

# Digital Signal Processing for the APS Transverse and Longitudinal Damping System\*

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## Abstract

The Advanced Photon Source (APS) at Argonne National Laboratory will be a 7-GeV machine. It is anticipated that for beam operations beyond the baseline design of 100 mA stored beam current, a transverse and longitudinal damping system is needed to damp instabilities. A key part of this digital damping system is digital signal processing. This digital system will be used to process samples taken from the beam and determine appropriate correction values to be applied to the beam. The processing will take the form of a transversal digital filter with adaptable filter weights. Sampling will be done at 176 MHz with a possible correction bandwidth of 88 MHz. This paper concentrates on the digital processing involved in this system, and especially on the adaptive algorithms used for determining the digital filter weights.

## I. INTRODUCTION

If there are no interactions between circulating bunches of a synchrotron, the motion of each bunch can be described by three harmonic oscillators corresponding to the three tune frequencies. In real synchrotrons, coupling will be present, and proper description is in terms of the normal modes. If there are  $N$  bunches, there will be  $N$  modes and  $N$  tunes. It turns out however, that in many practical cases the tune shift from the non-coupled frequency is either nearly the same for all modes, or very small. Thus, all coupled modes can be taken as having the same frequency. All coupled bunches can thus be described, as in the uncoupled case, as three harmonic oscillators characterized by the same three tune-shifted frequencies.

This is the case in the APS storage ring [1]. In one simulation of the resistive wall instability, 54 evenly spaced bunches were assumed circulating in the ring to achieve the maximum design current of 300 mA. The fractional vertical tune,  $\nu_y=0.3$ , was reduced by an average of 6% for the 54 modes. The tune spread however, was only 0.7%. Under the same circumstances, a growth rate of 400/s in the longitudinal motion due to a single cavity higher-order mode (HOM) will result in a maximum of 2% tune shift in an affected coupled bunch mode. In the APS ring, HOM-induced longitudinal growth rates are expected to usually be below 200/s resulting in a maximum tune spread of 2%.

Since all bunches can be treated as having the same three tune frequencies, a bunch-by-bunch damping system can be implemented with one filter for each of the two transverse

tunes and a third for the longitudinal tune. For APS the same digital transversal filter design can be used for all three tunes. This paper deals with the design of filter coefficients.

It is natural to use a single pickup for this type of system. One stripline can readily be used to measure the two transverse displacements and longitudinal phase of each bunch on each turn. Two turns of information can be used to calculate the kick which needs to be applied to the bunch in order to cancel its transverse velocity. Let

$$y_1 = A \sin \phi \quad (1)$$

be the displacement of a bunch on one turn. On the next turn the displacement will be

$$y_2 = A \sin(2\pi\nu + \phi), \quad (2)$$

where  $\nu$  is the fractional tune. The transverse velocity of this bunch at the kicker will be

$$y_3 = B \cos(2\pi\nu + \phi + \kappa), \quad (3)$$

where  $\kappa$  and  $B$  are determined from the betatron amplitude and phase considerations. It is straightforward to show that

$$(A/B)y_3 = a_1 y_1 + a_2 y_2, \quad (4)$$

where

$$a_1 = [\sin(2\pi\nu + \kappa) - \cot(2\pi\nu) \cos(2\pi\nu + \kappa)] \quad (5)$$

$$a_2 = \cos(2\pi\nu + \kappa) / \sin(2\pi\nu). \quad (6)$$

Thus, a two-term transversal filter is adequate for dealing with the transverse motion.

There are a number of reasons why more than two terms are desirable. One is that the detrimental effects of noise and digitizing granularity can be reduced. A second is that offset errors can be reduced. A third is that greater flexibility in the coefficient set is achieved, thus allowing a greater amount of adaptability due to the larger number of coefficients.

## II. TRANSVERSAL FILTER

### A. Theory

A general way to arrive at a useful set of coefficients is suggested by the following. Suppose there is a continuous signal

$$s(t) = s_0 \cos(\omega t + \phi). \quad (7)$$

At  $t=0$  this will have the value

$$s(0) = s_0 \cos(\phi). \quad (8)$$

In order to phase shift the value of the signal at  $t=0$  by  $\theta$ , one can take a second signal

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$$r(t) = \frac{2}{T} \cos(\omega t + \theta), \quad (9)$$

where  $\omega=2\pi/T$  ( $T$  is the period of the cosine wave). Now compute the integral

$$I = \int_{-T}^0 s(t) r(t) dt = s_0 \cos(\phi - \theta) \quad (10)$$

and the desired phase shift is accomplished. By changing  $r(t)$ , the amplitude of  $I$  can also be controlled.

If the signal is sampled, one can achieve the phase shifting by a sum of products. This is accomplished by using a transversal filter. Thus, if a signal were taken for  $N$  revolutions,

$$x[n] = D \cos(2\pi v n + \phi) \quad n = -(N-1), \dots, 0 \quad (11)$$

and we use weights

$$b_j = \frac{2}{N} \cos(2\pi v j + \theta), \quad (12)$$

then

$$I = \sum_{n=-(N-1)}^0 b_n x[n] \quad (13)$$

should give us the desired phase-shifted signal. To calculate the proper kick, one simply chooses the appropriate  $\theta$  and adjusts the amplitude to take the beta function into account. Thus, suppose the transverse velocity at the kicker is

$$v = v_0 \sin(2\pi v n + \phi + \kappa) = v_0 \cos\left(2\pi v n + \phi + \kappa - \frac{\pi}{2}\right). \quad (14)$$

The phase shift is

$$\theta = \frac{\pi}{2} - \kappa. \quad (15)$$

The term  $v_0/D$  is determined from the beta functions at the pickup and kicker.

### B. Application of Filter Design

The main goal of the digital signal processing, or DSP, is to develop a transversal filter to process the incoming data [2]. The filter should be adaptive in order to deal with changes in the beam. Specifically, tune shifts could warrant an update to the filter. The main goal of the filter is to provide the proper phase and amplitude shift to the incoming signal that will produce the desired output for the kicker. Any DC offset must also be minimized. The input signal is of the form

$$x_i[n] = D_i \cos(2\pi n v + \phi_i) + E_i, \quad (16)$$

where  $i$  is the bunch number,  $n$  is the turn number,  $v$  is the fractional tune,  $\phi$  is the reference phase, and  $D$  and  $E$  are constants. It is desired for the filter to produce some output  $x'_i[n]$  such that

$$x'_i[n] = D_i F \sin(2\pi n v + \phi_i + \kappa). \quad (17)$$

$F$  is known *a priori* and so is  $\kappa$ . The value of  $\kappa$  is related to the change in the betatron phase from the pickup to the kicker. It is possible to synthesize  $x'_i[n]$  independently from  $x_i[n]$  if  $\phi_i$  is known accurately enough. This would allow for

perfect DC offset cancellation ( $E_i$  in Eq. (16)). The problem is that  $D_i$  in Eq. (16) is not known and must be derived from multiple measurements of  $x_i[n]$ . At least two measurements would be required and probably more would be used in practice. Unfortunately, it will take too long to solve for  $D_i$  and synthesize  $x'_i[n]$ . This leaves some sort of real-time filtration of  $x_i[n]$  to produce  $x'_i[n]$ .

The transversal filter will take the form of

$$x'_i[n] = \sum_{j=0}^{N-1} b_j x_i[n-j], \quad (18)$$

where  $N$  is the number of filter weights. The filter operates on data from past turns as well as the present turn (assuming  $N>1$ ). Each bunch in the ring will have to be dealt with separately, but will use the same filter.

The goal now is to design the filter weights (the  $b_j$ 's in Eq. (18)) in order to model Eq. (17). The filter must effectively implement the gain, the phase shift (this also includes transforming the cosine to a sine), and the DC offset rejection. From the derivation in the previous section, a sine wave in the filter coefficients will be used. From Eqs. (12) and (15), take the weights to be

$$b_j = \frac{2}{N} \cos\left(2\pi j v + \frac{\pi}{2} - \kappa\right). \quad (19)$$

Using Eqs. (16) and (19), Eq. (18) becomes

$$x'_i[n] = D_i F \frac{2}{N} \sum_{j=0}^{N-1} \cos\left(-2\pi j v + \frac{\pi}{2} - \kappa\right) \left[ \cos\left(2\pi(n-j)v + \phi_i\right) + E_i \right]. \quad (20)$$

If Eq. (20) is simulated using a digital computer, it is seen that the chosen filter effectively accomplishes the desired goal. Note that the sum of  $b_j$  over all  $j$  produces the attenuation factor for  $E_i$ . In other words if all the filter weights summed to 0.2, the DC offset would be reduced to 20% of its previous value.

## III. SIMULATIONS AND RESULTS

Figure 1 shows the calculated and simulated data using four weights with  $D_i=F=1.0$ ,  $\phi=0.0$ ,  $\kappa=0.7$ , and  $v=0.3$ . The graphs show the actual calculated beam position at the kicker (calculated) versus the filtered prediction at the kicker (simulated). This result is typical in that small but significant errors result by using weights defined by Eq. (19). These errors can be eliminated by introducing an additional multiplicative constant,  $B$ , for each coefficient and requiring that the calculated and simulated results be equal. In particular, letting  $n=0$ ,  $D_i=F=1.0$ ,  $E_i=0.0$ , and  $\phi_i=\phi$ , one gets, using Eqs. (14), (17), and (20),

$$\cos\left(\phi + \kappa - \frac{\pi}{2}\right) = \frac{2}{N} \sum_{j=0}^{N-1} B_j \cos\left(-2\pi j v + \frac{\pi}{2} - \kappa\right) \cos(-2\pi j v + \phi). \quad (21)$$

Expanding this and requiring that this be true for any  $\kappa$  and any  $\phi$ , one gets three equations:

$$\sum B_i \cos^2(2\pi v_i) = \frac{N}{2}, \quad (22)$$

$$\sum B_i \sin^2(2\pi v_i) = \frac{N}{2}, \quad (23)$$

$$\sum B_i \sin(4\pi v_i) = 0. \quad (24)$$

Thus for  $N=2$ , Eqs. (5) and (6) are used for calculating the weights. For  $N=3$ , Eqs. (22), (23) and (24) are used. For  $N>3$ , the above expressions, combined with additional conditions would be used. In particular, for  $N \geq 4$  one can impose the additional condition that

$$\sum B_i \cos\left(-2\pi v_i + \frac{\pi}{2} - \kappa\right) = 0. \quad (25)$$

This assures complete DC offset cancellation. Figure 2 compares the calculated and simulated results for  $N=4$  after solving for the  $B_i$ 's from Eqs. (22), (23), (24), and (25). As expected, there is complete overlap.

As discussed earlier, the largest expected tune spread will be about 2% of the fractional tune. Figure 3 was generated by using the same weights as in Figure 2 ( $v=0.3$ ), but letting  $v=0.3 \times 98\%$  for generating the measured positions of the bunch (for the simulated data). The ratio of the sum of the magnitude of the difference at each turn, over the sum of magnitudes is 6%.

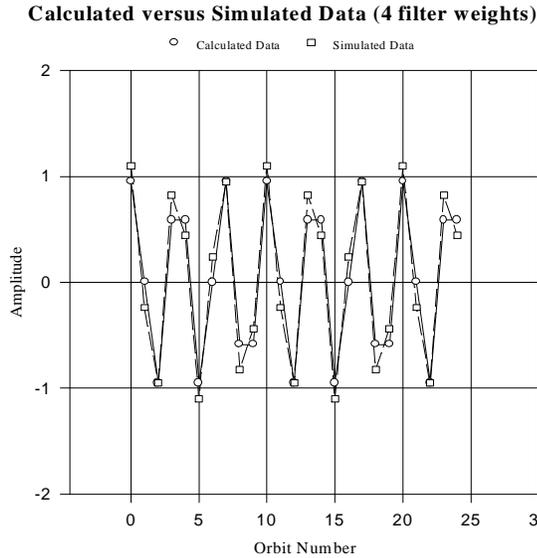


Figure 1. Calculated versus Simulated Data (4 filter weights)

#### IV. CONCLUSIONS

Multi-tap programmable transversal filters are useful for calculating the required kick in damping systems. The larger the number of taps (or filter weights), the greater the flexibility. In particular, four or more taps can be used to assure the elimination of the detrimental effects associated with DC offsets. Effects of noise and digitizing granularity can be reduced. Large tune shifts can be accommodated. A

single filter design can accommodate both transverse and longitudinal damping.

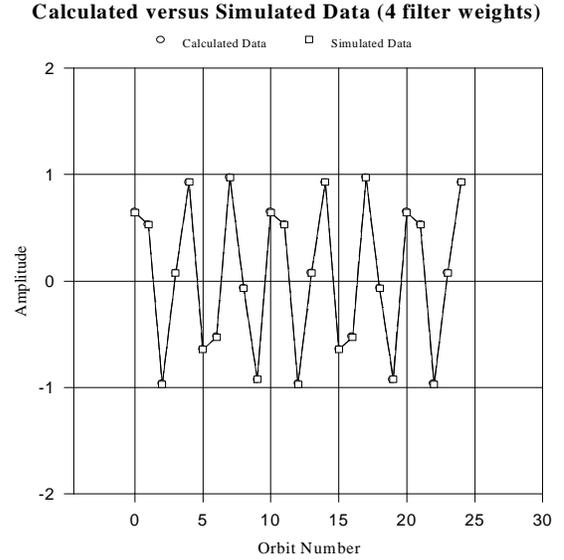


Figure 2. Corrected Calculated versus Simulated Data ( $v=0.3$ )

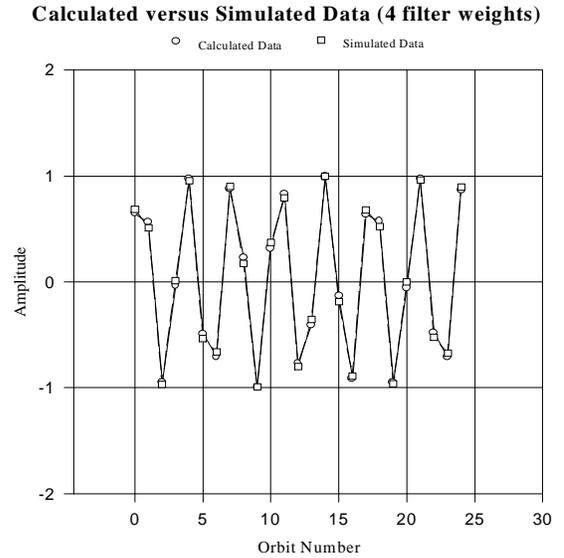


Figure 3. Corrected Calculated versus Simulated Data ( $v=0.3 \times 98\%$ )

#### V. REFERENCES

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