INFERENCE OF WAKE FIELD STRUCTURE BY DRIVING LONGITUDINAL COUPLED BUNCH MODES IN MAIN RING*1

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Here, we present a qualitative study of coupled bunch effects for a partially filled Main Ring. Individual bunches interact with their environment via wake fields generated by the fundamental and higher parasitic structural modes. Bunches at the front of the train influence the motion of the following bunches. We employ a two-point correlation method [1]to measure the phase oscillation between the reference bunch (bunch one) and a given bunch within the train, as a function of the bunch separations. The result of this analysis is a measure of the short range wakefield generated by the bunches.

I. ANALYSIS

Wake fields generated by a longitudinal kick to a train of proton bunches can be measured by the method of two-point correlation technique. Data are collected by observing signals from a resistive wall current monitor with a Tektronix 2 GHz digitizer. The digitizer is triggered every 32 turns to capture ~10 synchrotron periods worth of longitudinal profiles of the bunches (Fig. 1).

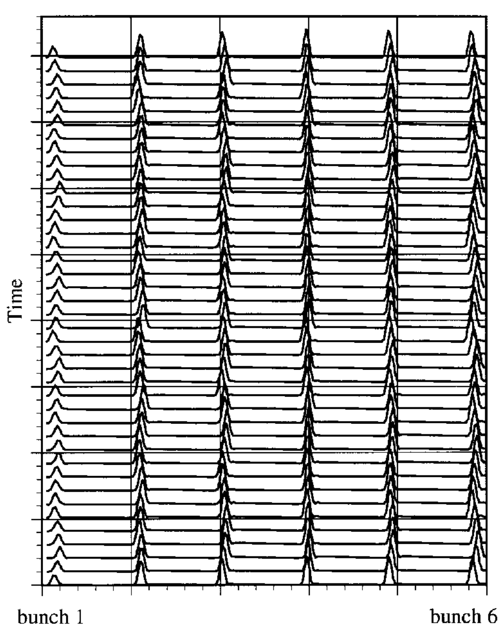

Fig. 1) Waterfall plot of the first 6 bunches.

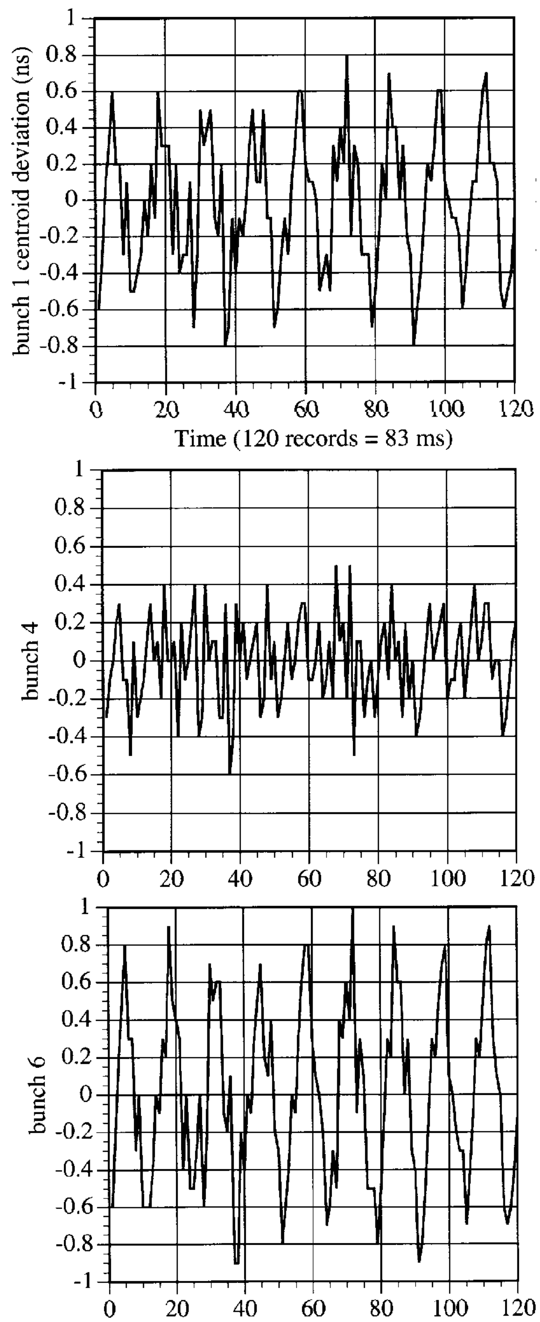

Fig. 2) The deviation of bunch centroid from bucket center for bunches 1,4,6. For the RF frequency of 53 MHZ, 1 ns is equal to 19 degrees of phase.

*1 Operated by University Research Association Inc., under contract with the U.S. Department of Energy.

Readout and the storage of the data is done by Macintosh computer using Labview software. The beam is given an initial kick by a programmable digital phase shifter which shifts the phase of all 18 RF cavities by 20 degrees for 200 μ sec (10 turns). The centroid position of each bunch can be found and longitudinal dipole oscillations at the synchrotron frequency are evident (Fig. 2).

The resulting time series is then broken up into 1000 realizations. Each realization contains the turn by turn data for each bunch (Fig. 3) which are then individually Fourier transformed. A cross power estimate $C_j(f)$ for each realization j is formed using the formula $C_j(f) = \phi_{j,1}\,\phi^*_{j,2}$ where $\phi_{j,1}$ is the Fourier transform of the signal from realization j, bunch 1 and frequency f and similarly for bunch "n" where n >1. The cross power spectrum $Cj(f)$ is in general a complex quantity and can be written alternatively as $Cj(f) = |C_j(f)|\exp[\theta j(f)]$ where $\theta j(f)$ is known as the cross phase. An estimate of the wavenumber $kj(f)$ is formed from each realization using $kj(f)=\theta j(f)/\Delta x$. Here Δx is the longitudinal separation of the bunches in centimeters. The spectral power $S(k,f)$ is then formed by summing the power $|C_j(f)|$ according to its wavenumber k and frequency f, $S(k,f) = \Sigma j(\delta\kappa,\kappa j\ \delta f,fj)\ |Cj(f)|$ where $\delta k,l$ is the Kronecker delta function and the sum runs over all realizations j. Normalizing $S(k,f)$ by the total power $S_{tot} = \Sigma k,f\ S(k,f)$ produces the spectral density function $s(k,f)\Delta k\ \Delta f$ which can be thought of as the probability of finding a fluctuation in the wavenumber and frequency band [k, $k + \Delta k$] and [$f + \Delta f$]. Finally, the average value of the wavenumber $k_{\theta j}$ is produced using the spectral density $s(k,f)$:

$$k_\theta = \sum_k kS(k)$$

where the sum runs over all wavenumbers k and

$$s(k) = \sum_f S(k, f)$$

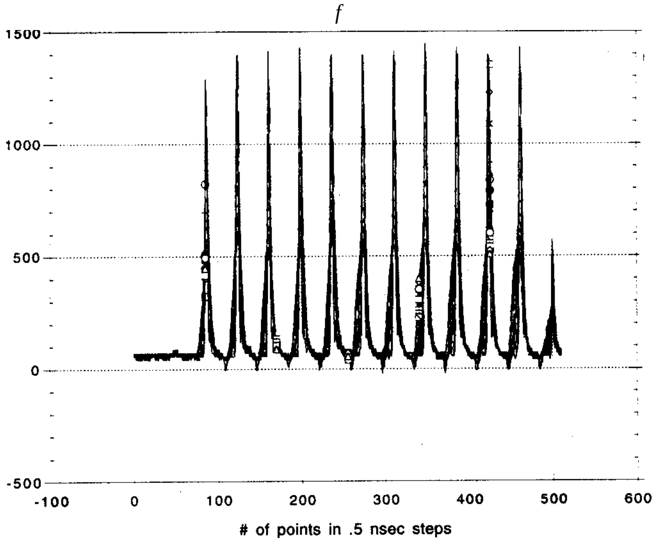

Fig. 3) A digitized batch of 12 bunches is shown.

II. RESULTS

Figure 4 shows an individual bunch oscillating within the RF bucket. The two-point correlation is done by determining the phase difference between the reference bunch (bunch one) and another bunch. For example, Figure 5 shows bunch one profile (top) and bunch 3 profile (bottom).

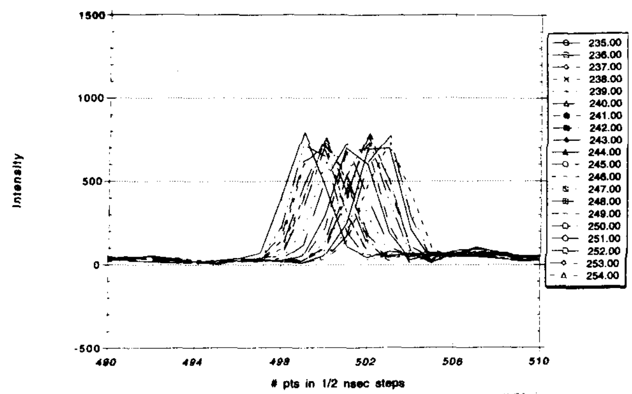

Fig. 4) Bunch oscillation within the bucket for many turns is shown.

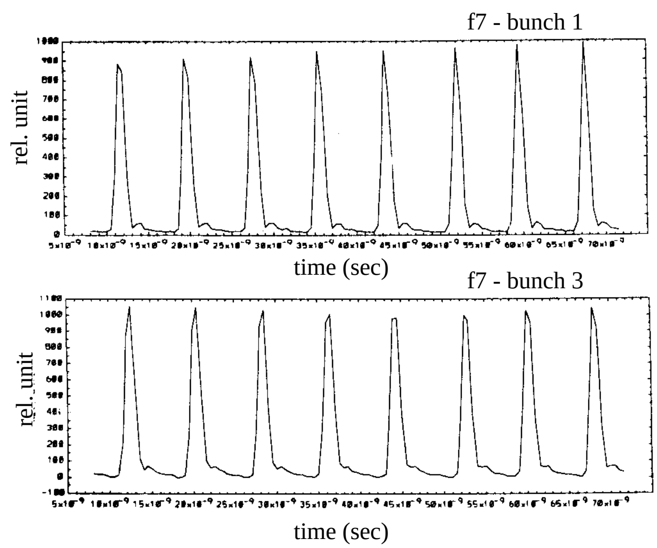

Fig. 5) Train of bunch one (top) is digitally formed from the turn by turn profiles. Same is performed for bunch three (bottom).

The wavenumber spectrum between bunch one and bunch two is shown in Figure 6. Only one peak, the dipole mode is observed. In figure 7, the wavenumber spectrum between bunch 2 and bunch 4 shows wavenumbers associated with dipole and quadrupole mode. A small peak associated with sextupole moment is also present but it is within the noise level of the measurement.

Figures 8 and 9 show the spectrum for other bunch combinations. Figure 9 shows a lack of the quadrupole mode which may have been eliminated by the superposition of the wakefields generated by the leading bunches.

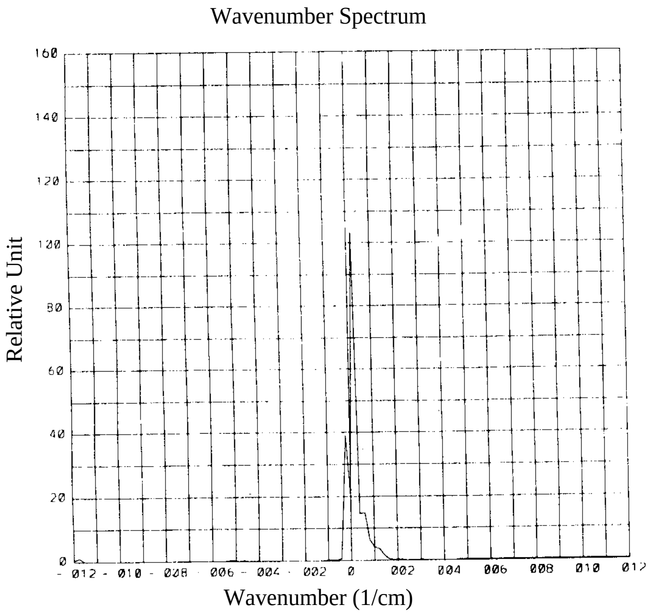

Fig.7) Shown peak associated with dipole mode is measured from the two-point correlation between the two adjacent bunch 1 and 2.

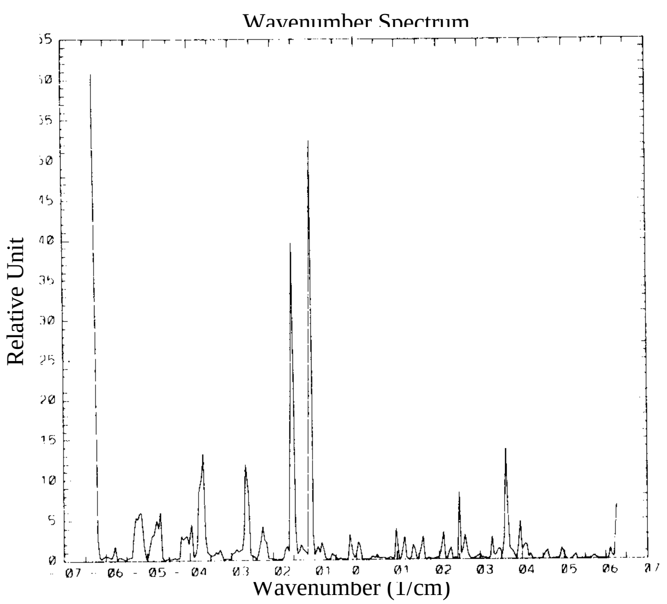

Fig.8) Measured two peaks associated with dipole and quadrupole oscillations between bunch 1 and 3 is shown.

III. CONCLUSION

From the difference in phase oscillation between bunches, the two-point correlation technique can estimate the wavenumber associated with low order short range wakefields.

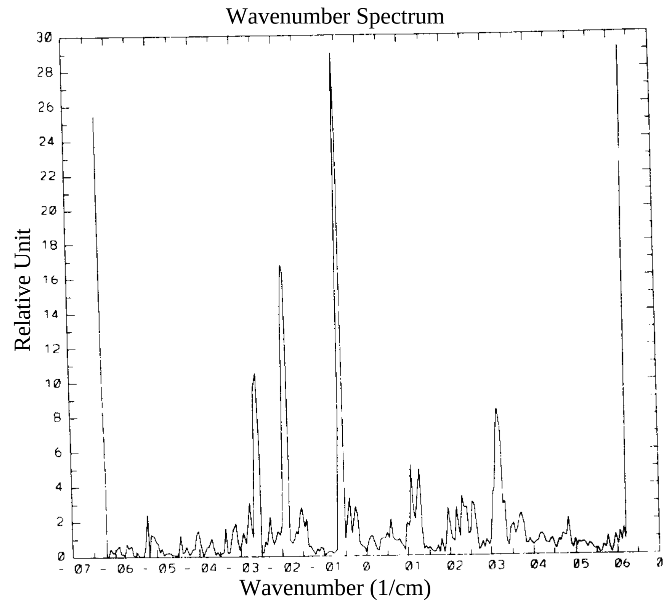

Fig.8) Three modes measured between bunch one and bunch five. m=2. Note, quadrupole mode is missing

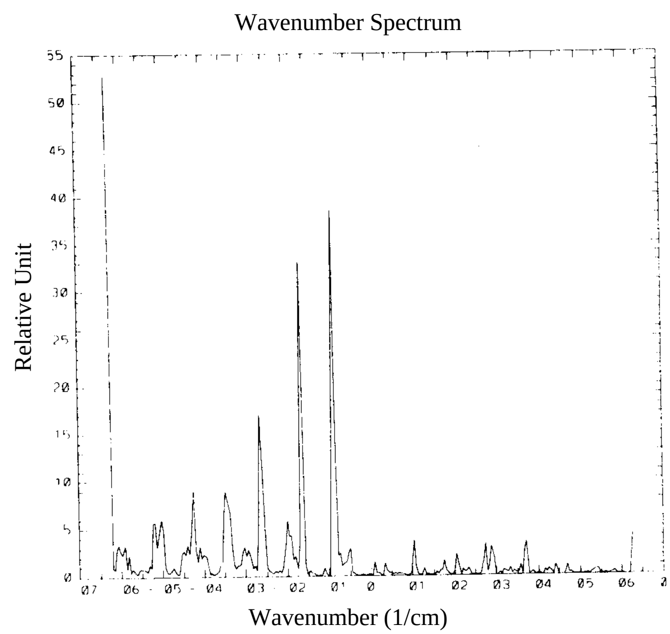

Fig.9) Three modes between bunches one and six are measured.

IV. REFERENCES

1) J.M. Beall, et al., Journal of Applied Physics, **53**, 3933 (1982).