2,5-DIMENSIONAL NUMERICAL SIMULATION OF PROPAGATION OF THE FINITE SEQUENCE OF RELATIVISTIC ELECTRON BUNCHES (REB) IN TENUOUS AND DENSE PLASMAS*

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Abstract

Particle simulation results on the wake–field excitation with the programmed finite sequence of REB in tenuous and dense plasmas are presented. The REB/background plasma configuration is described by full set of 2D3V relativistic Vlasov equations for each plasma species and nonlinear Maxwell equations for self–consistent electromagnetic field. The physical parameters in runs carried out are close to those used in laboratory experiment at Kharkov IPT (Ya. Fainberg at all, 1994). The simulation points to the fact that the reached value of electric field depends on charge distribution and choice of intervals between bunches in programmed sequence.

I. INTRODUCTION

Charged particle acceleration by means of charge density waves both in plasmas and in noncompensated charge particle beams is the major trend in collective acceleration methods [1], [2]. The charge density variable part can be made very high (up to n_e); consequently, the accelerating field can reach values on the order of $10^7 - 10^9 V/cm$. Chen et al. [3] suggested a modification of Fainberg's acceleration method [1] consisting of the use of wave trains. Katsouleas [4] considered this problem for different electron cluster profiles: a wedgelike one with a slow start growth and sharp terminal drop as well as a Gaussian-type distribution with different rise and fall rates. He presented proof [4] that the use of such inhomogenious nonsymmetric clusters, instead of the homogeneous-density ones, might ensure particle accelerating field E_{ac} values many times (10-20) greater than the electric field stopping the cluster. The so-called transformation coefficient $T = E_{ac}E_{st}^{-1} = \gamma_{ac}\gamma_b$ is equal to $T = 2\pi N - \pi N$, where N represents the number of wave-lengths along the cluster length. Excitation of nonlinear stationary waves in plasma by a sequence of periodically spaced electron bunches was studied in [5], [6], where the electric field of the wave was shown to increase as γ rises in the case of plasma n_e and bunch n_b of comparable density. Nonlinear mode experiments on acceleration by means of wake fields emphasized the importance three-dimensional effects.

There are two distinct regimes where the plasma wake fields can provide large fields which may be of use in accelerator physics. If one utilizes an appropriate density plasma the short, wide beam can be used to drive large amplitude waves with high–gradient electric fields useful for accelerating other particle bunches. On the other hand, the long, narrow beam can be strongly focused by its self–magnetic field which are left unbalanced when the plasma response neutralizes the beams space charge density.

The wake field excitation was studied using 2D3V axially symmetric electromagnetic code COMPASS [7]. Previously, this code was used to simulate an induction accelerator [8], modulated REB [9] and a single relativistic electron bunch or a sequence of such bunches in plasma [10]. Note that, as in experiments [6], the bunch initial transversal R_0 and longitudinal L_0 dimensions were smaller than c/ω_p at the REB density $n_b = \frac{1}{2}n_e$ (n_e is plasma density). The computer simulation [10] showed the transversal dimension of a bunch, propagating in plasma, to vary over a wide range.

This caused substantial changes in its density (more then order of magnitude) as well as a change in the excited wake field. It is shown too that the amplitudes of longitudinal and transversal fields increase upon additional bunch injection. However these amplitudes are not proportional to the number of injected bunches (as it should be in case of "rigid" bunches). For future experimental researches in the domain of intense microwave (wake) field excitation in plasmas and for use of these fields for charged particle acceleration it is expected to employ a new electron accelerator being constructed at KhIPT. Its parameters are as follows: energy $W = 200MeV$; number of electrons per bunch $N \sim 10^{10}$; number of bunches from 10 to 20; bunch repetition rate is $2797.16MHz$. The electron bunches will be injected into plasma of the following sizes: the length $L = 1m$ and the radius $R = 10cm$ (plasma density will be variable within $n_e = 10^{10} - 10^{14}cm^{-3}$) with a minimum density longitudinal gradient. In our numerical simulation we shall keep in mind these parameters.

II. MATHEMATICAL MODEL

The REB dynamics is described by Vlasov relativistic equations (the Belyaev–Budker equations) for the distribution function of each plasma specie and by the Maxwell equation set for self–consistent electric $\vec{E}$ and magnetic $\vec{B}$ fields. The two–component main plasma ($m_e/m_i = 1840$, where m_i and m_e are the ion and electron mass respectively) is initially cold and fills completely the considered region $[0, L] \times [0, R]$. Usually, L and R are chosen to be $100cm$ and $10cm$ respectively. A finite sequence of cold REB is injected in the plane $z = 0$ in accordance with $n_b\theta(R_0 - r)\theta(V_bt - z + (n-1)\lambda_p)\theta(z - V_bt + Z_0 + (n-1)\lambda_p)$.

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Here n denotes the number of injected bunch. The beam velocity is $V_b = c\sqrt{1-1/\gamma^2}$; c is the velocity of light; the initial bunch sizes $L_0 \times R_0$ were equal to $0.4cm \times 0.5cm$; $\lambda_p = 2\pi c/\omega_p$, ω_p is the Langmuir plasma frequency; and n_b is the REB average density. The quantity $m_e\omega_p c/e$ served as a scale for electric and magnetic fields. Bunches and plasma components can leave the considered region through two boundary surfaces: $z = 0$ and $z = Z$. The plasma components can also reenter the region. On the inner boundary the following conditions exist: a metal surface at $r = R$ and open for electromagnetic wave radiation face and rear butt-end surfaces. Explicit scheme were used in calculations.

Four runs were considered to study the dependence of the exited fields: upon the number N of bunches injected into plasma; on the density ratio of bunch and plasma; on the bunch repetition rate; on the ratio of the bunch size R_0 and skin–depth c/ω_p. The parameters of these runs are listed in the table:

run id		Var1	Var2	Var3	Var4
bunch density	n_b cm^{-3}	$2\cdot 10^{10}$	$2\cdot 10^{10}$	$4.86\cdot 10^{10}$	$4.86\cdot 10^{10}$
plasma density	n_e cm^{-3}	$4\cdot 10^{10}$	$4\cdot 10^{11}$	$9.72\cdot 10^{10}$	$8.75\cdot 10^{11}$
plasma frequency	ω_p c^{-1}	$1.13\cdot 10^{10}$	$3.57\cdot 10^{10}$	$1.76\cdot 10^{10}$	$5.27\cdot 10^{10}$
skin-depth	c/ω_p cm	2.66	0.84	1.71	0.57
particles per bunch	N	$6.28\cdot 10^{9}$	$6.28\cdot 10^{9}$	$1.53\cdot 10^{10}$	$1.53\cdot 10^{10}$

The weight of model particles was assumed to be a function of their radial position. The plasma was assumed to have smaller numbers of particles in the less disturbed region distant from the axis. The total number of macro particles was about 10^6. Note that all the calculations were carried out using a PC/Pentium–66 computer and a high-speed particle–in–cell technique.

III. RESULTS

The computer simulation showed the transversal dimension of the bunch, propagating in plasma, to vary over a wide range at the conditions $R_0 < c/\omega_p$ and $L_0 < c/\omega_p$. Contrary to popular consideration (with $L_0 \gg c/\omega_p > R_0$ or $R_0 \gg c/\omega_p > L_0$) we considered the conditions $L_0 \sim R_0 < c/\omega_p$ or $L_0 \sim R_0 \sim c/\omega_p$, which is corresponding to the experimental situation [11]. In these cases significant nonlinearities in both plasma and beam behavior have been observed. As we shall see subsequently in our simulation the ion motion plays significant role in the REB propagation in the plasmas. The dependence of the ion density n_i upon the radial coordinate r is presented in Fig.1 and Fig.2. One can see that ions form the plasma channel due to their transversal motion in self–consistent fields. The channel parameters are determined by density ratio of bunch and plasma, and by size ratio of bunch radial size R_0 and skin–depth c/ω_p.

The spatial distribution of the longitudinal E_z and radial E_r electric field is presented on Fig.3, 4, 5 and 6. One can see that amplitudes of E_z and E_r increase with injection of each additional bunch. However, these amplitudes are not proportional to the number of injected bunch as this should be in the case of "rigid" bunches. This is due to transversal oscillations of bunch particles in self–consistent fields caused by the lack of charge and current compensation.

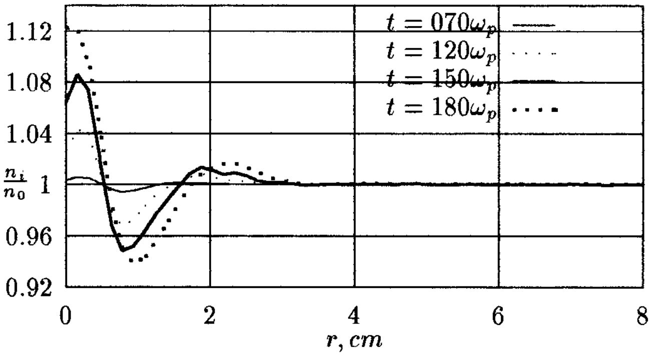

Figure 1. Var1: Dependence of the ion density n_i upon the radial coordinate r ($z = 10.cm$)

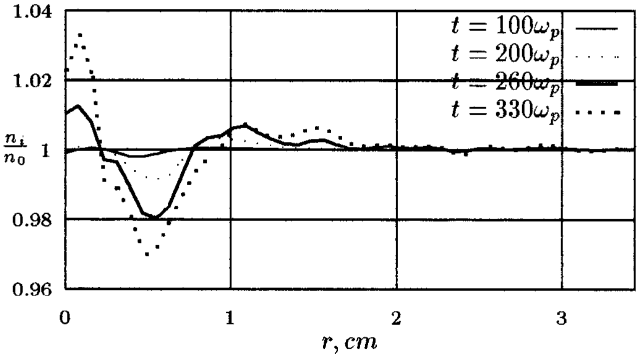

Figure 2. Var4: Dependence of the ion density n_i upon the radial coordinate r ($z = 10.cm$)

The shape of bunches are presented in Fig. 7. One can see that the envelope shape in not close to Bennet equilibrium case. However the REB expansion is appreciably retarded by the formation of the plasma channel with the ion radial motion. The conduct consideration of the three–dimensional nonlinear bunch–plasma behavior is useful for better understanding of fundamental physics of the plasma wake–field acceleration and focusing.

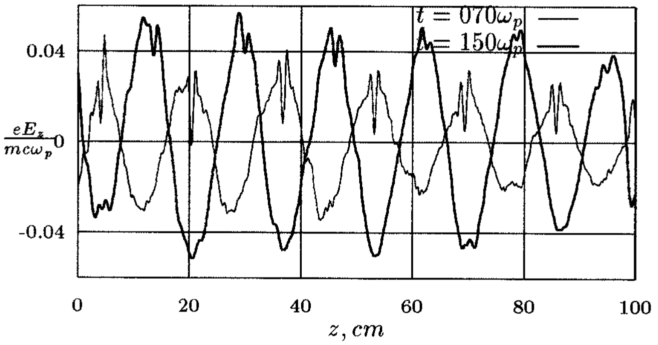

Figure 3. Var1: Dependence of the longitudinal electric field E_z on z ($r = R_0 = 0.5cm$)

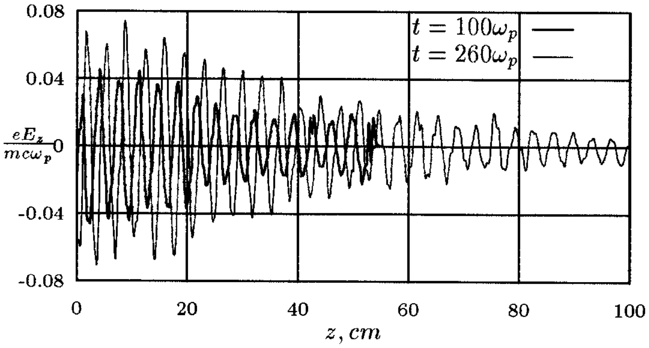

Figure 4. Var4: Dependence of the longitudinal electric field E_z on z ($r = R_0 = 0.5cm$)

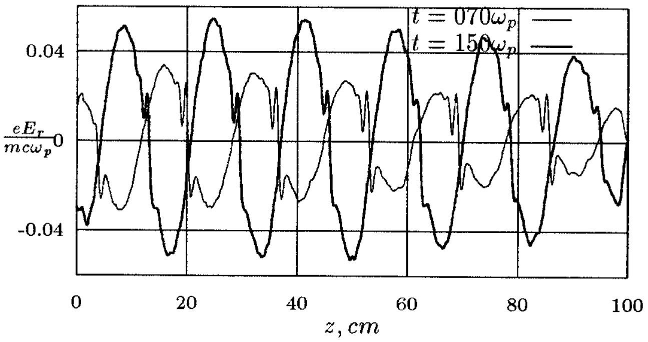

Figure 5. Var1: Dependence of the radial electric field E_r on z ($r = R_0 = 0.5cm$)

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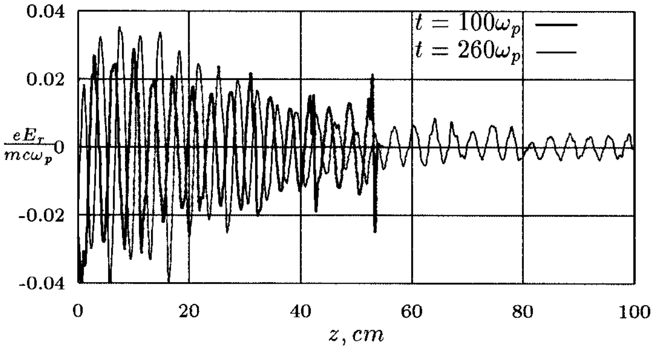

Figure 6. Var4: Dependence of the radial electric field E_r on z ($r = R_0 = 0.5cm$)

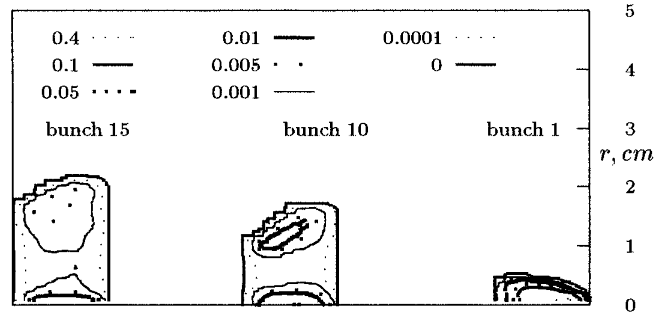

Figure 7. Var1: Density cross–section of bunches (number 1,10 and 15) at moment of passing over $z = 60$.

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