EMITTANCE GROWTH DUE TO DECOHERENCE AND WAKEFIELDS *

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Abstract

Suddenly induced coherent centroid oscillations about the closed orbit will decohere due to nonlinearities in the magnetic optics—at the expense of a stored beam's emittance. Collective effects mediated by the vacuum chamber wakefield and dependent on the beam current, can however damp the coherent oscillations—ameliorating the emittance growth. Closed form expressions for both the beam centroid and the beam size are obtained in the absence of collective effects. Simultaneous turn-by-turn measurements of beam centroid and size in the SLC damping ring are presented, and the importance and intricacy of collective effects is discussed.

I. NONLINEAR DETUNING AND CHROMATICITY

In the absence of collective effects, decoherence is dominated by nonlinear detuning and chromaticity. The evolution of the beam centroid has been described in [1-4]. We here extend these results to the rms beam size. Consider a beam with a gaussian distribution in the (x, x') phase space. At turn $M = 0$, the beam is kicked by an angle $\Delta x'$. We normalize the coordinates by the unperturbed rms beam size σ_x as $\bar{x} = x/\sigma_x$ and $\bar{p} = (\alpha_x x + \beta_x x')/\sigma_x$, where β_x and α_x are the Courant-Snyder parameters. We normalize the kick by defining $Z = \frac{\beta_x}{\sigma_x}\Delta x'$. The amplitude $a = \sqrt{\bar{x}^2 + \bar{p}^2}$, and ϕ is the betatron phase. The beam distribution after the kick is

$$\rho_k(\phi, a) = \frac{a}{2\pi} e^{-(a^2+Z^2+2Za\sin\phi)/2} \tag{1}$$

The nonlinearity is assumed to result from an amplitude-dependent betatron tune and a relative energy offset δ of a particle which modifies the betatron tune through the chromaticity ξ:

$$\Delta\nu = -\mu a^2 + \xi\delta \tag{2}$$

with detuning μ; μ/σ_x^2 is determined by the lattice.

For single particle motion the amplitude a is an invariant. The time dependence of the energy offset is

$$\delta(M) = \delta_0 \cos(2\pi\nu_s M + \phi_0), \tag{3}$$

while the betatron phase advances [1] as

$$\Delta\phi = 2\pi M(\nu_0 - \mu a^2) + \frac{2\xi}{\nu_s}\delta_0 \sin(\pi\nu_s M)\cos(\pi\nu_s M + \phi_0). \tag{4}$$

*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

The beam centroid motion after the kick is given by

$$\begin{aligned}[\langle\bar{x}\rangle + i\langle\bar{p}\rangle] &= \int_0^\infty da \int_0^{2\pi} d\phi\; a e^{-i\phi - i\Delta\phi(a,M)}\; \rho_k(\phi, a) \\ &= \frac{iZF_1}{(1-i\theta)^2}\exp\left(-i2\pi M\nu_0 + \frac{Z^2}{2}\frac{i\theta}{1-i\theta}\right), \end{aligned} \tag{5}$$

where we have defined a time variable in units set by the nonlinearity,

$$\theta = 4\pi\mu M \tag{6}$$

and a form factor differing from 1 when $\xi \neq 0$

$$F_1 = \int_0^{2\pi} d\phi_0 \int_0^\infty d\delta_0 \rho(\delta_0) e^{-i2\frac{\xi}{\nu_s}\delta_0 \sin(\pi\nu_s M)\cos(\pi\nu_s M + \phi_0)} \tag{7}$$

Assuming a gaussian distribution for δ_0 with rms σ_δ,

$$F_1 = \exp\left[-2(\frac{\xi\sigma_\delta}{\nu_s})^2 \sin^2(\pi\nu_s M)\right] \tag{8}$$

Equation (5) gives the decoherence behavior of the beam centroid.[1-4]

We next compute the rms beam size after the kick,

$$\begin{aligned}\begin{bmatrix}\langle\bar{x}^2\rangle \\ \langle\bar{x}\bar{p}\rangle \\ \langle\bar{p}^2\rangle\end{bmatrix} &= \begin{bmatrix}1+\frac{Z^2}{2} \\ 0 \\ 1+\frac{Z^2}{2}\end{bmatrix} + \frac{F_2 Z^2/2}{(1+4\theta^2)^{3/2}}\exp\left(-\frac{2Z^2\theta^2}{1+4\theta^2}\right) \\ &\times \begin{bmatrix} -\cos(4\pi M\nu_0 - \frac{Z^2\theta}{1+4\theta^2} - 3\tan^{-1}(2\theta)) \\ \sin(4\pi M\nu_0 - \frac{Z^2\theta}{1+4\theta^2} - 3\tan^{-1}(2\theta)) \\ \cos(4\pi M\nu_0 - \frac{Z^2\theta}{1+4\theta^2} - 3\tan^{-1}(2\theta)) \end{bmatrix}\end{aligned} \tag{9}$$

where $F_2 = F_1^4$ for the gaussian δ_0 distribution; therefore the rms size is more strongly modulated by the chromaticity than the centroid.

The instantaneous beam size is given by $\sigma_{\bar{x}} = \sqrt{\langle\bar{x}^2\rangle - \langle\bar{x}\rangle^2}$. This gives

$$\begin{aligned}\sigma_{\bar{x}}^2 = 1 + \frac{Z^2}{2}\Bigg\{ & 1 - \frac{F_2}{(1+4\theta^2)^{3/2}}\exp\left(-\frac{2Z^2\theta^2}{1+4\theta^2}\right) \\ & \times\cos\left[4\pi M\nu_0 - \frac{Z^2\theta}{1+4\theta^2} - 3\tan^{-1}(2\theta)\right] \\ & - \frac{2F_1^2}{(1+\theta^2)^2}\exp\left(-\frac{Z^2\theta^2}{1+\theta^2}\right) \\ & \times\sin^2\left[2\pi M\nu_0 - \frac{Z^2\theta}{2(1+\theta^2)} - 2\tan^{-1}\theta\right]\Bigg\}\end{aligned} \tag{10}$$

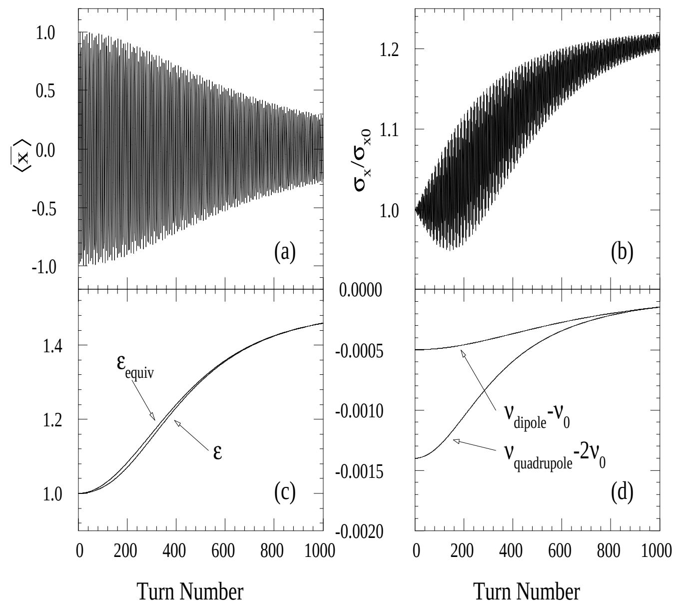

Figure 1. Beam evolution; the first 1000 turns after a kick: (a) $\langle\bar{x}\rangle$, (b) $\sigma_{\bar{x}}$, (c) $\epsilon_{\rm equiv}$ and ϵ, (d) $\nu_{\rm dipole} - \nu_0$ and $\nu_{\rm quadupole} - 2\nu_0$. Parameters used are $Z = 1$, $\xi = 0$, $\nu_s = 0.01$, $\sigma_\delta = 0.001$, $\nu_0 = 0.18$, and $\mu = 1 \times 10^{-4}$.

The amplitude of the beam centroid is, from Eq. (5),

$$A_{\bar{x}} = \sqrt{\langle\bar{x}\rangle^2 + \langle\bar{p}\rangle^2} = \frac{ZF_1}{1+\theta^2} \exp\left[-\frac{Z^2\theta^2}{2(1+\theta^2)}\right] \qquad (11)$$

For small θ this amplitude decoheres approximately as a gaussian in time. For large θ, it decoheres roughly $\sim \frac{1}{\theta^2}$. When the kick is weak and the chromaticity is small, the beam filaments on a time scale of $\frac{1}{4\pi\mu}$ turns.

Note from Eq. (9) that $\langle\bar{x}^2\rangle + \langle\bar{p}^2\rangle = 2 + Z^2$ is an invariant after the kick. If one defines a 'matched equivalent' beam emittance [5] as $\epsilon_{\rm equiv} = \frac{1}{2}(\sigma_{\bar{x}}^2 + \sigma_{\bar{p}}^2)$, then

$$\epsilon_{\rm equiv}(M) = \frac{1}{2}(\langle\bar{x}^2\rangle - \langle\bar{x}\rangle^2 + \langle\bar{p}^2\rangle - \langle\bar{p}\rangle^2) = 1 + \frac{Z^2}{2} - \frac{A_{\bar{x}}^2}{2} \quad (12)$$

One may also define an instantaneous emittance as

$$\epsilon(M) = \sqrt{\sigma_{\bar{x}}^2\sigma_{\bar{p}}^2 - (\langle\bar{x}\bar{p}\rangle - \langle\bar{x}\rangle\langle\bar{p}\rangle)^2} \qquad (13)$$

When $M = 0$, we have $\epsilon_{\rm equiv} = \epsilon = 1$. When $M \to \infty$, we have $\epsilon_{\rm equiv} = \epsilon = 1 + \frac{Z^2}{2}$.

One can define an 'instantaneous' dipole tune as $\frac{1}{2\pi}\times$ (phase advance per turn of the centroid oscillation when $\xi = 0$):

$$\nu_{\rm dipole} = \nu_0 - \frac{\mu}{1+\theta^2}\left[4 + \left(\frac{1-\theta^2}{1+\theta^2}\right)Z^2\right] \qquad (14)$$

Note that if one measures the dipole tune by kicking the beam and analyzing its subsequent centroid motion, the measured dipole tune will be a function of time.

The 'instantaneous' quadrupole tune can likewise be defined as $\frac{1}{2\pi}\times$ (phase advance per turn of the beam size oscillation when $\xi = 0$),

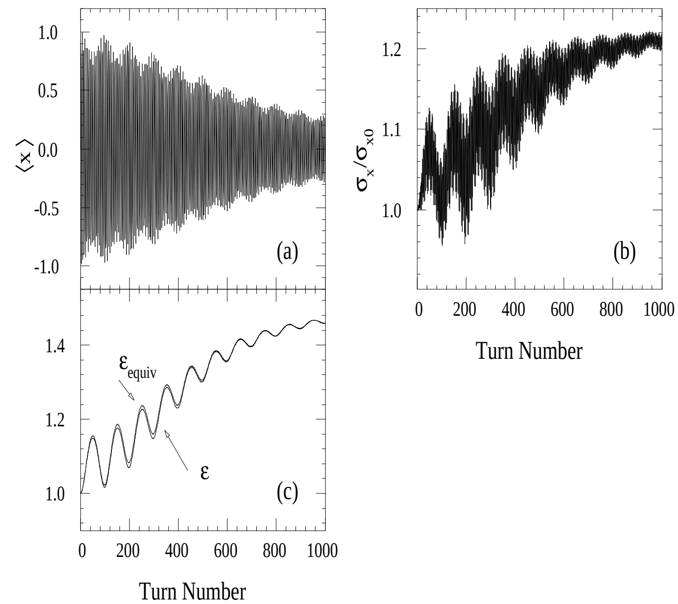

Figure 2. $\xi = 3$, otherwise as in Fig. 1.

$$\nu_{\rm quadrupole} = 2\nu_0 - \frac{2\mu}{1+4\theta^2}\left[6 + \left(\frac{1-4\theta^2}{1+4\theta^2}\right)Z^2\right] \qquad (15)$$

In general, the quadrupole tune is close, but not equal, to twice the dipole tune. For $M = 0$, we have $\nu_{\rm dipole} = \nu_0 - (4 + Z^2)\mu$ and $\nu_{\rm quadrupole} = 2\nu_0 - 2(6 + Z^2)\mu$. When $M \to \infty$, we have $\nu_{\rm dipole} = \nu_0$ and $\nu_{\rm quadrupole} = 2\nu_0$.

Figures 1 – 2 show the time behavior of various quantities after a kick using the analytic expressions. The beam size modulation at the synchrotron frequency is a result of "recoherence" [2,3]. Despite the prominent β-beat evident in Figs. 1(b) and 2(b), the difference between the instantaneous and the matched equivalent emittances is small.

II. COLLECTIVE EFFECTS AND EXPERIMENT

Both the horizontal centroid and beam size were measured by digitizing the synchrotron light image [6] of the positron beam in the SLC damping ring. A fast-gated camera detected the radiation emitted on a single pass of the particle bunch, although each image corresponds to a different machine pulse because of the limited bandwidth of the data acquisition system. Observations were made in the neighborhood of a time in the SLC damping cycle during which the beam is accidentially kicked by spurious transients in the injection/extraction fast kicker pulses. Data for various beam currents and chromaticities are shown in Fig. 3.

The data were analyzed by the method of [5] to find $\epsilon_{\rm equiv}$, which is plotted in Fig. 4 in the ratio $X = (\epsilon_{\rm equiv} - 1)/\frac{1}{2}Z^2$, which we expect to asymptote to 1 for $M \to \infty$ in the case of pure decoherence (*cf.* Eq. 12). But when the chromaticity is positive, as in the data, there will be collective "head-tail" damping of the centroid motion. As the coherent motion damps, rather than decoheres, there is less motion to filament and the emittance growth may be significantly inhibited, as seen decisively in the data. The extent to which $X < 1$ as $M \to \infty$ indicates that the

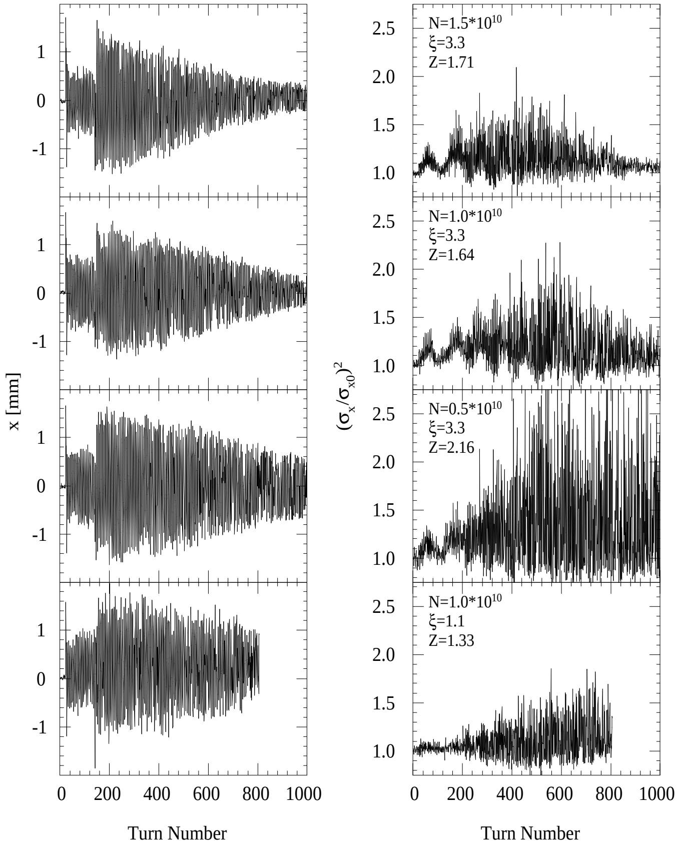

Figure 3. Measured horizontal centroid and rms size as functions of turn number in the SLC positron damping ring. The beam was kicked transversely at turns 25 and 135.

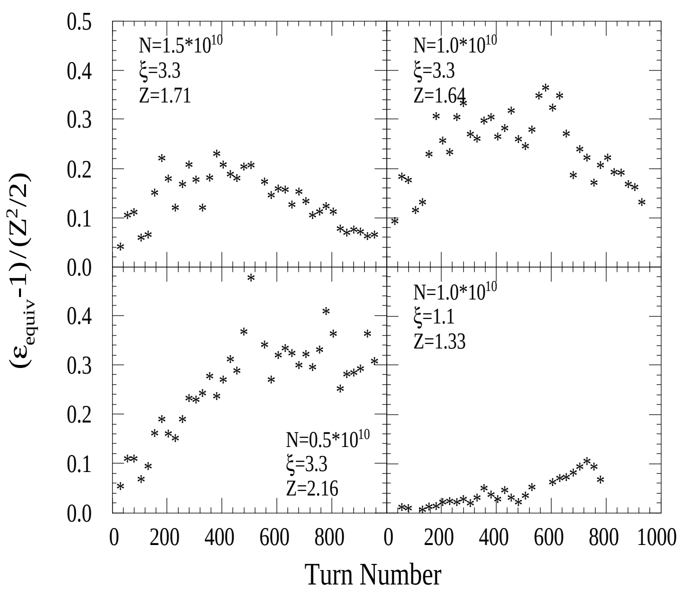

Figure 4. Matched equivalent emittance growth determined from the data in Fig. 3, relative to the maximum expected in the absence of collective damping.

time scale for collective damping is relatively 'fast' compared to that for decoherence. The decoherence in turn has a quenching influence on the collective damping in that the detuning phase competes with the accumulating head-tail phase causing the instantaneous damping rate to decrease. (For $\xi < 0$ this raises the instability threshold [7].) Thus naively we do not expect filamentation once-occuring, to be reversed; however the data in Fig. 4 appear to show an emittance drop at higher current. It may be notable that the "strong" head-tail strength $\Upsilon = \frac{Nr_0\beta_x\sigma_z W_0}{4\sqrt{2}\nu_s\gamma}$ [8] takes on values of 0.27, 0.18, 0.09, and 0.18 in our four cases—below the instability threshold $\Upsilon = 2$. More work, both in theory and experiment, is needed to completely understand the collective aspects of these phenomena.

If head-tail damping dominates the centroid damping, the SLC damping ring wakefield $W_\perp(z) = W_0 z$, $(z < 0)$ (reasonable for short bunches) follows from the data since the damping rate [8]

$$\frac{1}{M_{\rm D}} \approx \frac{4}{\pi^2}\frac{Nr_0\beta_x\xi\sigma_\delta\sigma_z W_0}{\nu_s\gamma}. \tag{16}$$

A rough fit yields $W_0 = 6 \times 10^7 \mathrm{m}^{-3}$, giving damping times of 670, 1000, 2000, and 3000 turns for the four cases of Fig. 3. (We use $\sigma_\delta = 0.73 \times 10^{-3}$, $\sigma_z = 6$mm, $\nu_s = 0.01275$, the β-function at the impedance source $\beta_x = 3$m, and $\gamma = 2350$.) The expected β-tron tune shift with current

$$\frac{d\nu_x}{dN} = -\frac{r_0\beta_x\sigma_z W_0}{8\sqrt{2}\pi\gamma} = -3.6 \times 10^{-14} \tag{17}$$

then is -0.0007 at $N = 2 \times 10^{10}$, *e.g.*.

We thank Franz-Josef Decker and Marc Ross for useful discussions and support throughout the course of this work, and the Operations section of the SLAC Accelerator Department for their assistance during the data-taking.

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