Low Energy Regime for Optical Transition Radiation Emission

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Abstract

OTR has been widely investigated in the literature as an electron beam diagnostic tool in the high energy region (E > 50 MeV). At lower energy both the model of OTR emission and the experimental observation of the phenomena require a different and more complex approach. Nevertheless the information which can be drawn from OTR spectra at these low energies play a relevant role in new injectors. In this paper we will present and discuss the theoretical work carried out in the last year to provide a suitable background to OTR measurements at energies below 50 MeV.

Introduction

OTR based beam diagnostics are beginning to be used more and more at accelerator facilities around the world. There is a wide spread of data in the high energy region where the phenomena is more relevant. At low energies the situation is quite different for the difficulties in the measure of the emitted radiation, due to its low level, as well as for the lack of an homogeneous approach to the theoretical description of the emission .
The fundamental characteristics of the transition radiation emission shows both a rather complicated relation between a lot of different parameters and the coexistence of different regimes. This latter feature is mainly related to the dependence of the emission from the energy of the charged particles. The emission models used to describe the phenomena at high energies must be deeply modified in order to understand the behavior at lower energies. Moreover, the incidence angle of the beam plays also a relevant role in the characteristics of the emission.
Observation at the focal plane of the detector or at infinite will give different informations, but may results in completely different design of the experimental apparatus.
The purpose of this brief paper is to present and discuss the results of an homogeneous theoretical approach to the model of OTR emission which will include the different regimes and the possible different experimental configurations. We will demonstrate the feasibility of OTR measurements at electron energies of the order of few MeV and we will discuss the most important relations between OTR spectra and beam characteristics.

Analytical treatment

Our aim has been to obtain a general expression for the OTR emission using the simplest and more classic approach based on Maxwell equations and considering all the variables involved. The results obtained are fully compatible with the analytical expressions already available in the literature, but evaluated only for particular conditions (1).
The assumptions which have been considered may be so summarized:

- the materials have been described and modeled according to their macroscopic properties (ε,μ) and they are supposed homogeneous and isotropic
- the electron motion has been considered uniform and along a straight path
- the damping of the emitted electromagnetic waves depends only on the conductivity σ of the materials and on the distance from the emission point.

Starting from these hypothesis, we have considered two semi-infinite media with a plane interface between them. The propagation axis forms an angle ψ with the normal to the separation interface. Maxwell equations have been written for such a geometry and with the boundary conditions at the interface which describe the presence of a charged particle and the response of the material to an external electric field.
Computations are quite cumbersome and take into account the presence of fields due to the particle and radiation fields, which arises from the need to satisfy boundary conditions. Electric and magnetic fields may be computed in the two media and symmetry between them may be easily proved. Naming θ the angle between the normal to the interface and the wave vector κ, and φ the angle between the projection of the velocity on the interface and the projection of the wave vector κ on the interface, we obtain from the Poynting theorem the complete expression for the emitted radiation (Appendix 1) .
The expression so far obtained shows all the dependencies from the different physical parameters involved in a measure: energy of the particle, materials, incidence and observation angles. Nevertheless the formula is rather complex to handle and is still referenced to a model of two semi-infinite planes. It is still possible to use this result also in a more realistic model for the target, i.e. one of finite thickness, providing that the material is a perfect conductor (σ very high).

These expressions are still valid for all the energies and all the geometric arrangements of the experimental setup. It may be shown that the errors introduced by the perfect conductor limit are of the order of 10% (in excess) for incidence angle of 45°, observation in the incidence plane and observation angle less than 0.5 rad. At low energies ($\gamma = 3$) the error percentage may grow up to 100% for observation angles greater than 1 rad. At high energies (γ=100) the error is negligible.

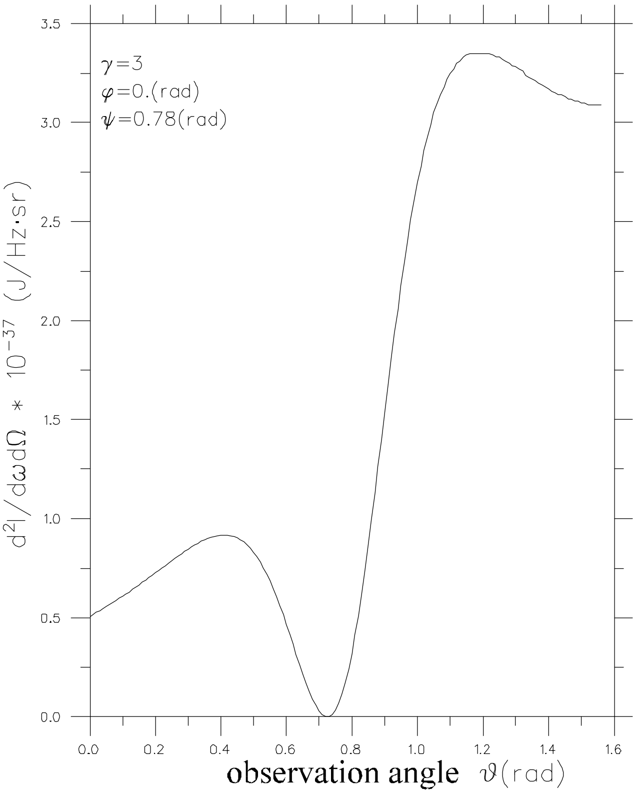

Fig. 1 Distribution of the emitted radiation at γ=3

Results and discussion

The relations so far obtained can be used to compute the spatial distribution of the radiation emitted from an interface for given conditions of the incident beam and of the detector geometry. This would provide a powerful tool for the design of a good experimental setup for an OTR measure. We have examined such relation in order to outline all the existing relations between the OTR spectra and the characteristics of a beam.

Fig. 1, 2, 3 show the distribution of the emitted radiation at $\varphi = 0$ for different values of energy at incidence angles of 0 and 0.78 rad. In the following, we will consider only the so called backward emission, since it is the easiest to observe in diagnostics experiments. The maximum collection of radiation takes place under the following conditions:

- incidence angle centered around 0.78 rad for energies higher than 5 MeV and in the region 0.4-0.78 rad for lower energies (in this region the dependence from the energy is strong)
- observation angle centered around 0.78 rad for energies higher than 5 MeV and for lower energies with a strong dependence from γ^{2}.

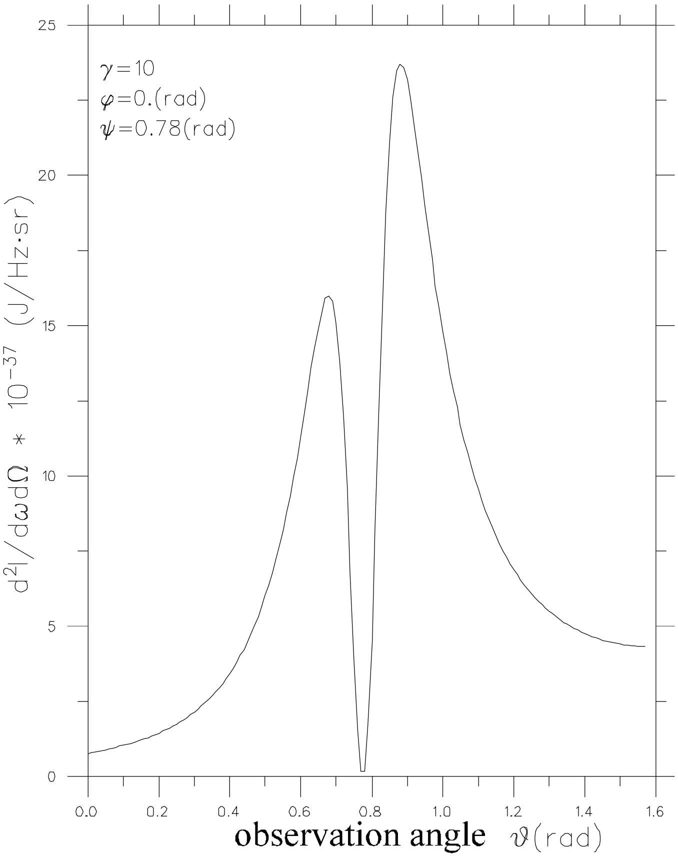

Fig. 2 Distribution of the emitted radiation at γ=10

These results are particularly interesting since the best geometric arrangement for radiation collection is also the easier from the experimental point of view. In fact the most usual configuration of the target with respect to the beam propagation axis is at 45°, in order to extract the emitted radiation in a simple way from the beam pipe.

The analysis of the relations between the spatial distribution of the emitted radiation and the energy of the beam, has given the following results:

- the two characteristics peaks in the OTR spectra presents a maximum of 10% of amplitude unsymmetry with respect to variations in observation angle at energies higher than 10 MeV. At 2-3 MeV the unsymmetry is of the order of 400%. Symmetry is completely maintained with respect to variations in φ (fig. 4).
- the distance of the two peaks (Δ) is related to γ of the particle according to different expressions with respect to beam energy and measuring conditions. In particular it may be shown that Δ, at θ fixed, is proportional to $2.8\gamma^{-1}$ and, at φ fixed, Δ is proportional to $2.0\gamma^{-1}$ (in the range $\gamma > 20$) and to $1.5\gamma^{-1.1}$ (in the range $\gamma < 20$).

The minimum in the emission distribution is related to the energy of the beam. It may be shown that θ_{min} overlaps with the reflection axis for $\gamma > 20$ and it is proportional to $0.5\ \gamma^{-2}$ for $\gamma < 20$.The amplitude of the maximum peak in the distribution at $\varphi = 0$ is related to γ^2.

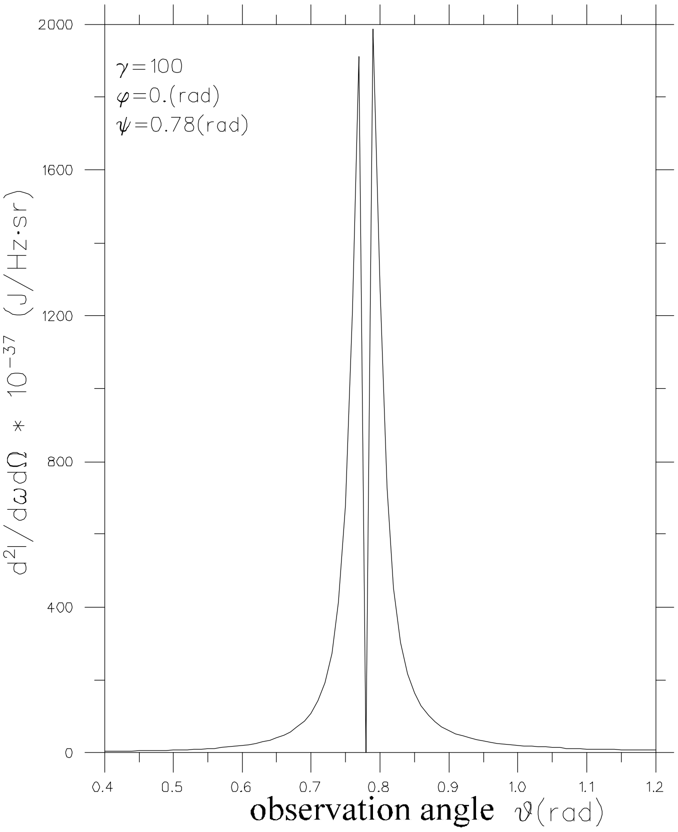

Fig. 3 Distribution of the emitted radiation at γ=100

As far as the intensity level is concerned our study has shown that at low energy (2-5 MeV) and in the condition of oblique incidence it is impossible to collect informations about spatial distribution. This is due to the large spread of the spectra (fig. 1) with respect to the geometrical constraints of the detector. Nevertheless it is possible to measure the radiation emitted regardless of its distribution. The measure can be carried out using a normal camera.

At higher energies (γ = 10) it is possible to collect a discrete level of intensity (of the order of 10^{-4} lux and which can be measured using an intensified camera) and the distribution may be analyzed according to the relations so far discussed.

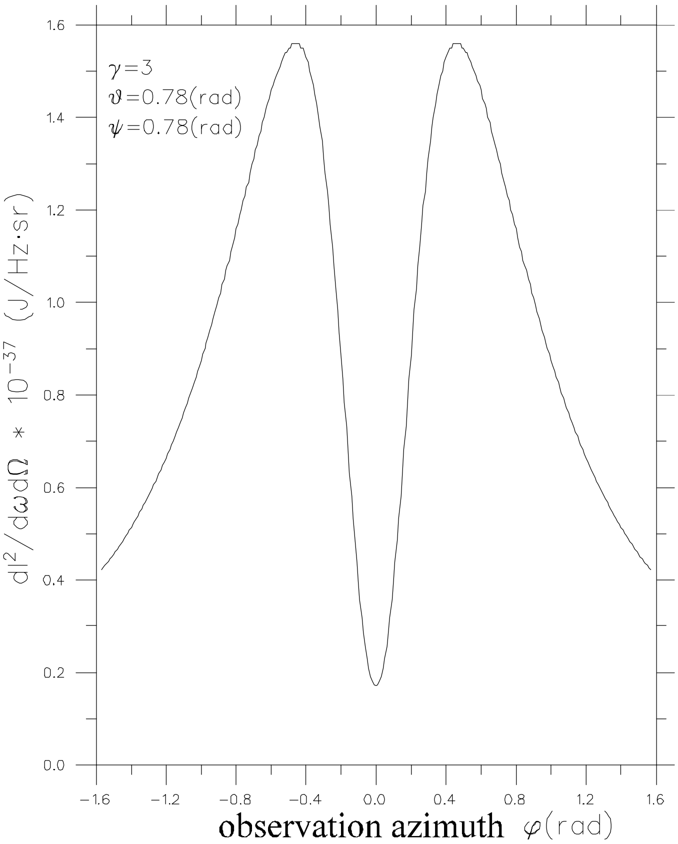

Fig. 4 Distribution of the emitted radiation at γ=3 vs. azimuth of observation

References

[1] M. L. Ter Mikaelian, High energy electromagnetic processes in condensed media, ed. Wiley, 1972

Appendix 1

$$\frac{dI_{\parallel}}{d\Omega d\omega} = \frac{e^2\sqrt{\varepsilon_1}}{\pi^2 c}\frac{\beta^2\cos^2\psi\cos^2\vartheta_1|\varepsilon_2-\varepsilon_1|^2}{[(1-\sqrt{\varepsilon_1}\sin\vartheta_1\cos\varphi\beta\sin\psi)^2-\varepsilon_1\cos^2\vartheta_1\beta^2\cos^2\psi]^2}\frac{1}{\sin(\vartheta)^2}\cdot$$
$$\left|\frac{\sin^2\vartheta_1(1-\sqrt{\varepsilon_1}\sin\vartheta_1\cos\varphi\beta\sin\psi+(\varepsilon_2-\varepsilon_1\sin^2\vartheta_1)^{1/2}\beta\cos\psi-\varepsilon_1\beta^2\cos^2\psi)}{[1-\sqrt{\varepsilon_1}\sin\vartheta_1\cos\varphi\beta\sin\psi+(\varepsilon_2-\varepsilon_1\sin^2\vartheta_1)^{1/2}\beta\cos\psi][(\varepsilon_2\varepsilon_1-\varepsilon_1^2\sin^2\vartheta_1)^{1/2}+\varepsilon_2\cos\vartheta_1]}-\frac{\sqrt{\varepsilon_1}\sin\vartheta_1\cos\varphi\beta\sin\psi(\varepsilon_2-\varepsilon_1\sin^2\vartheta_1)^{1/2}\beta\cos\psi}{}\right|^2$$

$$\frac{dI_{\perp}}{d\Omega d\omega} = \frac{e^2\sqrt{\varepsilon_1}}{\pi^2 c}\frac{\beta^6\cos^4\psi\sin^2\psi\sin^2\varphi\cos^2\vartheta_1|\varepsilon_2-\varepsilon_1|^2}{[(1-\sqrt{\varepsilon_1}\sin\vartheta_1\cos\varphi\beta\sin\psi)^2-\varepsilon_1\cos^2\vartheta_1\beta^2\cos^2\psi]^2}\cdot$$
$$\left|[1-\sqrt{\varepsilon_1}\sin\vartheta_1\cos\varphi\beta\sin\psi+(\varepsilon_2-\varepsilon_1\sin^2\vartheta_1)^{1/2}\beta\cos\psi][(\varepsilon_2-\varepsilon_1\sin^2\vartheta_1)^{1/2}+\cos\vartheta_1]\right|^{-2}$$