ELECTRON COOLER IMPEDANCES

A. Burov, Budker INP, 630090, Novosibirsk, Russia

Abstract

An electron beam of a cooler in an ion storage ring can be considered as a medium which responds to fields generated by an ion beam. Electron density perturbations awaken by the ion beam act back on it, which can give rise to instabilities. This reaction can be described in terms of an impedance introduced in the ring by the electron beam. Longitudinal and transverse impedances of an electron cooler are derived here, increments and thresholds of corresponding instabilities are estimated.

I. Introduction

The interaction of an ion beam with different elements of the vacuum chamber can give rise to coherent instabilities of the beam. If the beam temperature is sufficiently high, its collective modes are stabilized due to the Landau damping. Under cooling, the temperature is going down and the Landau damping is switching off; but the cooling brings about its own decrement in the collective motion of the beam. However, if the decrement of some mode is smaller than its increment caused by the interaction with the environment, the beam is either stopped at the instability threshold or lost. So, these increments determine a minimum cooling rate needed to achieve a beam temperature below the Landau damping threshold.

In a relativistic case, a significant contribution in the beam - surrounding interaction is given by a broad-band wall impedance. For a moderate relativism $\gamma - 1 \leq 1$, this impedance is shown to be exponentially damped , $\propto \exp(-4.8/(\beta\gamma))$ [3]. The influence of the wall resistivity is too small to play any role in the cooling process. Therefore, the electron cooler itself can be the main reason of instabilities; this problem was discussed in Ref.[4], [5], [6], [7].

The longitudinal and transverse impedances of the cooler are calculated here and shown to be normally orders of magnitude higher then the resistive wall ones. In the result, increment rates of the corresponding instabilities can be higher than typical cooling rates [8], [9], [10], even for rather low ion currents (e. g. $\sim$ few μA).

II. Main Equations

The dynamics of the magnetized electron medium of a cooler excited by fluctuations of the cooled ion beam can be described in a reference frame by the following set of equations [11]:

$$\begin{aligned} \frac{\partial \tilde{n_e}}{\partial t} + \bar{n_e}\frac{\partial \tilde{v_e}}{\partial z} &= 0 \\ \frac{\partial \tilde{v_e}}{\partial t} - \frac{e}{m}\frac{\partial \Phi}{\partial z} &= 0 \\ \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{\partial^2 \Phi}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \phi^2} &= 4\pi e(\tilde{n_e} - \tilde{n_i}) \end{aligned} \quad (1)$$

Here $\bar{n_e}$ is an electron density, $\tilde{n_e}, \tilde{n_i}$ are electron and ion density perturbations, $\tilde{v_e}$ is a perturbation of electron velocity, Φ is an electrostatic potential. In the impedance calculations ions are assumed to be protons, the impedances are linear responce functions and do not depend on the charge of exciting particles. Unperturbed velocities of the beams are supposed to be equal, the solenoidal magnetic field assumed to be directed along the longitudinal axis z everywhere. The former assumption is justified if the velocities coincide with a sufficiently good accuracy, the criterium is discussed in the next chapter. The later assumption is only warranted for long-wave perturbations, $ka \ll 1$, where k is a longitudinal wavenumber, a is a radius of the electron beam. It is demonstrated in the following, that cooler impedances reach their maxima somewhere in the intermediate region, $ka \simeq 1$. In this wave band the results obtained from Eqs.(1) can serve only as estimations with accuracy about $\simeq 100\%$. More accurate results in the intermediate band could be found with the electron beam curvature in the ions entrance to be taken into account, this rather complicated problem is not considered here.

The solution of the problem (Eqs.1) with proper boundary conditions can be presented in the following form:

$$\begin{aligned} \Phi &= \sum_{\mu\pm} A_{l\mu\pm} J_l(\kappa_{l\mu} r/a)\cos(l\phi)\exp(iq_{l\mu\pm}z \mp i\omega_{l\mu}t) \\ \tilde{v_e} &= -\frac{e}{m}\sum_{\mu\pm} \pm\frac{A_{l\mu\pm}}{u_{l\mu}} J_l(\kappa_{l\mu} r/a)\cos(l\phi)\exp(iq_{l\mu\pm}z \mp i\omega_{l\mu}t) \\ \tilde{n_e} &= \tilde{n}_{il}\cos(l\phi)\exp(ikz) - \\ &\quad -\frac{\omega_e^2}{4\pi}\sum_{\mu\pm} \frac{A_{l\mu\pm}}{u_{l\mu}^2} J_l(\kappa_{l\mu} r/a)\cos(l\phi)\exp(iq_{l\mu\pm}z \mp i\omega_{l\mu}t) \end{aligned} \quad (2)$$

Here the subscript $\mu = 1, 2, ...$ is a counter of radial wavenumbers, $\pm$ corresponds to waves travelling along and against the beams, $\tilde{n}_{il}$ is an amplitude of the ion perturbation, $\kappa_{l\mu}$ are zeroes of a Bessel function J_{l-1}, with $\kappa_{01} \approx \sqrt{2/\ln(1/(ka))}$, if an aperture radius $b \gg 1/k$. In the opposite case of adjoining aperture, $b - a \ll a$, the radial numbers $\kappa_{l\mu}$ are zeroes of J_l.

The connection between electron and ion longitudinal wavenumbers is determined by the Doppler condition, which gives:

$$q_{l\mu\pm} = k/(1 \pm \alpha_{l\mu}), \quad \alpha_{l\mu} = \omega_e a/(\kappa_{l\mu} v), \quad (3)$$

where v is the beams velocity in the laboratory frame. The eigenfrequencies are described by sound-like dispersion equations:

$$\omega_{l\mu\pm} = \pm q u_{l\mu}, \quad u_{l\mu} = \alpha_{l\mu} v = \omega_e a/\kappa_{l\mu}. \quad (4)$$

It follows from a zero boundary condition on $\tilde{v_e}$ at the ion entrance, that $A_{l\mu+} = A_{l\mu-} = A_{l\mu}$. The amplitudes $A_{l\mu}$ have to be found from the zero boundary condition imposed on the density perturbation $\tilde{n_e}$.

III. Longitudinal Impedance

In this section longitudinal perturbations are considered, $l = 0$. Making use of the orthogonality properties of the Bessel functions, the amplitudes of excited modes can be found:

$$A_\mu = 2\tilde{\rho_{i0}}/(\kappa_\mu^2 F_\mu^2), \quad (5)$$

$\tilde{\rho_{i0}}$ is a linear density perturbation of the ion beam. An average of the electric field over a time τ of the flight through the cooler

$$\langle E_z \rangle = -\frac{1}{\tau}\int_0^\tau \left.\frac{\partial \Phi}{\partial z}\right|_{z=0} dt \quad (6)$$

gives cooler's longitudinal impedance $Z^{\|}$ [1]:

$$\langle E_z \rangle \tau = -e\tilde{\rho_{i0}} Z^{\|} \quad (7)$$

Real parts of impedances responsible for instabilities are of the main interest:

$$\mathrm{Re} Z^{\|} = 2\sum_\mu \frac{(1-\cos(\omega_{\mu+}\tau)) - (1-\cos(\omega_{\mu-}\tau))}{\kappa_\mu^2 F_\mu^2 u_\mu}. \quad (8)$$

The contributions in the real part of the impedance (8) from the positive $(+)$ and negative $(-)$ waves can be seen to have opposite signs: an emission of the positive wave takes away a longitudinal momentum of ions, i. e. decelerates them; on the contrary, radiation of the negative wave with a negative momentum accelerates the ion beam. The phase advances of the positive and negative waves $\omega_{\mu\pm}\tau$ normally can be rather close: $(\omega_{\mu-} - \omega_{\mu+})\tau/2 \ll 1$. In this case the $(+)$ and $(-)$ contributions in the impedance almost cancel each other, the series over radial modes (8) converges as $1/\mu^2$, which gives:

$$\mathrm{Re} Z^{\|} = -4\omega\tau\frac{\sin(\omega\tau)}{\kappa^2 F^2 v} = -\frac{Z_0 \omega\tau}{\pi\beta\kappa^2 F^2}\sin(\omega\tau), \quad (9)$$

where the counter $\mu = 1$ is omitted, $Z_0 = 4\pi/c = 377\Omega$, $\beta = v/c$ and $\omega = \omega_e ka/\kappa$ is the frequency of the first radial mode. The impedance (Eq.9) linearly increases with the longitudinal wave number k up to $k \simeq 1/a$.

In the short wave band ($ka > 1$), the part of the electron beam with the transverse size $\simeq 1/k$ only effectively interacts with ions. It follows, that the impedance achieves the maximum at $ka \simeq 1$, the later can be estimated from (Eq.9) with $\kappa = 2.4,\ F = 0.5,$:

$$|\mathrm{Re} Z^{\|}|_{\max} \simeq 0.1 Z_0 \frac{\omega_e \tau}{\beta}, \quad (10)$$

where $\omega\tau \gg 1$ was assumed.

Substituting, for example, $n_e = 3\cdot 10^7$ cm^{-3}, $g = 1$m, $\beta = v/c = 0.06$ (Li_7^{+1} in the CRYSTAL ring [13]), it gives the cooler impedance

$$\mathrm{Re} Z^{\|} = 10\mathrm{K}\Omega,$$

which is three orders of magnitude more than typical values of the resistive-wall impedance.

Velocities of the electron and ion beams are assumed to be zero in this calculations. It is warranted, if an additional difference between phase advances of the $(+)$ and $(-)$ modes introduced by a discrepancy of the velocities δv is smaller then this difference for equal velocities: $k\delta v < \alpha^2 kv$, or $\delta v/v < \alpha^2$.

The real part of the impedance is responsible for a coherent instability with an increment [12]:

$$\begin{aligned} \Lambda_k &= \left|ku_i \mathrm{Re} Z^{\|}/(2Z_{sc})\right|, \quad Z_{sc} = iZ_0 L_i kR/\beta, \\ L_i &= \ln(1/(ka_i)), \quad u_i = c\sqrt{2\bar{\rho}_i r_0\left(\gamma^{-2} - \gamma_t^{-2}\right)}, \end{aligned} \quad (11)$$

where r_0 is a classical radius of the ion, R is a radius of the storage ring, a_i is a radius of the ion beam, $\bar{\rho}_i$ is its linear density. The increment rate for the coasting Li beam with $2\cdot 10^7$ particles, $L_i = 4$, $\mathrm{Re} Z^{\|} = 10\,\mathrm{K}\Omega$, is found to be: $\Lambda = 5\mathrm{s}^{-1}$.

To damp the instability and to continue a cooling process, the cooling rate $\lambda_{\|}$ needs to be twice more than the increment [4], [7], $\lambda_{\|} > 10\mathrm{s}^{-1}$ in the last example.

IV. Transverse Impedance

An excitation of transverse dipole modes ($l = 1$) of the electron beam and their back action on ions can be described in terms of a transverse impedance $Z^\perp$ [2]:

$$\langle E_x \rangle \tau = -\frac{1}{\tau}\int_0^\tau \left.\frac{\partial \Phi}{\partial x}\right|_{z=0} dt = ie\bar{\rho}_i x_i Z^\perp, \quad (12)$$

where $\bar{\rho}_i$ is a linear density of the ion beam, x_i is an amplitude of its deviation along the $x-$ direction. The potential Φ is given by Eq.(2) with $l = 1$. The wave amplitudes A_μ are found from the zero boundary conditions on $\tilde{v_e}, \tilde{n_e}$ at the entrance, which give: $A_\mu = -2\bar{\rho}_i x_i/(\kappa_\mu a F_\mu^2)$ Asymptotically, at $\kappa_\mu \gg 1$, $F_\mu = (\pi\kappa_\mu/2)^{-1/2}$ and A_μ tends to be a constant: $A_\mu \simeq A = -\pi\bar{\rho}_i x_i/a$. From here, the real part of the transverse impedance is found:

$$\mathrm{Re} Z^\perp = -\frac{Z_0}{4\beta a}\sum_\mu \frac{\kappa_\mu v}{a}\int_0^\tau dt\,\{\sin(\omega_{\mu-}t) - \sin(\omega_{\mu+}t)\}. \quad (13)$$

This series is diverging. It means a large number of the contributing modes and approves an application of the asymptotic $\mu \gg 1$.

An upper limit of the summation $\mu_{max} = m$ is determined by an account of a finite transverse size of the ion beam a_i, it gives $m \simeq a/a_i$. If the difference of $\pm$ phase advances at $\mu \simeq m$ is small, $(\psi_- - \psi_+)/2 = (\omega_-\tau - \omega_+\tau)/2 < 1$, the expression (13) can be simplified:

$$\mathrm{Re} Z^\perp = -\frac{Z_0 \omega_e \tau}{4\beta a} S, \quad S = \sum_{\mu=1}^m \begin{cases} \sin(\omega_\mu \tau) & \text{if } \omega_\mu\tau \gg 1 \\ \omega_\mu\tau/2 & \text{if } \omega_\mu\tau \le 1 \end{cases} \quad (14)$$

The result of the summation depends on the phase advance ψ and its difference between neighbor modes $\Delta\psi$ at the upper limit of the summation $m \simeq a/a_i$:

$$|S| \simeq \begin{cases} \sqrt{\dfrac{m}{2}}, & \text{if } m < m_r \\ \sqrt{\dfrac{m_r}{2}} + \dfrac{m^2}{m_r^2}, & \text{if } m_r < m < m_r^2 \\ \sqrt{\dfrac{m_r}{2}} + m_r^2 + \dfrac{m_r^2}{2} \ln\left(\dfrac{m}{m_r^2}\right) & \text{if } m > m_r^2 > 1 \\ \dfrac{m_r^2}{2} \ln m & \text{if } m_r^2 < 1 \end{cases}$$

$$m \simeq \quad a/a_i, \quad m_r \simeq \sqrt{\omega_e \tau k a/\pi}. \tag{15}$$

Turning back to Eq.(14) and assuming, for instance, the electron density $\bar{n_e} = 3 \cdot 10^7$ cm^{-3}, the cooler length $g = 1$ m, the velocity $v = 2 \cdot 10^9$ cm/s, the electron beam radius $a = 1$cm, taking the sum factor $S = 10,$ the transverse impedance is found:

$$|\mathrm{Re}Z^{\perp}| \simeq 30 \text{ M}\Omega/m. \tag{16}$$

Generally, a real part of an impedance is responsible for an instability, an increment for the coasting beam is [2]:

$$\Lambda^{\perp} = \frac{\bar{\rho}_i r_0 c}{\gamma Q_b} \frac{|\mathrm{Re}Z^{\perp}|}{Z_0}, \tag{17}$$

where Q_b is a betatron tune, r_0 is the classical radius of an ion of the beam.

Taking as an example the mentioned Li_7^{+1} beam, $Q_b \simeq 2,$ assuming the transverse impedance of $\mathrm{Re}Z^{\perp} = 30$ MΩ/m, the increment is calculated: $\Lambda^{\perp} = 3$ s^{-1}.

To suppress the instability at low temperatures, where the Landau damping does not exist, a transverse cooling rate of the electron cooler $\lambda_c^{\perp}$ has to be twice higher than the increment [4]: $\lambda_c^{\perp}/2 > \Lambda^{\perp}$, for an example above $\lambda_c^{\perp} > 6$ s^{-1} has to be achieved.

V. Conclusions

Longitudinal and transverse impedances introduced in a storage ring by the electron beam of an electron cooler have been found here; both of them occurs to be orders of magnitude higher than resistive-wall ones. The corresponding values of coherent increments can be comparable with the cooling decrements even for such small ion currents, which are usually suggested for crystallization.

VI. Acknowledgments

The report has been done in the framework of the CRYSTAL project. The author is thankful to N. S. Dikansky, V. V. Parkhomchuk, L. Tecchio, G. Bisoffi, M. Grieser, G. Lamanna and S. Gustafsson for discussions and information.

References

[1] A. Sessler and V. Vaccaro, CERN Report ISR-RF/67-2 (1967)

[2] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, J. Wiley & Sons, Inc., New York, 1993.

[3] A. V. Burov, *Wall Impedance for Low and Moderate Beam Energies*, CSR Note, Dec. 1994.

[4] N. S. Dikansky, D. V. Pestrikov, *The Physics of Intense Beams and Storage Rings*, AIP Press, New York, 1994.

[5] V. V. Parkhomchuk, private communication, 1992.

[6] V. V. Parkhomchuk, D. V. Pestrikov, in *Proc. of the Workshop on Beam Cooling and Related Topics*, p. 327, Montreux, 1993.

[7] A. V. Burov, in *Proc. of the Workshop on Beam Cooling and Related Topics*, p. 230, Montreux, 1993.

[8] G. I. Budker et al. In Proc. of 10 Int. Conf. on High Energy Accel., v. 1, p. 498, Serpukhov, 1977.

[9] M. Grieser, private communication.

[10] D. Reistad, private communication.

[11] L. S. Bogdankevich, A. A. Rukhadze, Sov. Uspekhi Phys. Nauk, **103**, 4, p. 609 (1971).

[12] P. L. Morton, V. K. Neil and A. M. Sessler, J. Appl. Phys. **37**, 3875 (1966).

[13] L. Tecchio, G. Bisoffi et al., *CRYSTAL. A Storage Ring for Crystalline Beams. Feasibility Study.* LNL-INFN Int. Rep., 1994.