TRANSVERSE INSTABILITY ANALYSIS FOR THE IPNS UPGRADE *

K. Harkay and Y. Cho, Argonne National Laboratory, Argonne, IL 60439 USA

Abstract

The proposed 1-MW spallation neutron source upgrade calls for a 2-GeV rapidly-cycling synchrotron (RCS) with an intensity of 1.04x10^{14} protons per pulse [1]. The potential exists for the excitation of collective, intensity-dependent transverse instabilities. These can normally be controlled by introducing a betatron tune shift or spread, where care is exercised to avoid single-particle resonance effects. Adjusting the chromaticity using sextupoles to vary the head-to-tail phase shift is compared to introducing Landau damping by octupoles. An option for a feedback system is also examined. The momentum spread used for the transverse analysis was obtained from the requirements for longitudinal stability [2].

I. INTRODUCTION

The head-tail effect has been observed at other low energy, rapidly-cycling synchrotrons: the 500-MeV IPNS [3], KEK Booster [4], and ISIS [5]. Therefore, it is analyzed in some detail for the IPNS Upgrade. An estimation of the coupling impedance, which characterizes the interaction between the beam and its surroundings, is given first. The head-tail instability, driven by the resistive wall and kicker impedance, is discussed next using Sacherer's bunched beam formalism [6]. Further discussion of damping transverse instabilities considers the coasting beam stability criterion. The requirements for stability are dominated by the impedance due to space charge effects. It is found that passive means, which include minimizing the coupling impedance and introducing a tune shift or spread, are sufficient to ensure stability. Finally, the requirements for an active feedback kicker are given as an option.

II. COUPLING IMPEDANCE ESTIMATION

The transverse coupling impedance is estimated by using the standard approximations [7]. The impedance is dominated by space charge effects. The rf shield, extraction kickers, and the beam position monitors (BPMs) also contribute to the impedance. The kickers are traveling-wave structures with a characteristic impedance of 3.28 Ω. The impedance due to other components, such as vacuum ports and bellows, is expected to be negligible because these are isolated from the beam by the rf shield. The contribution to the transverse impedance due to the rf cavity higher-order modes was found by URMEL-T calculations to be negligible. The coupling impedance of interest for instability analysis corresponds to the frequencies $\omega = (p+\nu)\omega_0$, or about 1 MHz for the RCS, where p is a negative integer near ν, the horizontal or vertical tune. The results are summarized in Table 1 and correspond to injection energy (400 MeV), unless otherwise noted. The space charge impedance is purely capacitive, while the others are inductive and include a resistive term.

Table 1: RCS Transverse Impedance Estimation (at 1 MHz)

	$\mathrm{Re}(Z_\perp)$ (kΩ/m)	$\mathrm{Im}(Z_\perp)$ (kΩ/m)
Space charge (injection)	–	– 2800
Space charge (extraction)	–	– 1500
Rf shield	20	20
Extraction kickers	30	50
BPMs	0.003	110

To minimize the impedance due to space charge, the vacuum chamber is constructed with a special rf shield, similar in principle to that used at ISIS [1,2]. The shield consists of a Be-Cu wire cage which follows the beam envelope. The space charge impedance is calculated using the standard assumption of a uniform, round, unbunched beam of radius a in a vacuum chamber of radius b. The geometrical factor is given by $g_\perp = 1/a^2 - 1/b^2$. Compared to a fixed-radius rf shield, this contour-following scheme reduces the transverse space charge impedance by 35% at injection and 10% at extraction.

A number of corrections to the geometrical factor have been proposed to account for the wires and the more realistic elliptical beam cross section. The electrostatic fields due to a uniform beam propagating inside a round rf-screening wire cage have been derived by T. Wang [8]. A second correction, derived by H. Okamoto, takes into account the varying elliptical shape of beam in a smooth, metallized vacuum chamber without wires and with a fixed radius [9]. Wang's method results in a +15% correction in the transverse impedance and Okamoto's method results in a –20% correction [10]. Until a rigorous derivation is performed for a contour-following wire rf shield, the above corrections are treated as an uncertainty.

III. HEAD-TAIL INSTABILITY

In the absence of a tune spread, the growth rate for the head-tail instability, assuming a time dependence of $\exp(j\Omega_m t)$, is given by the imaginary part of [6]

$$\Omega_m = \frac{j}{(1+m)} \frac{c\,I\,Z_\perp(\omega_p)}{4\pi\nu(E/e)} F'_m \,,$$

$$\text{where } F'_m = \frac{1}{B} \frac{h_m(\omega_p - \omega_\xi)}{\sum_p h_m(\omega_p - \omega_\xi)}$$

is a form factor and m is a mode number denoting the number of nodes along the bunch. This expression assumes that the instability is dominated by the contribution from the impedance at a single frequency, $\omega_p = (p+\nu)\omega_0$. The total growth involves a convolution of this impedance with the envelope (form factor) of the bunch spectra, h_m, which are

* Work supported by U.S. Department of Energy, Office of Basic Energy Sciences under Contract No. W-31-109-ENG-38.

shifted by the chromatic frequency, $\omega_\xi = (\xi/\eta)\nu\omega_0$. The chromaticity is $\xi = (\Delta\nu/\nu)/(\Delta p/p)$ and B is the ratio of bunch length to ring circumference.

For the RCS at 400 MeV, the form factor for $m = 0$ has a half-width of about 2 MHz, and the form factor for $m = 1$ peaks around 1 MHz. The chromaticity is normally corrected to zero in order to avoid a large betatron tune spread in the large-momentum particles. When the chromaticity is zero, the bunch spectra are centered at zero frequency, where the rf shield and kicker impedances are important.

In the vertical plane, the largest instability growth due to the rf shield and kicker impedances occur when p = –6 and –7, respectively. Figure 1 shows the impedances and the frequency, ω_p, for the two values of p. The BPM impedance does not contribute significantly to the growth. The growth rates of the first few head-tail modes are given in Table 2 for the corrected and natural chromaticities at the end of injection. The impedance and the frequency giving the largest growth are also listed. Similar results are found for the horizontal plane.

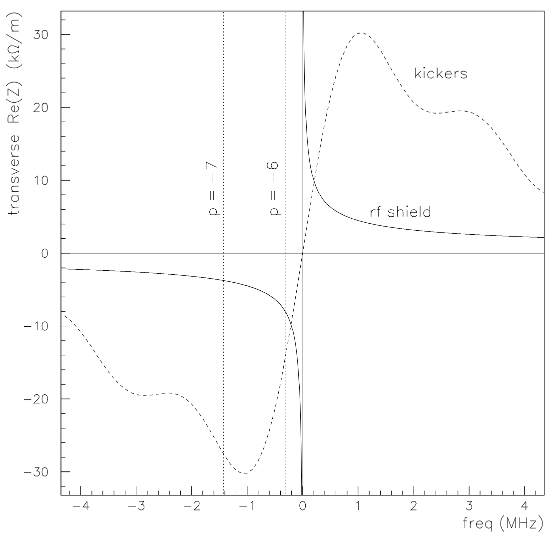

Figure 1: Real part of the transverse impedance due to the rf shield and kickers.

Table 2: Head-tail instability growth rate vs. chromaticity.

mode number, m	$\mathrm{Re}(Z_\perp)$ (kΩ/m)	frequency (MHz)	source of imped-ance	growth rate (s)$^{-1}$ $\xi = 0$	$\xi = -1.2$
0	8	0.3	rf shield	350	0
0	14	0.3	kickers	620	0
1	28	1.4	kickers	730	0
2	28	1.4	kickers	590	0

The lowest modes are stabilized at the natural chromaticity, $\xi = -1.2$. Using the maximum value of $\Delta p/p$ of 1.2% obtained from the longitudinal tracking [2], the corresponding betatron tune spread is 0.1. With tune values of ν_x=6.821 and ν_y=5.731 and an incoherent Laslett space charge tune shift of about 0.15, one must consider the half-integer resonance. Therefore, the chromaticity is adjusted to be between 0 and –1.2 to shift the bunch spectrum to higher-order head-tail modes, which have lower growth rates, while at the same time ensuring that the tune spread remains inside a resonance-free working region. The stability of modes $m \le 2$ is assured with $\xi \le -0.35$. This gives a betatron tune spread of 0.024. The inverse growth rate of the first unstable mode, m = 3, is then about 14 ms. This gives less than two e-folding times during the total acceleration cycling time of 25 ms.

IV. STABILITY CRITERION

Discussed next is the transverse stabilization of the beam, achieved by using a betatron tune spread or shift. An amplitude-dependent tune spread is introduced by octupoles and second-order effects in the sextupoles, leading to Landau damping. A momentum-dependent tune shift results from a finite chromaticity, leading to stability as discussed above. The stability criterion derived for coasting beams is extended heuristically to bunched beams, and is given by

$$(\Delta\nu)_{\mathrm{thresh}} = \frac{c\, I_{pk} |Z_\perp|}{4\pi\nu\omega_0 (E/e)} \ .$$

Since the impedance is dominated by space charge effects, this expression is equivalent to the coherent Laslett tune shift (due to image charges only.) For the RCS, the expected growth rates (Table 2) are of the order of the synchrotron frequency, ω_s; therefore, we use the peak current, I_{pk}, to be conservative. The results for the threshold tune spread are presented in Table 3 for the vertical plane at 400 MeV and 2 GeV. The impedance is taken from Table 1.

Table 3: Threshold tune spread according to the coasting beam stability criterion

time in cycle (ms)	$\mathrm{Im}\, Z_\perp$ (MΩ/m)	$(\Delta\nu)_{\mathrm{thresh}}$
0	2.7	0.06
25	1.4	0.03

The threshold tune spread can be met using the chromatic contribution and/or the octupolar contribution. Our strategy requires that each term satisfy the stability criterion independently, if possible.

First, we consider a solution using the chromatic term only, where $\Delta\nu_\xi = |(p-\nu)\eta + \nu\xi|(\Delta p/p)$. Using the condition $\Delta\nu_\xi \ge \Delta\nu_{\mathrm{thresh}}$ and the natural chromaticity, with p = 6 (vertical plane), the required momentum spread becomes:

$$\frac{\Delta p}{p} \ge \begin{cases} 0.8\% & 400\ \mathrm{MeV} \\ 0.6\% & 2\ \mathrm{GeV} \end{cases} .$$

This is to be compared to a $\Delta p/p$ of about 1% obtained from the longitudinal tracking study. Conversely, using the $\Delta p/p$ from the tracking, the required chromaticity becomes:

$$\xi \le \begin{cases} -0.97 & 400 \text{ MeV} \\ -0.66 & 2 \text{ GeV} \end{cases} .$$

Next, we consider a solution using octupoles only with the chromaticity corrected to zero. The required octupole tune spread is $\Delta\nu_{\rm oct} \ge \Delta\nu_{\rm thresh}$, where $\Delta\nu_{\rm oct} = \left(\beta_y a_y^2 / 32\pi\right) k_{\rm oct}$, and $k_{\rm oct} = \left(B''' l / B\rho\right)$ is the octupole strength. Using a beta-function, β_y, of 12 m and beam size, a_y, of 0.07 m at 400 MeV and 0.04 m at 2 GeV, the integrated octupole strength per super-period (= 4) required for stability is given by

$$k_{\rm oct} \ge \begin{cases} 24 \text{ m}^{-3} & 400 \text{ MeV} \\ 44 \text{ m}^{-3} & 2 \text{ GeV} \end{cases} .$$

This assumes that the octupoles are located at each defocusing quadrupole (QD). The approximate limiting value of the octupole strength at injection is shown in a plot of the dynamic aperture in Figure 2. The dynamic aperture with full-strength sextupoles and no octupoles is also shown.

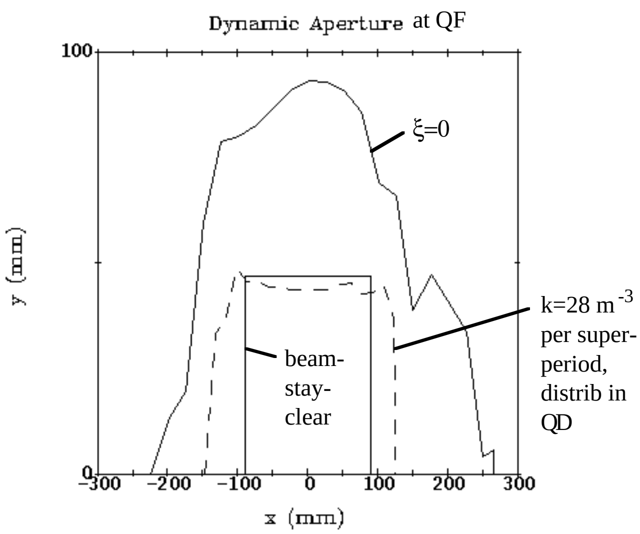

Figure 2: Dynamic aperture at a focusing quadrupole (QF).

V. BEAM FEEDBACK SYSTEM

Active beam feedback is an option to suppress or damp the head-tail instability. A typical feedback system consists of a beam position detector, processing electronics and a delay unit, a power amplifier, and a deflection kicker. Other considerations for the system include noise sensitivity and maintaining synchronicity with the rapidly-varying revolution frequency. Only the kicker strength is analyzed. The parameters of the feedback system have not been optimized, nor have space limitations been studied.

Assuming an instability rise time of 1 msec and a revolution period of 1 µs, the instability amplitude growth is $\Delta x / x_0 = \tau_{\rm rev} / \tau_{\rm growth} = 10^{-3}$, where x_0 is the minimum detected beam deviation, assumed to be 0.5 mm. The electric field, E_k, for the kicker can be written as:

$$E_k \ge 10^{-3} \frac{x_0}{\beta_x} \frac{2\beta^2 \left(E_b / e\right)}{l} ,$$

where E_b is the beam energy and l is the length of the kicker. This gives 230 V/m for a 0.5 m kicker length at 400 MeV, and 950 V/m at 2 GeV, using the average value β_x = 6 m. It is customary in practice to allow a factor of 5 to 10 increase in the electric field.

VI. SUMMARY

An analysis of transverse instabilities in the RCS for the IPNS Upgrade is performed, including an estimation of the coupling impedance. The results show that stability is achievable through a choice of the chromaticity between the natural value and zero or through the addition of octupoles. Beam feedback is also an option. It has long been observed in other low-energy accelerators that the current required in the correction octupoles is found to be less than that predicted by the theory [4,11]. In other studies, the head-to-tail phase shift and form factors observed are not consistent with the theory [5]. Further refinement of the theory for bunched beams is required.

The authors would like to thank E. Crosbie for the dynamic aperture calculation and figure.

REFERENCES

[1] "IPNS Upgrade: A Feasibility Study," ANL-95/13, (April 1995).

[2] K. Harkay, Y. Cho, E. Lessner, "Longitudinal Instability Analysis for the IPNS Upgrade," these proceedings.

[3] Y. Cho and A.V. Rauchas, "Observations and Cure of the Head-Tail Effect in the Argonne Rapid Cycling Synchrotron," *IEEE Trans. Nucl. Sci.*, **NS-28** (3), 2585, (1981).

[4] Y. Kimura, et al, "Transverse Coherent Instability in the KEK Booster," *Proc. 10th Int'l Conf. High Energy Accel.*, **2**, 30, (1977).

[5] G.H. Rees, "Interpretation of the Higher-Order Mode, Head-Tail Motion Observed on ISIS," *Particle Accelerators*, **39**, 159, (1992).

[6] B. Zotter and F. Sacherer, “Transverse Instabilities of Relativistic Particle Beams in Accelerators and Storage Rings,” CERN 77-13, (1977).

[7] K.-Y. Ng, “Fields, Impedances, and Structures,” *The Physics of Particle Accel.*, AIP Conf. Proc. 184, (1989).

[8] T-S. F. Wang, “Electrostatic Field of a Perturbed Beam with RF-Screening Wires,” CERN/PS 94-08 (DI), (1994).

[9] "AUSTRON Accelerator Feasibility Study," AUSTRON/PS/PJB, Chapter 4, p. 145, (October 1994).

[10] K.C. Harkay, "Study of Corrections to the Geometrical Factor in the Space Charge Impedance for the IPNS Upgrade," ANL Report NS-95-4, (March 1995).

[11] D. Möhl and H. Schonauer, “Landau Damping by Non-Linear Space-Charge Forces and Octupoles,” *Proc. 9th Intl. Conf. High Energy Accel.*, 380, (1974).