may not be true for a realistic machine; first, a smaller K means a weaker wakefield, which may not be reasonable in practical cases. Second, because the real impedance cannot be correctly described sufficiently by the simple resonator impedance model, a small K means that the system appears to be more capacitive for a short natural bunch length; it may also not be true. It was pointed out in Ref. [5] that the wakefield of a realistic machine is always determined by many small discontinuities, and most of them are inductive. It is quite different from the resonator impedance. Thus, in practice, most realistic machine parameters are located at the right-hand side of the picture i.e., $K > 0.9$.

A real example is given here by using the wake field of the old vacuum chamber of the SLC damping ring[5]. Figs. 5(a) and 5(b) give the microwave thresholds for both cases; the result coincides with the analysis mentioned above.

Another very interesting phenomenon is that even above the threshold the bunch length behavior is very similar to that described by a pure potential well distortion; about half the bunch lengthening is contributed by PWD in most cases.



(a) $\alpha < 0$ (b) $\alpha > 0$

Figure. 5. Threshold and bunch length behaviour for $\alpha < 0$ and $\alpha > 0$ with the wake field of the old vacuum chamber of SLC damping ring.

Conclusion

The longitudinal bunch shape is less deformed and bunch lengthening is less serious in electron rings with negative momentum compaction factor. A natural "negative feedback" mechanism can explain the above mentioned phenomenon. Even above the threshold the intrinsic shape of the potential well field plays an important role.

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