THE EFFECTS OF TUNING AND TERMINATING ON THE OPERATING MODE OF MULTI-CELL COUPLED CAVITY*

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I. INTRODUCTION

The coupled resonator chain is widely used as an accelerating unit. Many people have successfully treated this kind structure with different ways in lossless case to understand the steady state behavior of the chain. A chain of N coupled resonators has N dispersion resonate modes which has different properties. In general, when losses and frequency error are present in cavity the solution are no longer the simple eigenfunction of the homogeneous equations, but are superpositions of all the eigenfunctions. Many such structures, especially superconducting coupled cavity, have been operated in the " π mode" as accelerator elements. According to the theoretical analyze in lossless case, pai-mode operating means group velocity, which is relative to the power flowing in the structure and the energy stored per unit length of the structure, is zero and in steady state no power of this mode can flow in the cavity. Strictly speaking, operation in the π mode is not possible. In practical case any cavity has losses, the energy must be supplied through excitation of adjacent modes to compensate for losses and there are resultant phase changes in the cavity. The phase deviation from π radians per cell is given by [Nagle (1964), Knapp (1964), and Smith (1964)].[1]

$$\Delta\phi_{(n,n-1)} = [2(1-k)^{\frac{1}{2}} / Q_k](N-n+\tfrac{1}{2})$$

Any machining error also will produce frequency and phase deviation from theoretical modes. As many papers described what was so called " π mode" only means the field in the cavity has been flatted by tuning individual cell frequency.[2,3,4,5] It is true the π mode has flat field distribution, but the cavity with flat field may not exactly operate at π mode. This paper discusses the affection of the terminated cell and tuning on operating mode of coupled cavity at an ideal steady state by using eigenequations and perturbation theory.

II. THE SOLUTION OF THE EIGENEQUATION FOR TWO KIND TERMINALS

For the coupled resonator chain there are two different kind terminals, two half end cell terminal which put the shorted plat at a symmetric plan and two full end cell terminal. Many of the electrical properties of a chain with N+1 coupled resonators have been investigated by considering the properties of N+1 coupled circuits. For N+1 cell cavity the circuit equations are

$$X_0(1+\frac{\omega_0}{jQ\omega}-\frac{\omega_0^2}{\omega^2})+m X_1=I_0$$

$$X_N(1+\frac{\omega_0}{jQ\omega}-\frac{\omega_0^2}{\omega^2})+m X_{N-1}=I_N$$

$$X_n(1+\frac{\omega_0}{jQ\omega}-\frac{\omega_0^2}{\omega^2})+\frac{k}{2}(X_{n-1}+X_{n+1})=I_n \quad (n=1,2\cdots N-1)$$

Here ω_0^{-2}=2LC; $X_n=I_n(2L_n)^{½}$; QR=2 ω_0L, In the lossless and no generator included case Q $\Rightarrow \infty$ and I_n=0. There are N+1 solutions to the homogeneous equation of the form. For half end cell terminated, m=K, the solutions are

$$X_n^{(q)} = (const)\cos\frac{qn\pi}{N}e^{j\omega_q t};\ \omega_q^2=\omega_0^2/(1+k\cos\frac{q\pi}{N});$$

$$\omega_q=\omega_0(1+k\cos\varphi)^{-½};\ \varphi=q\pi/N\ (q=0,1,2\cdots N)$$

The lowest mode is q=0, $\omega_{q=0}^2=\frac{\omega_0^2}{1+k}$; the highest mode is q= π, $\omega_{q=\pi}^2=\frac{\omega_0^2}{1-k}$ (0, π mode). The group velocity is

$$v_g = L\frac{d\omega}{d\varphi}=\frac{1}{2}Lk\,\omega_0(1+k\cos\varphi)^{-3/2}\sin\varphi$$

(L the length of single cell)

It indicated that for half end cell terminated without loss or other adjustment the chain does have zero and π mode, but for both zero and π modes (φ=0, π) the group velocity v_g=0.

As above, for full end cell terminated, m=k/2 the solutions are

$$X_n^q=(const)\sin\frac{qn\pi}{(N+2)}e^{j\omega_q t};$$

$$\omega_q^2=\omega_0^2/(1+k\cos\frac{q\pi}{(N+2)});\ \omega_q=\omega_0(1+k\cos\varphi)^{-½};$$

$$\varphi=\frac{q\pi}{(N+2)}\quad (q=1,2,3\cdots N+1).$$

The lowest mode q=1, $\omega_{q=1}^2=\omega_0^2/(1+k\cos\frac{\pi}{(N+2)})$; and the highest mode q=N+1, $\omega_{q=n+1}^2=\omega_0^2/(1+k\cos\frac{(n+1)\pi}{(n+2)})$ There is no zero and π mode, and the field amplitude along axis of the cavity operating in the highest mode is no longer uniform and tilts from the center of the cavity to ends, but if the single cell is the same, both with full and half end cell terminal the dispersion functions have same form. At an unperturbed case the field along axis of the cavity is flatness only when exactly operating at zero or π mode, this is why after the field was tuned flatness, one thinks cavity operating in the " π mode", as mentioned above it may be not right, because of perturbation.

III. PERTURBATION EQUATIONS AND FIRST ORDER SOLUTIONS OF COUPLED CIRCUITS

Since the π mode has a higher effective shunt impedance, most superconducting multi-cell cavities were demanded operating at " π mode". For full end cell terminated cavity there is no π mode, introducing some frequency error in single cell is needed to move the highest mode to " π mode". If the frequency ω_i error is small, it can be considered

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perturbation, the perturbation theory can be used to treat this kind problem.[6] For simple, five full cell cavity has been treated. In ideal case and steady state, the circuit eigenfuctions of the cavity can expressed as a matrix form as follow:

$$L=\begin{bmatrix}\lambda_1 & \frac{k}{2}\lambda_1 & 0 & 0 & 0\\ \frac{k}{2}\lambda_2 & \lambda_2 & \frac{k}{2}\lambda_2 & 0 & 0\\ 0 & \frac{k}{2}\lambda_3 & \lambda_3 & \frac{k}{2}\lambda_3 & 0\\ 0 & 0 & \frac{k}{2}\lambda_4 & \lambda_4 & \frac{k}{2}\lambda_4\\ 0 & 0 & 0 & \frac{k}{2}\lambda_5 & \lambda_5\end{bmatrix};\ X_n=\begin{bmatrix}X_{n1}\\X_{n2}\\X_{n3}\\X_{n4}\\X_{n5}\end{bmatrix};LX=AX;$$

Here $\lambda_i=\omega_i^{-2}$;A=ω^{-2}; L is an operator, A_n is the eigenvalues and X_n is the eigenvectors. In unperturbed case $\lambda_i=\lambda$ (i=1,⋯5), the solutions are $^0A_1=\lambda(1+\frac{\sqrt{3}}{2}k)$; $^0A_2=\lambda(1+\frac{1}{2}k)$; $^0A_3=\lambda$; $^0A_4=(1-\frac{1}{2}k)$; $^0A_5=\lambda(1-\frac{\sqrt{3}}{2}k)$

$$^0X_n=\begin{bmatrix}(\lambda-A_n)^4-3(\lambda-A_n)^2(\frac{k}{2}\lambda)^2+(\frac{k}{2}\lambda)^4\\ -(\lambda-A_n)^3(\frac{k}{2}\lambda)+2(\lambda-A_n)(\frac{k}{2}\lambda)^3\\ (\lambda-A_n)^2(\frac{k}{2}\lambda)^2-(\frac{k}{2}\lambda)^4\\ -(\lambda-A_n)(\frac{k}{2}\lambda)^3\\ (\frac{k}{2}\lambda)^4\end{bmatrix}$$

$$^0X_1=\begin{bmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\\ 1\\ \frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{bmatrix};\ ^0X_2=\begin{bmatrix}\frac{-1}{2}\\ \frac{-1}{2}\\ 0\\ \frac{1}{2}\\ \frac{1}{2}\end{bmatrix};\ ^0X_3=\begin{bmatrix}\frac{1}{2}\\ 0\\ \frac{-1}{2}\\ 0\\ \frac{1}{2}\end{bmatrix};\ ^0X_4=\begin{bmatrix}\frac{-1}{2}\\ \frac{1}{2}\\ 0\\ \frac{-1}{2}\\ \frac{1}{2}\end{bmatrix};\ ^0X_5=\begin{bmatrix}\frac{1}{2}\\ \frac{-\sqrt{3}}{2}\\ 1\\ \frac{-\sqrt{3}}{2}\\ \frac{1}{2}\end{bmatrix}$$

All eigenvectors are orthogonal each other, XiXj=0 (i≠j) and XiXj=const (i=j) (i,j=1,2...5), the solutions of 5-cell cavity are the same as results calculated by SUPERFISH shown in Fig.1.

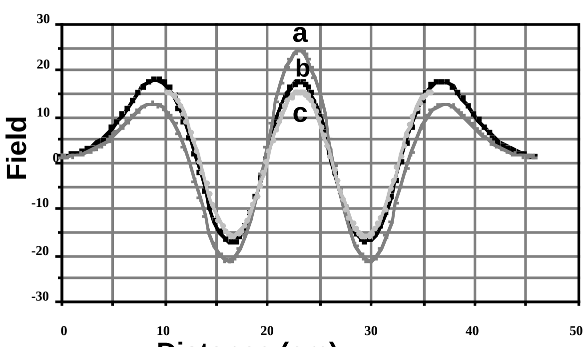

Fig.1a

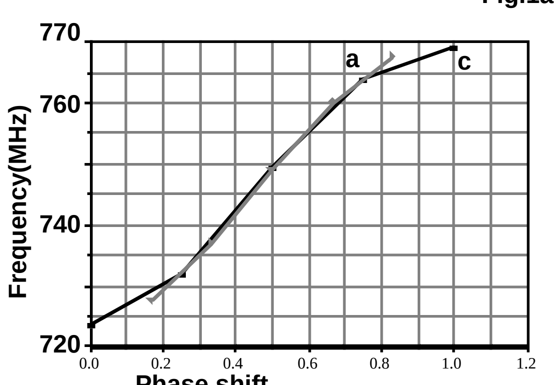

Fig.1b

Fig. 1: The field distribution and dispersion curve of a five-cell cavity calculated by SUPERFISH a) full end cell terminnated without tuning b) full end cell terminated with tuning axial field flatness. c) half end cell terminated.

When the individual cell frequencies are not equal to each other but the deviation $\varepsilon_i=\lambda_i-\lambda$ is small, $|\varepsilon_i|/\lambda$ «1, (such as a full end cell cavity after tuning field flatness. If the central cell frequencies are equal, only needed to tuning full end cells) the operator L can be written as $L=L_0+P$, L_0 is the unperturbed part of the matrix operator, P is the perturbed part, and the matrix elements of perturbation matrix

$Q_{ij}={}^0X_i^+$ P ^0X_j; and $^0X_i^+$, ^0X_j are unperturbed eigenvectors.

$$L_0=\lambda\begin{bmatrix}1 & \frac{1}{2} & 0 & 0 & 0\\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0\\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{2} & 1 & \frac{1}{2}\\ 0 & 0 & 0 & \frac{1}{2} & 1\end{bmatrix};\ P=\begin{bmatrix}\varepsilon_1 & \frac{k}{2}\varepsilon_1 & 0 & 0 & 0\\ \frac{k}{2}\varepsilon_2 & \varepsilon_2 & \frac{k}{2}\varepsilon_2 & 0 & 0\\ 0 & \frac{k}{2}\varepsilon_3 & \varepsilon_3 & \frac{k}{2}\varepsilon_3 & 0\\ 0 & 0 & \frac{k}{2}\varepsilon_4 & \varepsilon_4 & \frac{k}{2}\varepsilon_4\\ 0 & 0 & 0 & \frac{k}{2}\varepsilon_5 & \varepsilon_5\end{bmatrix}$$

Since most single periodic couple cavities operate in the highest mode, here i.e. mode 5 which so-called "π mode", only this mode has been calculated with first order approximation.

$Q_{15}=(\frac{1}{4}-\frac{\sqrt{3}}{8}k)(\varepsilon_1-3\varepsilon_2+4\varepsilon_3-3\varepsilon_4+\varepsilon_5)$

$Q_{25}=(\frac{1}{4}-\frac{\sqrt{3}}{8}k)(-\varepsilon_1+\sqrt{3}\,\varepsilon_2-\sqrt{3}\,\varepsilon_4+\varepsilon_5)$

$Q_{35}=(\frac{1}{4}-\frac{\sqrt{3}}{8}k)(\varepsilon_1-2\varepsilon_3+\varepsilon_5)$

$Q_{45}=(\frac{1}{4}-\frac{\sqrt{3}}{8}k)(-\varepsilon_1-\sqrt{3}\,\varepsilon_2+\sqrt{3}\,\varepsilon_4+\varepsilon_5)$

$Q_{55}=(\frac{1}{4}-\frac{\sqrt{3}}{8}k)(\varepsilon_1+3\varepsilon_2+4\varepsilon_3+3\varepsilon_4+\varepsilon_5)$

The first order correction is obtained using

$$^1A_n=Q_{nn};\quad ^1X_n=\sum_{j\neq n}\frac{Q_{jn}\,{}^0X_j}{({}^0A_n-{}^0A_j)}$$

$^1A_5=Q_{55}=(\frac{1}{4}-\frac{\sqrt{3}}{8}k)(\varepsilon_1+3\varepsilon_2+4\varepsilon_3+3\varepsilon_4+\varepsilon_5)$

$$^1X_5=\frac{Q_{15}\,{}^0X_1}{{}^0A_5-{}^0A_1}+\frac{Q_{25}\,{}^0X_2}{{}^0A_5-{}^0A_2}+\frac{Q_{35}\,{}^0X_3}{{}^0A_5-{}^0A_3}+\frac{Q_{45}\,{}^0X_4}{{}^0A_5-{}^0A_4}$$

$$=\frac{(1-\frac{\sqrt{3}}{2}k)}{8k\lambda}\begin{bmatrix}\sqrt{3}(-3\varepsilon_1-\varepsilon_2+3\varepsilon_4+\varepsilon_5)\\ \varepsilon_1+9\varepsilon_2-4\varepsilon_3-3\varepsilon_4-3\varepsilon_5\\ 2\sqrt{3}(\varepsilon_1-2\varepsilon_3+\varepsilon_5)\\ -3\varepsilon_1-3\varepsilon_2-4\varepsilon_3+9\varepsilon_4+\varepsilon_5\\ \sqrt{3}(\varepsilon_1+3\varepsilon_2-\varepsilon_4-3\varepsilon_5)\end{bmatrix}$$

First order approximation

$A_n={}^0A_n+{}^1A_n;\ X_n={}^0X_n+{}^1X_n$

Here $\frac{\varepsilon_i}{\lambda}\cong\frac{-2\delta\omega_i}{\omega}$, $\delta\omega_i$ is the frequency error of the individual cell (normally $\Sigma\delta\omega_i\neq0$). Substitute $\delta\omega_i$ to the equation one can get new operating mode ω' and relevant phase φ'. If the perturbation $\delta\omega_i$ is very small, and $\Sigma\delta\omega_i$=0 one still can use unpertuabed dispersion curve to approximately calculate the group velocity v_g'

$v_g'=L\frac{d\omega}{d\varphi}=\frac{1}{2}Lk\,\omega_0(1+k\cos\varphi')^{-3/2}\sin\varphi'$.

When tuning, in order to tune cavity field flatness, ,one can let all parts of the eigenvector are equal and using symmetry, and then calculate the needed ε_i. Practically it is opposite, after tuning $\varepsilon_i \cong 0$ (i=1,2·····N-1) (middle cells) and $\varepsilon_0 = \varepsilon_N = \varepsilon_e$ (end cells); i.e. only two end cells have frequency error, in the case the perturbed operator is simple, one can directly solve the eigenequation to get eigenvalues and eigenvectors.

IV. DISCUSSION

In fact almost all superconducting multi-cell cavities are terminated by full end cell, when the half end cell terminated replaced by full end cell, the full end cell only had one side coupled, the change of the boundary condition broke the symmetry, and the field penetrated into two end beam pipes to extend field area, sequentially, the field amplitude tilted from center of the cavity to the ends, relevant phase shift between cells decreased and the π mode vanished, the highest mode was (N+1)π/(N+2) (here N+1 are total cell numbers of the cavity). From perturbation theory the individual cell frequency error will cause the operating mode and its properties change, such as phase shift and field distribution, it was used for tuning cavity.

The cavity tuning includes tuning middle individual cell (reduce frequency error ε_i) to eliminate field non-uniform caused by mechanical tolerance and tuning end cells (add suitable frequency error $\varepsilon_0 = \varepsilon_N = \varepsilon_e$) to compensate the field tilt caused by full end cell terminated and move the operating mode to close "π mode" in which the field along the axis of cavity is flatness. Fig.2 is the field distribution of the 5-cell cavity calculated by P. Fernandes and R.Parodi using OSCAR2D code.

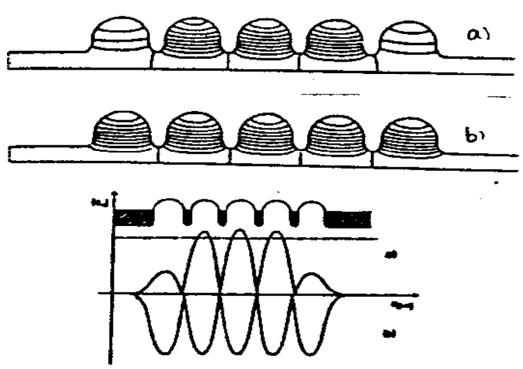

Fig.2 Inner field and axial field distribution of a five cells accelerating structure calculated using OSCAR2D code a)uncompensated, b) compensates.

From Fig 1 and Fig 2 one can see the effects of tuning and terminating on the field distribution and phase shift (modes). In this way after tuning $\Sigma\delta\omega_i$=2 $\varepsilon_e \neq 0$, the average individual cell frequency ω_0' was changed $\omega_0' = \omega_0 + \delta\omega = \omega_{\pi/2}$ and $\delta\omega = \Sigma\delta\omega_i$/(N+1), it means the dispersion curve should parallel move up $\delta\omega$, which also can directly be got by measuring the π/2 mode frequency $\omega_{\pi/2}$ The dispersion function and the group velocity still can be described as

$$\omega_q^2 = \omega_0'^2/(1+k\cos\varphi')$$

$$v_g' = L\frac{d\omega}{d\varphi} = \frac{1}{2}Lk\,\omega_0(1+k\cos\varphi')^{-3/2}\sin\varphi'$$

In theoretical one can tune cavity with $\Sigma\delta\omega_i$=0 by lower middle cell and higher end cell frequencies or reverse, it also can be checked by measuring $\omega_{\pi/2}$. In practical tuning case the beadpull was used to check the field amplitude, when the field amplitude along the axis of the cavity is flat, the tuning is done. After tuning with the average single cell frequency ω_0, which equals new π/2 mode $\omega_{\pi/2}$ of the tuned cavity (for $\Sigma\delta\omega_i$=0), and coupling constant k which is same as unperturbed k, one can calculate the dispersion curve and then substitute the operating mode frequency measured (the highest mode) into the dispersion function to calculate mode properties, such as phase shift between cells and group velocity.

REFERENCE

1, Pierre M. Lapostolle and Albert L. Septier "Linear Accelerators" P71, 1970

2, A. Marziali and H. A. Schwethman " Structure tuning and its effect on high order modes". P707 1992 Linac conference.

3, B. D Wersteg et.al. "Structure Studies for an S- Band linear Collider" P516 1992 Linac Conference.

4, D.J. Laska "Design Fertures of a Seven-Cell High gradient Superconducting cavity" 1992 Linac Conference.

5, P. Terwandes "OSCAR2D A computer code for the design of RF cavities and structures" P 330 1990 Linac Conference.

6, T.P Wangler "Calculations for three coupled oscillators-Including eigenvalue problem, perturbation theory and losses" AT-1:84-299 LANL