A WEAK MICROWAVE INSTABILITY WITH POTENTIAL WELL DISTORTION AND RADIAL MODE COUPLING

Alex Chao, Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309 USA
Bo Chen, Superconducting Super Collider Laboratory, Dallas, TX 75237 USA
Katsunobu Oide, KEK, National Laboratory for High Energy Physics, Tsukuba, Japan

I. INTRODUCTION

In attempts to minimize the impedance of an accelerator by smoothing out its vacuum chamber, improvements are typically first made by reducing the inductive part of the impedance. As the inductance is reduced, however, the impedance becomes increasingly relatively resistive, and as a consequence, the nature of potential well distortion changes qualitatively. An inductive impedance lengthens the bunch (above transition) while maintaining more or less a head-tail symmetry of the bunch longitudinal distribution. A resistive impedance does not change the bunch length as much, but tends to cause a large head-tail asymmetry.

The details of how potential well is distorted, particularly the head-tail asymmetry, affects the mechanism of the longitudinal microwave instability. Without a head-tail asymmetry, the microwave instability mechanism relies on the coupling among the "azimuthal" modes. The coupling is strong but the mode frequencies have to shift by large amounts (comparable to the synchrotron frequency ω_s) before the instability threshold is reached. With a head-tail asymmetry, the instability can be triggered by coupling of the "radial" modes. The coupling is weak, but the mode frequency shifts involved are small ($\ll \omega_s$). One then may have the following situation: as one tries to minimize the impedance, the impedance becomes resistive; the longitudinal bunch shape acquires a large head-tail asymmetry; the nature of microwave instability changes from a strong one (that involves azimuthal mode coupling) to a weak one (that involves head-tail asymmetry and radial mode coupling), but the threshold of the instability is not raised or is even lowered.[1,2] The gain of reducing the impedance is reflected only in the fact that the instability growth rate above threshold is slower.

The instability effect due to potential-well distortion and radial mode coupling has been analyzed before.[3-7] Our analysis is based on a technique[7,8] developed for the treatment of the longitudinal head-tail instability effect. To treat the coupling among radial modes, we introduce a "double water-bag" model for the simplicity of analysis.

The analysis is applied to the SLC Damping Ring. The wake function, as shown in Fig.1, is the present model used [9] taking into account the recent changes made on the vacuum chamber.[10] The calculated bunch shape distortion (particularly the head-tail asymmetry), as well as the calculated instability threshold, seem to agree with the observations.[10,11]

We explored two ways which might in principle alleviate this instability mechanism. (i) add a higher harmonic cavity: A higher harmonic rf voltage with appropriate phase and amplitude may compensate for the head-tail asymmetry and thus raise the instability threshold. (ii) operate the accelerator with a negative
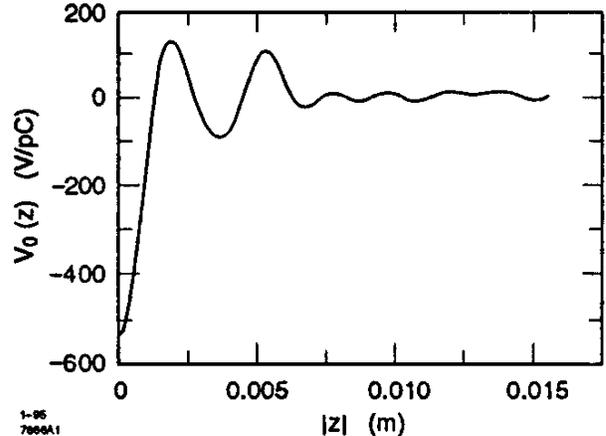
Figure 1. Wake function (in volts/pC) versus $|z|$ (in meters) used in the analysis for the SLC Damping Ring.

momentum compaction factor η:[12] With $\eta > 0$, the distorted beam distribution leans toward the head of the bunch; the bunch tail sees large wake fields. Operating with $\eta < 0$ could conceivably help because the beam distribution now leans toward the tail of the bunch. Both ways (i) and (ii) were explored in this paper. We found that a higher harmonic cavity of a modest voltage can indeed eliminate this instability, while the advantage of operating with $\eta < 0$ is less obvious.

II. SUMMARY OF ANALYSIS

Details of the analysis has been given in [2]. A brief summary is given below. We need to first compute the potential well distortion effects. Let $\psi_0(z, \delta)$ be the potential-well distorted beam distribution in the longitudinal phase space (z, δ). The corresponding wake potential is

$$V_0(z) = e \int_{-\infty}^{\infty} dz' W_0'(z - z') \int_{-\infty}^{\infty} d\delta \, \psi_0(z', \delta) \qquad (1)$$

We have assumed that the wake function $W_0'(z)$ is short, i.e. we consider single-bunch, single-pass instabilities. Later when we add a higher harmonic rf voltage to counteract the potential-well distortion, we will add it to V_0. The Hamiltonian for the potential-well distorted beam is

$$H(z, \delta) = \frac{\eta}{2}\delta^2 + \frac{\omega_s^2}{2\eta c^2}z^2 - \frac{e}{T_0 E c}\int_0^z dz' V_0(z') \qquad (2)$$

where ω_s is the unperturbed synchrotron frequency, T_0 is the revolution period, E is the beam energy, and c is the speed of light.

We now apply the technique developed in [7,8] and change variables from (z, δ) to (Φ, H) by a canonical transformation,

where H is given by Eq.(2), and Φ is the canonical variable conjugate to H:

$$\Phi = \frac{\partial F(\delta, H)}{\partial H} = -\int_0^\delta d\delta' \frac{\partial z(\delta', H)}{\partial H} \tag{3}$$

where $z(\delta, H)$ is obtained by inverting Eq.(2). The motion of a particle is periodic in Φ with period

$$\Phi_0(H) = \oint d\delta' \frac{\partial z(\delta', H)}{\partial H} \tag{4}$$

Note this period depends on the value of H of the particle under consideration.

In the double water bag model, ψ_0 has the form

$$\psi_0(H) = 2\mathcal{N}[(1 - \Gamma)\Theta(\hat{H}_1 - H) + \Gamma\Theta(\hat{H}_2 - H)] \tag{5}$$

where $\Theta(x)$ is the step function, Γ is a parameter between 0 and 1 that specifies the relative amount of particles in each of the waterbags, and

$$\mathcal{N} = \frac{N/2}{(1 - \Gamma)\int_0^{\hat{H}_1} dH \, \Phi_0(H) + \Gamma \int_0^{\hat{H}_2} dH \, \Phi_0(H)} \tag{6}$$

with N the number of particles in the beam bunch. We choose $\Gamma = 0.45$, and $\hat{H}_1$ and $\hat{H}_2$ to correspond to one- and two-sigma particles, such that the weak-beam limit of ψ_0 approximates a gaussian distribution.

Consider the ℓ-th azimuthal mode ($\ell = 1, 2, 3$ means dipole, quadrupole, sextupole modes) in the longitudinal phase space. There are two radial modes allowed in the double water bag model, one at $H = \hat{H}_1$, another at $H = \hat{H}_2$. The two radial mode frequencies are determined by the solutions of

$$\det \begin{bmatrix} \Omega^{(\ell)} - \frac{2\pi\ell c}{\Phi_0(\hat{H}_1)} + M_{11} & M_{12} \\ M_{21} & \Omega^{(\ell)} - \frac{2\pi\ell c}{\Phi_0(\hat{H}_2)} + M_{22} \end{bmatrix} = 0 \tag{7}$$

where we have defined the matrix elements

$$M_{ij} = -\frac{4r_0\eta\mathcal{N}}{T_0\gamma\Phi_0(\hat{H}_i)} \int_0^{\frac{\Phi_0(\hat{H}_i)}{2}} d\Phi \delta(\Phi, \hat{H}_i) \sin\left[2\pi\ell\frac{\Phi}{\Phi_0(\hat{H}_i)}\right]$$

$$\int_0^{\frac{\Phi_0(\hat{H}_j)}{2}} d\Phi' W_0'(z(\Phi, \hat{H}_i) - z(\Phi', \hat{H}_j)) \cos\left[2\pi\ell\frac{\Phi'}{\Phi_0(\hat{H}_j)}\right] \tag{8}$$

It can be shown that all elements M_{ij} are real. The beam is stable if both solutions for $\Omega^{(\ell)}$ are real. The instability growth rate is given by the imaginary part of $\Omega^{(\ell)}$.

In writing down Eq.(7), we have assumed that azimuthal mode coupling (coupling among different ℓ's) can be ignored. This assumption is valid if the mode frequencies do not shift much away from the unperturbed value $\ell\omega_s$ (i.e. the mode frequency shifts $\ll \omega_s$).

The potential-well distortion can be considered to have two effects on the particle motion. First, it causes a "detuning" effect, i.e., Φ_0 now depends on H. Second, it causes a distortion of the phase space topology, i.e., the constant-H contours in phase space are no longer ellipses. It can be shown that the instability

is a result of the second effect alone. In other words, *distortion of phase space from elliptical contours is a necessary condition for instability*. This observation suggests that one way to alleviate this instability is to introduce an external higher harmonic rf to reduce the net phase space distortion.

III. APPLICATION TO SLC DAMPING RING

We have applied the analysis to the SLC Damping Rings. The following assumptions are made: (a) synchrotron radiation damping can be ignored; (b) the linearized Vlasov equation applies below the instability threshold; (c) this is a single-bunch, single-turn instability; (d) the wake field is as shown in Fig.1; (e) coupling among the azimuthal modes can be ignored; (f) we include two and only two radial modes with a double water-bag beam.

Unless specified otherwise, the parameters we used for the Damping Ring are $\eta = 0.0145$, $V_{\rm rf} = 1.0$ MV, $\nu_s = 0.01275$, $cT_0 = 35.268$ m, $E = 1.19$ GeV. The unperturbed gaussian beam is assumed to have $\sigma_\delta = 0.73 \times 10^{-3}$. We mostly have studied the case of the quadrupole azimuthal mode with $\ell = 2$. The $\ell = 1$ case is determined by the Robinson damping mechanism and is not the subject of our study.

Figure 2 shows one set of results of our calculations. Figure 2(c) shows the complex mode frequency shifts $Y = \left(\frac{\Omega^{(\ell)}}{\omega_s} - \ell\right)$ as functions of the beam intensity N. The solid curves show the real part of Y. The two radial modes have separate frequencies for small beam intensities. At a threshold value of $N_{\rm th} = 1.4 \times 10^{10}$, the two mode frequencies merge, and the beam becomes unstable. The instability growth rate τ^{-1}/ω_s is given by the dotted curve above threshold. The portion of the solid curve below threshold in Fig.2(b) shows the relative bunch lengthening factor σ_z/σ_{z0} versus N, where σ_{z0} is the unperturbed rms bunch length. The dotted curve above threshold is an under-estimate because the calculated σ_z took into account of potential-well distortion but ignored bunch lengthening due to microwave instability. (The solid curve above threshold will be explained later.) Figure 2(a) shows the shift of synchronous phase z_s versus N. The dotted portion of the curve gives an over-estimate of z_s.

The longitudinal radiation damping rate of the Damping Ring gives $\tau_{\rm rad}^{-1}/\omega_s = 0.00095$. The effect of radiation damping on $N_{\rm th}$ is presumably small.

The instability threshold was studied as a function of the rf Voltage $V_{\rm rf}$. It was found that $N_{\rm th} = 1.7 \times 10^{10}$ when $V_{\rm rf} = 0.8$ MV and 2.1×10^{10} when $V_{\rm rf} = 0.6$ MV.

We have also calculated the case for the sextupole azimuthal mode $\ell = 3$ and $V_{\rm rf} = 1.0$ MV. The instability threshold is found to be $N_{\rm th} = 1.6 \times 10^{10}$, slightly higher than the threshold for $\ell = 2$. The beam is first unstable in its quadrupole motion, but the sextupole mode threshold is not far away. The behavior is similar when $V_{\rm rf}$ is lowered to 0.6 MV. At 0.6 MV, the $\ell = 3$ threshold is found to be $N_{\rm th} = 2.6 \times 10^{10}$.

Our analysis describes the beam behavior at or below the instability threshold. By an ad hoc consideration, however, we may try to extend its application to cases *above* threshold by conjecturing that, above threshold, the bunch would lengthen just enough to stablize the beam. The beam is therefore constantly staying at the edge of instability. The extension of the solid curve
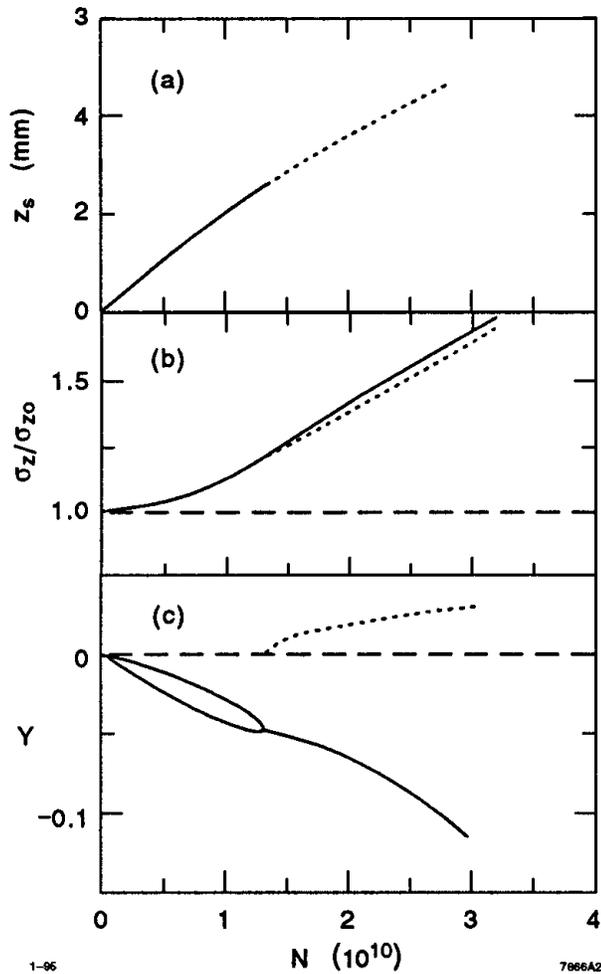
Figure 2. (a) Shift of synchronous phase z_s in mm as a function of beam intensity N. (b) Bunch lengthening factor (due to potential-well distortion) versus N. (c) The complex mode frequency shifts versus N.

in Fig.2(b) beyond threshold represents the conjectured bunch lengthening due to microwave instability. Note that the region between the dotted and the solid curves is relatively small, indicating that this instability is weak and a small increase of the bunch length beyond the potential-well distortion stabilizes the beam. One also expects that the same small relative increase would occur in the energy spread above threshold. Furthermore, if there is a mechanism which causes the beam to execute a saw-tooth oscillation, as observed in the Damping Ring,[10,11] the amplitude of the sawtooth oscillation is likely to correspond to the region between the dotted and solid curves of Fig.2(b).

To further study the instability mechanism, and to explore possible cures, we considered the following two possibilities: (i) add a high harmonic rf voltage to counteract the potential-well distortion, and (ii) operate the accelerator below transition with $\eta < 0$.[12]

We found that a higher harmonic rf is quite effective in raising the instability threshold. For example, by introducing a 12 GHz rf system (considered e.g. for the NLC at SLAC), which is phased 4 mm ahead of the main rf, a voltage of 6.5 kV pushes the threshold intensity to 3×10^{10}.
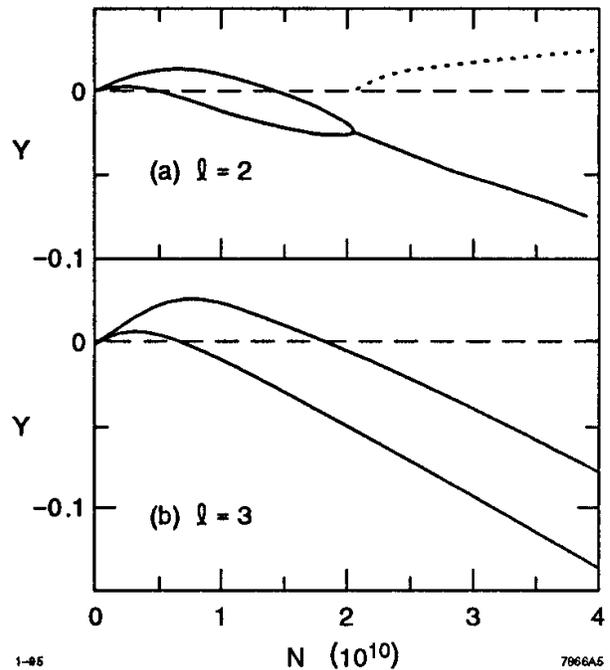
Figure 3. Mode frequencies when $\eta = -0.0145$.

Operating the accelerator with $\eta < 0$ turned out less conclusive. Figure 3(a) shows the mode frequencies with $\eta = -0.0145$. The instability threshold is raised from 1.4×10^{10} to 2.0×10^{10}. Figure 3(b) shows what happens to the $\ell = 3$ modes. Operating with $\eta < 0$ seems to improve the instability threshold somewhat in the present study. However, whether this is a general trend needs more investigation.

References

[1] K. Oide, private communications, 1994, to be published.

[2] A. Chao, B. Chen, and K. Oide, to appear in Proc. Workshop on Beam Instabilities in Storage Rings, NSRL, Hefei, China, 1994.

[3] K. Oide, AIP Proc. 45, Nonlinear Dynamics and Particle Acceleration, Tsukuba, Japan, 1990, p.266.

[4] K. Oide and K. Yokoya, KEK Preprint 90-10 (1990).

[5] X.T. Yu and J.S. Wurtele, IEEE Part. Accel. Conf., Washington D.C., 1993, p.3327.

[6] R. Baartman and M. D'yachkov, IEEE Part. Accel. Conf., Washington D.C., 1993, p.3330. See also these proceedings.

[7] Bo Chen, Ph.D. thesis, Univ. of Texas at Austin, 1995.

[8] B. Chen and A. Chao, Part. Accel. **43**, 77 (1993).

[9] K. Bane, private communications, 1994.

[10] K. Bane et al., these proceedings.

[11] R.L. Holtzapple, R. Siemann, C. Simopoulos, these proceedings.

[12] S.X. Fang, private communications, 1994, to be published.