SIMULATION OF TRANSVERSE COUPLED BUNCH INSTABILITIES *

S. Khan, BESSY II, Rudower Chaussee 5, Geb.15.1, 12489 Berlin, Germany

Abstract

Numerical simulations of transverse coupled bunch instabilities caused by the resistive wall of the vacuum chamber are presented. These simulations confirm the results obtained analytically and extend them to cases which are not easily accessible by analytical methods.

I. INTRODUCTION

Transverse coupled bunch oscillations in a storage ring are driven by wake fields due to the interaction with cavity-like structures or with the resistive wall of the vacuum vessel. Concentrating on the latter case, the dipole wake function for a cylindrical vacuum chamber of length L, radius b and conductivity σ (in cgs units) is given by (see e.g. [1])

$$W_{\perp}(z) = -\frac{2}{\pi b^3}\sqrt{\frac{c}{\sigma}}\,\frac{1}{\sqrt{z}}\,L \tag{1}$$

where c is the velocity of light and z is the distance behind the charge causing the wake.

Due to its strong dependence on the chamber size, the resistive wall effect is in particular of interest for modern synchrotron light sources with small wiggler/undulator gaps. In this case, the radius b in equation 1 is replaced by an effective half-height of the flat vacuum chamber.

In a system of M bunches, each containing N particles, a bunch i oscillates according to the equation of motion

$$\ddot{x}_i(t) + \omega_i^2 x_i(t) = \frac{-N r_\circ c}{\gamma\, t_\circ}\sum_{j=1}^{M}\sum_{n=0}^{\infty} x_j\ \ (t - t_{ij} - n\, t_\circ) \cdot W_{\perp}\,(-c\, t_{ij} - n\, c\, t_\circ), \tag{2}$$

where $r_\circ$ is the classical electron radius, γ the Lorentz factor, $t_\circ$ the revolution time and $c\, t_{ij}$ the distance at which bunch j follows i. The term including the wake functions summed over all bunches and over all previous revolutions causes the oscillation to grow or decrease, depending on the phase relationship between the bunches.

In the following discussion, the BESSY II electron storage ring serves as an example. BESSY II is a 1700 MeV synchrotron light source currently under construction at Berlin [2]. The storage ring will be 240 m in circumference ($t_\circ = 0.8\ \mu$s) and can be filled with max. 400 bunches. In this case, the coupled bunch system has 400 oscillation modes, some of which may be excited and some damped. If the growth of any mode exceeds the rate of radiation damping or Landau damping, the beam will be unstable.

* Funded by the Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie and by the Land Berlin.

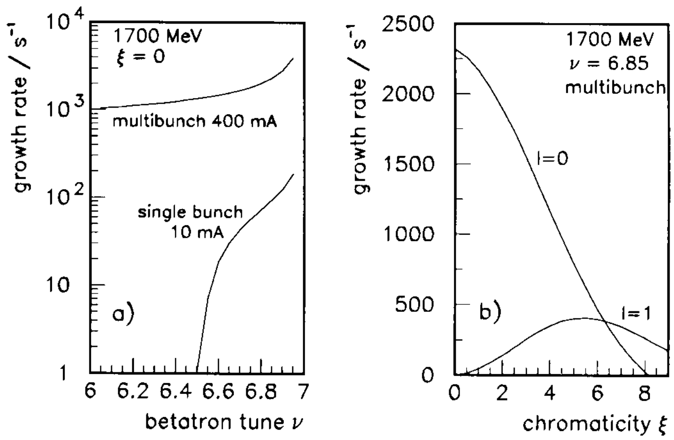

Figure. 1. Growth rate of transverse coupled bunch instabilities as function of the vertical betatron tune ν (left) and of chromaticity $\xi = \Delta\nu/(\Delta p/p)$ (right).

II. ANALYTICAL EVALUATION

Fourier transformation of equation 2 into the frequency domain yields an analytical expression for the growth rate of the most unstable multibunch mode, provided the bunches are equally spaced and their motion is a harmonic oscillation (ω_i constant). Application of this well-known theory [3] leads to the following general results:

A. Single Bunch Mode

A single bunch has only one oscillation mode which is driven by its own wake from previous revolutions. This mode is damped if the fractional part of the tune δ is below 0.5; for $\delta > 0.5$ the beam is unstable.

Considering the finite size of the bunch has several consequences: As an extended object, the bunch has internal modes l. Furthermore, a finite bunch length implies a finite spectrum. Its central frequency is a function of the chromaticity ξ. A small positive chromaticity is sufficient to stablize the $l = 0$ mode (head-tail effect), whereas the weaker $l > 0$ modes require larger values of ξ to be damped.

B. Multiple Bunches

For any tune, half of the multibunch modes are damped and half are unstable. The growth rate of the most unstable mode follows roughly $1/\sqrt{\delta}$. This is shown in the left part of figure 1 for the BESSY II storage ring, assuming an aluminium chamber in the insertion device region and a half-height of $b = 0.01$ m.

A large positive chromaticity reduces the growth rate and can even stabilize all multibunch modes for $l = 0$, while the $l = 1$ mode is still unstable, as shown in the right part of figure 1.

III. TIME DOMAIN SIMULATION (HARMONIC OSCILLATION)

A numerical simulation of the growing instability allows to study its time evolution, starting from different initial configurations and under the influence of different damping mechanisms. Furthermore, a simulation is required in cases where the growth rate cannot be determined analytically.

The simulation presented in this section maintaines the assumption of a simple harmonic motion. It is meant to study the case of a fractional bunch filling and the effect of two damping mechanisms: (i) Radiation damping, where a factor $\exp(-t_\circ/\tau_R)$ is applied to the oscillation amplitudes after each turn, thus neglecting the stochastic nature of the radiation process. (ii) A non-linear tune shift with the amplitude $\Delta\nu \sim x^2$.

Equation 2 is iterated for all bunches in time steps of 2 ns ($\sim$60 steps per oscillation period). If all bunches perform a harmonic oscillation with the nearly unperturbed betatron frequency, the force experienced by a particular bunch i (given by the sum in eq. 2) just oscillates with the same frequency. In the course of the simulation, its amplitude and phase have to be recalculated from time to time, which requires little computational effort.

A. Time Evolution

Starting from a random configuration, the 400 bunches considered in figure 2 behave very differently. Some grow in amplitude, while others nearly stop oscillating. It takes about 0.2 ms for the bunches to "find" the most unstable mode μ at which they oscillate with the same exponentially growing amplitude and a fixed relative phase of $2\pi\mu/M$ between adjacent bunches. Once this state is reached, the growth rate in the simulation is consistent with the analytical result. If the bunches are initiated with the proper phase relationship, the exponential growth starts without delay.

B. Damping

The figures 2b-2d show the influence of damping on the system described above. In figure 2b, the growth of the instability is slowed down by radiation damping.

In figure 2c, a tune shift with amplitude has been introduced. All amplitudes grow up to a certain value, at which they start to diverge and their phase relationship becomes distorted. The averaged amplitude, however, remains constant.

A combination of both damping effects leads to a highly irregular motion, as shown in figure 2d.

C. Fractional Bunch Filling

If the storage ring is fractionally filled with bunches in order to leave an ion clearing gap, the coupled bunch oscillations are not easily described analytically. It has been shown that – for a given total current – the complete filling represents the worst case [4].

In a time domain simulation with 320 bunches in 400 rf buckets (figure 3), the bunches oscillate with different amplitudes and the spectrum shows a fine structure which is not present in the case of a complete filling. However, the growth rate of the instability, the phase relationship between the bunches and the gross features of the spectrum are the same for both cases.

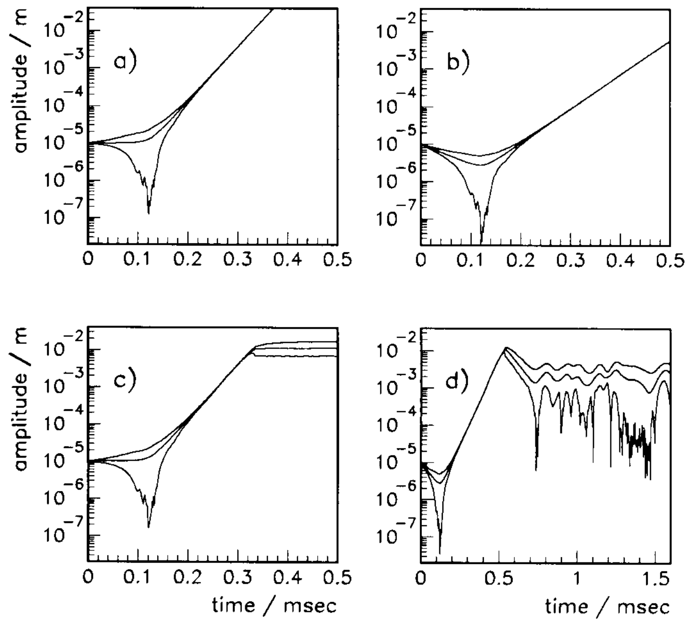

Figure. 2. Smallest, largest and average amplitude in a system of 400 evenly spaced bunches with a) no damping, b) radiation damping, c) tune shift with amplitude, d) both damping effects.

IV. TIME-DOMAIN SIMULATION (BETATRON OSCILLATION)

In a realistic betatron oscillation, the frequency ω_i in equation 2 depends on the position in the lattice. Therefore, the phase relationship between bunches changes all the time, which should reduce the effective driving force.

In order to simulate this effect, all bunches are tracked through the magnetic lattice using linear transfer matrices for each time step (2 ns, corresponding to 0.6 m). To prevent the effect in question from being obscured by non-linear lattice effects, sextupole kicks were not applied. The consequences of the tune shift caused by sextupole fields were studied in [5].

With the wake function of equation 1 inserted into equation 2, the driving force for bunch i is essentially given by

$$F_i(t) \sim \sum_{j=1}^{N} \sum_{n=0}^{\infty} \frac{1}{\sqrt{t_{ij}/t_\circ + n}}\, x_j(t - t_{ij} - nt_\circ). \qquad (3)$$

Keeping track of all bunch positions over many turns and evaluating the twofold sum for all bunches at each time step would clearly require too much computer memory and time. However, a simplification can be made by expessing the transverse position of bunch j at the $n^{\rm th}$ previous passage as

$$\begin{aligned} x_j(t - t_{ij} - nt_\circ) = x_j(t - t_{ij}) \left\{ \cos(n\delta_j) + \alpha \sin(n\delta_j) \right\} \\ + x'_j(t - t_{ij})\ \beta \sin(n\delta_j), \end{aligned} \qquad (4)$$

where α and β are the Twiss parameters at the location of bunch j, and δ_j is the phase advance over one revolution backward in time. Then, equation 3 can be rewritten as

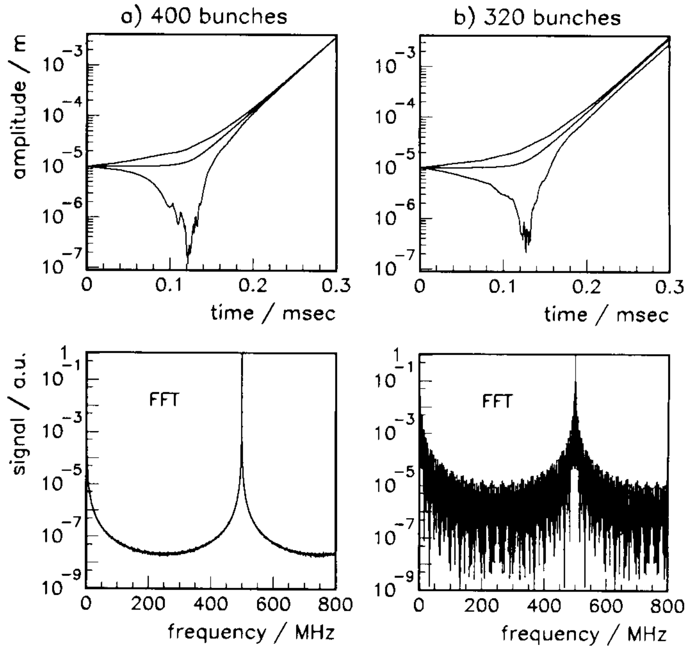

Figure. 3. Amplitudes (upper part) and the Fourier transformed signal seen at a fixed location of the storage ring (lower part) for a) 400 evenly spaced bunches, and b) 320 bunches in 400 rf buckets with the same total current.

$$F_i(t) \sim \sum_{j=1}^{N} \Bigg[\left\{ \sum_{n=0}^{\infty} \frac{\cos(n\delta_j)}{\sqrt{t_{ij}/t_\circ + n}} \right\} \cdot x_j(t - t_{ij}) + \left\{ \sum_{n=0}^{\infty} \frac{\sin(n\delta_j)}{\sqrt{t_{ij}/t_\circ + n}} \right\} \times \Big(\alpha\, x_j(t - t_{ij}) + \beta\, x'_j(t - t_{ij}) \Big) \Bigg]. \quad (5)$$

This allows to perform the sum over n beforehand and store the result for all required values of t_{ij} and δ_j in a look-up table. This is justified if δ_j is constant over a reasonable time scale – an assumption also made in the analytical theory.

A. First Results

In the simulation shown in figure 4, a system of 400 evenly spaced bunches was initiated with a phase relationship corresponding to the most unstable mode. The amplitudes first diverge and grow more slowly than they do in the harmonic oscillator model. The relative phases also diverge and fill a certain range (figure 4b), while the growth rate increases and settles at a value well below the theoretical rate (figure 4c).

In conclusion, an improved description of coupled bunch oscillations has been obtained and further refinements of the model are within reach, in particular the inclusion of damping mechanisms: (i) radiation damping, (ii) sextupole kicks to produce the tune shift that was included rather arbitrarily in the harmonic oscillator model, and (iii) the head-tail effect by splitting the bunches into macro-particles.

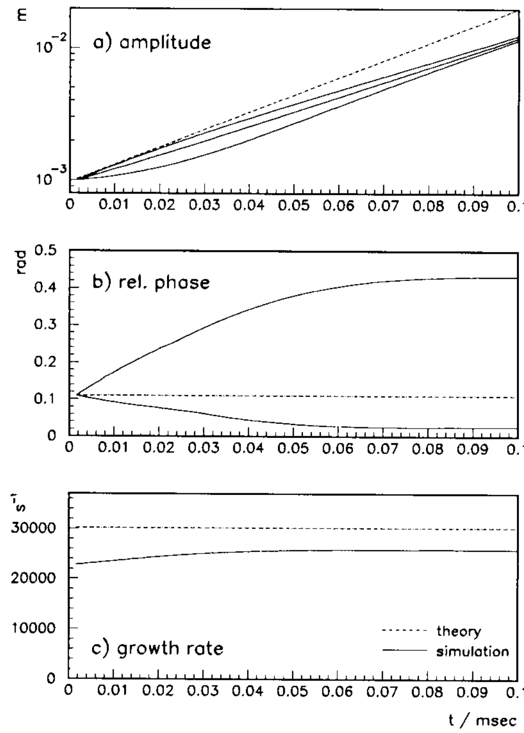

Figure. 4. Transverse resistive wall instability with 400 bunches, assuming a harmonic motion (dashed lines) and a more realistic betatron oscillation (solid lines): a) smallest, largest and average amplitude, b) smallest and larges phase difference between adjacent bunches, and c) growth rate versus time.

References

[1] Chao, A. W., *Physics of Collective Beam Instabilities in High Energy Accelerators*, New York, 1993.

[2] E. Jaeschke, *Status of the High Brillance Synchrotron Radiation Source BESSY II*, these proceedings.

[3] B. Zotter and F. Sacherer, in: *Theoretical Aspects of the Behaviour of Beams in Accelerators and Storage Rings*, CERN 77-13, 1977.

[4] R. D. Kohaupt, *On Multi-Bunch Instabilities for Fractionally Filled Rings*, DESY 85-139, 1985.

[5] M. Meddahi and J. Bengtsson, *Studies of Transverse Coherent Bunch Instabilities for the Advanced Light Source (ALS)*, LBL-35703, 1994.