INVARIABILITY OF INTENSE BEAM EMITTANCE IN NONLINEAR FOCUSING CHANNEL

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Abstract

New approach to keep emittance of a high current beam in a uniform focusing channel is presented. The matching conditions for a beam with arbitrary distribution function in a nonlinear focusing channel are examined. To obtain proper matching, it is necessary to accept that the potential of the external focusing field contains higher order terms than quadratic. The solution for external potential is obtained from the stationary Vlasov's equation for beam distribution function and Poisson's equation for electrostatic beam potential. An analytical approach is illustrated by results of a particle-in-cell simulation.

I. INTRODUCTION

The nonlinear space charge field of a beam is a serious concern for beam emittance growth in the low energy part of an accelerating facility. This effect is most pronounced in the injection region where particles are slow and space charge forces are significant. The problem of beam emittance growth due to nonstationary beam profile in a focusing channel with a linear focusing field was treated in many papers (see ref. [1-9] and cited references there). The general property of space charge dominated beam behavior is that a beam with an initial nonlinear profile tends to be more uniform and this process is associated with strong emittance growth and the appearance of beam halo.

The beam emittance is conserved if the beam is matched with the channel. The problem of matching of the nonlinear density profiled beam with linear uniform focusing channel was studied in detail in ref. [9-12]. The analytical approach is based on the fact that the Hamiltonian of the matched beam is a constant of motion, and therefore the unknown distribution function can be expressed as a function of the Hamiltonian. A general property of the solution to problem is that with increasing beam current, the profile of the matched beam has to be more and more flat while the phase space projection (beam emittance) has to be more and more close to a rectangle.

Laboratory beams are usually far from the above solution and suffer serious emittance growth. The purpose of this paper is to check whether it is possible to match the beam with a given distribution function with the uniform focusing channel. As is shown below, it is possible if we assume that the focusing field includes higher order terms than quadratic [13].

II. MATCHED CONDITIONS FOR THE BEAM WITH GIVEN DISTRIBUTION FUNCTION

The procedure to find the matching conditions for a beam with an arbitrary distribution function was discussed in ref. [13]. Let us assume that the beam is matched with the channel. Hence, the Hamiltonian is a constant of motion but no assumptions about linearity of focusing forces are adopted:

$$H = \frac{p_x^2 + p_y^2}{2m} + q\, U(x,y) = \text{const} \qquad (1)$$

The total potential of the structure is a combination of the external focusing potential, U_{ext}, and the space charge potential U_b of the beam, $U = U_{ext} + U_b$. The time-independent distribution function of a matched beam obeys Vlasov's equation:

$$\frac{df}{dt} = \frac{\partial f}{\partial x} v_x + \frac{\partial f}{\partial y} v_y - q \left(\frac{\partial f}{\partial p_x}\frac{\partial U}{\partial x} + \frac{\partial f}{\partial p_y}\frac{\partial U}{\partial y}\right) = 0 \qquad (2)$$

where the partial derivative of the distribution function over time is omitted due to initial matched conditions. The distribution function of the beam is supposed to be given from the source of particles of the beam. Therefore, the self potential of the beam U_b is also a known function derived from Poisson's equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial U_b}{\partial r}\right) = -\frac{\rho(r)}{\varepsilon_o} \qquad (3)$$

where $\rho(r)$ is the space charge density of the beam. Combining solutions of Vlasov's equation for total potential of the structure, U, and space charge potential of the beam, U_b, obtained from Poisson's equation, the external potential of the focusing structure can be found:

$$U_{ext} = U - U_b. \qquad (4)$$

The solution of this problem is unique for every specific particle distribution.

III. EXAMPLE OF THE MATCHED BEAM WITH NONLINEAR SPACE CHARGE FORCES

Let us consider a z-uniform beam with a "parabolic" distribution function in four -dimensional phase space:

$$f = f_o \left(1 - \frac{x^2 + y^2}{2R^2} - \frac{p_x^2 + p_y^2}{2p^2}\right) \qquad (5)$$

This distribution makes an elliptical phase space projection at every phase plane and produces a decrease function of space charge density from axis which is close to experimentally observed beam. The normalized root-mean-square (RMS) beam emittance is:

$$\varepsilon = \frac{4}{m\,c}\sqrt{<x^2><p_x^2> - <xp_x>^2} = \frac{R\,p}{m\,c} \qquad (6)$$

Substituting the distribution function (5) into Vlasov's equation yields an expression for the total unknown potential of the structure:

$$\frac{mc^2}{q}(x\,p_x + y\,p_y) = \frac{R^4}{\varepsilon^2}\left[p_x\frac{\partial U}{\partial x} + p_y\frac{\partial U}{\partial y}\right] \qquad (7)$$

Vlasov's equation can be separated into two independent parts for x- and y- coordinates respectively:

$$\frac{\partial U}{\partial x} = \frac{mc^2\,\varepsilon^2}{q\,R^4}\,x\,; \quad \frac{\partial U}{\partial y} = \frac{mc^2\,\varepsilon^2}{q\,R^4}\,y \qquad (8)$$

Combining solutions of eq. (8), the total potential of the structure is a quadratic function of coordinates which creates linear focusing:

$$U(x,y) = \frac{mc^2}{q}\,\frac{\varepsilon^2}{R^4}\left(\frac{x^2+y^2}{2}\right) \qquad (9)$$

The appearance of quadratic terms in the total potential of the structure is quite clear because phase space projections of the beam have elliptical shape and an ellipse is conserved in a linear field. The space charge field of the beam E_b is calculated from Poisson's equation using a known space charge density function of the beam ρ_b:

$$\rho_b = \frac{3\,I}{2\pi c\beta\,R^2}\left(1 - \frac{r^2}{2\,R^2}\right)^2 \qquad (10)$$

$$E_b = \frac{3\,I\,r}{4\pi\varepsilon_o\,\beta c R^2}\left[1 - \left(\frac{r}{\sqrt{2}R}\right)^2 + \frac{1}{3}\left(\frac{r}{\sqrt{2}R}\right)^4\right] \qquad (11)$$

where I is the beam current and β is the longitudinal velocity of particles. Subtraction of the space charge field from the total field of the structure gives the expression for the external focusing field of the structure which is required for conservation of beam emittance:

$$E_{ext} = -\frac{mc^2\,r}{q\,R^2}\left[\frac{\varepsilon^2}{R^2} + \frac{3\,I}{I_c\,\beta}\left(1 - \frac{r^2}{2\,R^2} + \frac{r^4}{12\,R^4}\right)\right] \qquad (12)$$

where $I_c = 4\pi\varepsilon_o mc^3/q$ = A/Z $3.13\cdot 10^7$ amp is a characteristic value of the beam current. The relevant potential of the focusing field is given by the expression:

$$U_{ext} = \frac{m\,c^2}{q}\left[\frac{r^2}{2R^2}\left(\frac{\varepsilon^2}{R^2} + \frac{3\,I}{I_c\,\beta}\right) + \frac{3\,I}{8\,I_c\,\beta}\left(-\frac{r^4}{R^4} + \frac{r^6}{9\,R^6}\right)\right] \qquad (13)$$

Let us note that the external potential of the structure consists of two parts: quadratic (which produces linear focusing) and higher order terms which describe nonlinear focusing. The linear part depends on the values of beam emittance and beam current while the nonlinear part depends on beam current only. This means that the external field has to compensate the nonlinearity of self-field of the beam and produce required linear focusing of the beam to keep the elliptical beam phase space distribution. Fig. 1 illustrates the relationships between space charge field of the beam, total field, and focusing field of the structure. The external focusing field obtained from the above consideration is a complicated function of radius which is linear near the axis and becomes nonlinear far from the axis. One of the ideal ways to create the required focusing potential is to introduce inside the transport channel an opposite charged cloud of heavy particles with the space charge density:

$$\rho_{ext} = \frac{I_c}{2\pi c\,R^2}\left[\frac{\varepsilon^2}{R^2} + \frac{3\,I}{\beta\,I_c}\left(1 - \frac{r^2}{2\,R^2}\right)^2\right] \qquad (14)$$

In fig. 2 the charged particle density of the transport beam and the external focusing beam are presented.

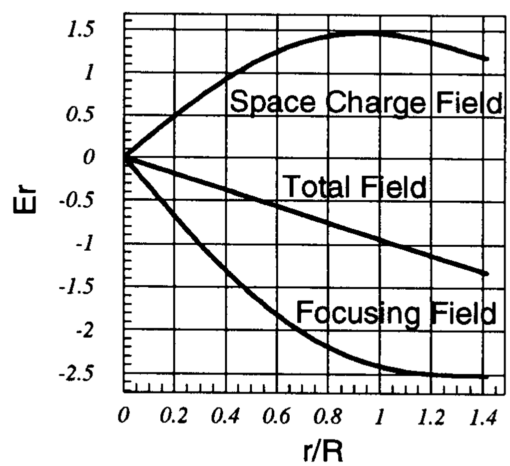

Fig.1. Space charge field of the beam, external focusing field, and total field of the structure.

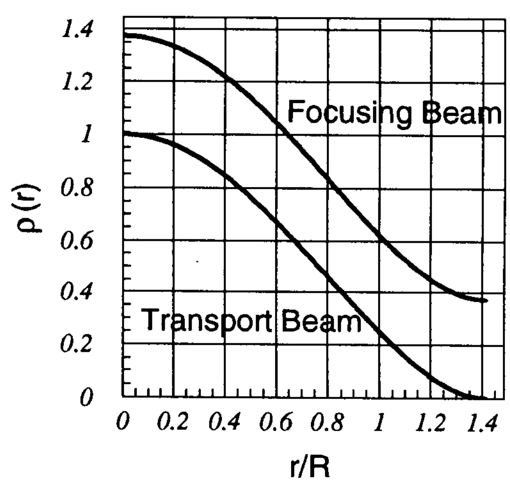

Fig. 2. Charged particle density of the transport beam and the external focusing beam.

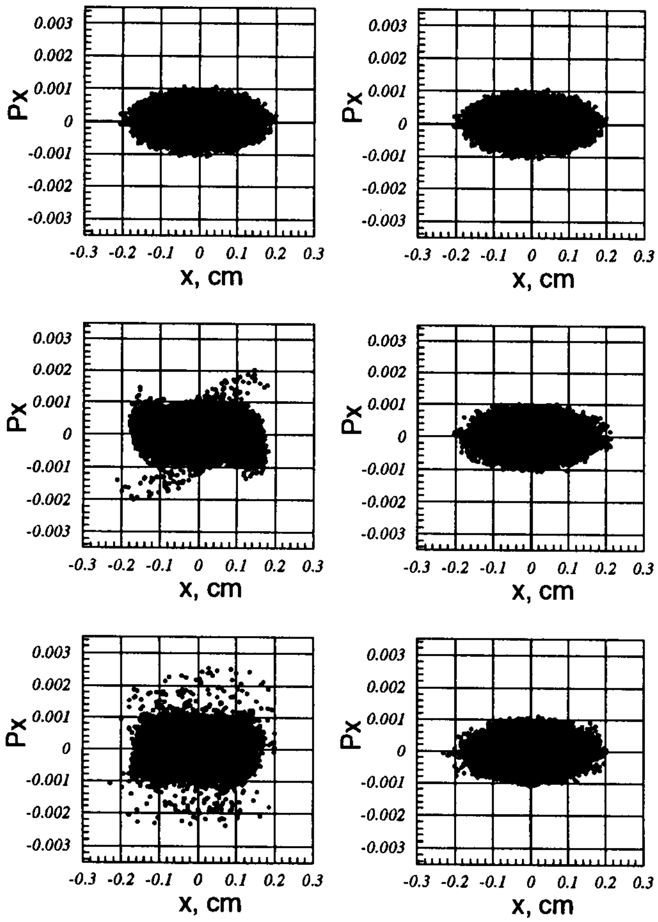

Fig. 3. Halo formation of the beam in focusing channel with linear focusing forces (left column) and perfect matching of the same beam with nonlinear focusing channel(right column).

At fig. 3 the results of particle-in-cell simulation of the beam in linear and nonlinear focusing channel using code BEAMPATH [14] are presented. A beam of particles was represented as a collection of 10000 trajectories. Space charge field of the beam was calculated from Poisson's equation on the uniform rectangular meshes of dimension NX x NY = 256 x 256. The external focusing potential for the linear focusing channel was chosen as

$$U_{ext}(r) = \frac{m\,c^2}{q}\left(\frac{\varepsilon^2}{2\,R^4} + \frac{I}{I_c\,\beta\,R^2}\right) r^2 \qquad (15)$$

which corresponds to the matched conditions for an equivalent KV beam with the same RMS beam emittance, ε, and RMS beam size, R. In the case of nonlinear focusing, the external potential is represented by eq. (13). Let us note that quadratic terms in potentials (13) and (15) are different.

From results of simulations, it is seen that in both cases the sizes of the beam in real space (beam envelopes) are close to constant which is typical for matching of the beam, taking into account RMS beam sizes. But in the case of linear focusing, the beam is mismatched in the phase plane which results in 25% emittance growth accompanied by halo formation. At the same time, the beam is completely matched with the nonlinear focusing channel, and this results in conservation of all beam characteristics and does not suffer any serious emittance growth.

IV. CONCLUSIONS

Conservation of beam emittance was treated as a problem of proper matching of the beam with a uniform focusing channel. Matched conditions for the beam with elliptical phase space projections but nonlinear space charge forces in a uniform focusing channel require the focusing field to include nonlinear terms of higher order than quadratic. The solution for the external potential is attained from the stationary Vlasov's equation for beam distribution function and Poisson's equation for electrostatic beam potential. The focusing field produces linear focusing near the axis of the structure but has to change non-linearly away from the axis. Example of the beam with "parabolic" distributions in 4D phase space was considered. Results of a particle-in-cell simulation confirms the conservation of beam emittance in a nonlinear external field.

V. REFERENCES

[1]. P.M.Lapostolle, IEEE Trans. Nucl. Sci. NS-18, 1101 (1971).
[2]. J.Struckmeier, J.Klabunde, and M.Reiser, Part. Accel. 15, 47 (1984).
[3]. T.P.Wangler, K.R.Crandall, R.S.Mills, and M.Reiser, IEEE Trans. Nucl. Sci. NS-32, 2196 (1985).
[4]. R.A.Jameson, Proceedings of the PAC93, Washington D.C., 3926 (1993).
[5]. J.S.O'Connell, T.P.Wangler, R.S.Mills and K.R.Crandall, Proceedings of the PAC93, Washington D.C., 3657 (1993).
[6]. R.D.Ryne, Proceedings of the PAC93, Washington D.C., 3229 (1993).
[7]. J.M.Lagniel, Nucl. Instr. and Meth. A 345, 46 (1994); Nucl. Instr. and Meth. A 345, 405 (1994).
[8]. R.L.Gluckstern, Proceedings of the 1994 International Linac Conference, Vol. 1, 333, Tsukuba, Japan (1994).
[9]. M.Reiser, Theory and Design of Charged Particle Beams, Wiley, New York, 1994.
[10]. I.M.Kapchinsky, Theory of Resonance Linear Accelerators, Moscow, Atomizdat, 1966, Harwood, 1985.
[11]. P.Lapostolle, YII Int. Conference on High Energy Accelerators, Yerevan, 205 (1969).
[12]. J.Struckmeier and I.Hofmann, Part. Accel., 39, 219 (1992).
[13]. Y.Batygin, Proceedings of the 1994 International Linac Conference, Vol. 1, 487, Tsukuba, Japan (1994).
[14]. Y.Batygin, Proceedings of the EPAC92, Berlin, 822 (1992).