BEAM PROFILE MEASUREMENT IN THE PRESENCE OF NOISE *

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Abstract

In measuring the distribution of a nominally Gaussian beam, it is generally necessary to sample the beam at a number of discrete positions. In the real world, these samples will be somewhat noisy. We present the results of simulations, showing the effects of noise in signal amplitude and noise in sample position on the calculated beam σ, as a function of sample spacing. This has implications for the wire spacing of multiwire profile monitors and for the sampling rate of flying wires and wall current monitors.

I. INTRODUCTION

Many situations occur where one wishes to measure the profile of a beam which is nearly Gaussian and to derive a good estimate of the beam σ. In generating design specifications for beam profile monitors and flying wires at the Superconducting Super Collider (SSC), it was necessary to find a quantitative relationship connecting the various sources of noise and the resultant errors in estimation of beam σ. Since we could not locate any direct treatments of this in the published literature, we decided to generate these relationships with some simple Monte Carlo simulations.

A Gaussian profile follows the relationship:

$$f(x)=A\,e^{\left(-(x-m)^2/2\sigma^2\right)} \tag{1}$$

which has three degrees of freedom: the amplitude A, the mean m, and the standard deviation σ. Thus, ideally, only three measurements at arbitrary points are necessary to perfectly reconstruct the distribution. But in the real world, noise is present and will lead to uncertainties in the reconstructed parameters. In practice, many more than three points are necessary to provide a good estimate of σ. A rule-of-thumb which has been used in the past is to try to sample at two or three points per σ [1].

II. ESTIMATION METHODS

The error in estimating σ depends greatly on how the estimation is done. Once a profile has been obtained, a number of options present themselves of calculating the beam's σ. One could simply calculate the mean and standard deviation of the data points [2], but this leads to large errors for sparsely sampled or noise profiles, because the contributions of points in the tails are overweighted. A much better method is to estimate σ by a least squares fit of the data to a Gaussian [2]. One could also use a "matched filter" approach, where one filters the data by multiplying the Fourier transform of the data by the Fourier transform of the expected profile, e.g. with a Gaussian or quasi-Gaussian profile. One could also "bin" the data, which is a special case of the matched filter approach. (It is equivalent to convolving the data with a rectangular pulse the width of a bin, then re-sampling this signal at points separated by the bin width.)

III. SIMULATIONS

Measurement noise can arise from two sources: noise in sample position (due primarily to mechanical jitter or misalignment) and noise in sample amplitude (due primarily to electrical noise on the signal). Simulations were done of each source separately.

The simulations were done as follows. First, a normalized Gaussian profile was generated with σ=1. This profile was sampled with a regular sampling grid of 60 points. The center of the sampling grid was randomly aligned (± one half of the sample spacing) with respect to the center of the Gaussian profile. Normally-distributed noise in either the sample positions or in the sample amplitudes was added. A Gaussian was fitted to the data by least squares regression (by varying A, m, and σ to minimize the squared errors or the χ^2). The resultant σ was compared to the original σ and the error was tabulated. This procedure was repeated a number of times, and the rms error in estimating σ was calculated for the given sample spacing and noise conditions.

This was repeated for different sample spacing and noise conditions, with the results shown in Figs. 1 and 2. In Fig. 1, amplitude noise was assumed to be independent of the signal amplitude at each sample point, and is normalized to the signal amplitude at the peak of the (theoretical) Gaussian profile. In Fig. 2, position noise is normalized to σ.

IV. DISCUSSION

For the case of noise in signal amplitudes (Fig. 1), the errors for a sampling frequency of one sample per σ are about what one might naively guess. The fractional errors

* Work primarily performed at the SSC Laboratory, operated by Universities Research Association, Inc. for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

† Operated by the Universities Research Association, Inc. under contract with the U.S. Department of Energy.

in σ are approximately equal to the normalized noise amplitude; i.e. an rms noise level of 1% (-40 dB) of the Gaussian peak causes an rms error of 1% in the estimated value of σ, and so forth.

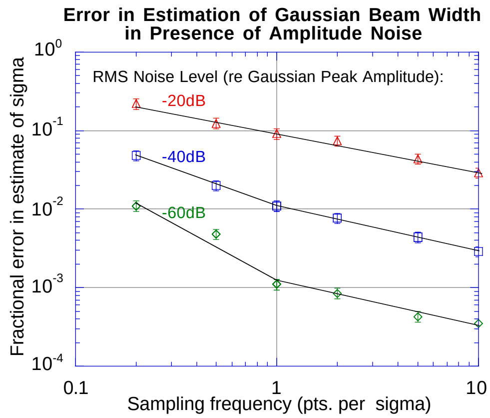

Figure 1: Simulation results of adding noise to signal amplitudes of a sampled Gaussian distribution.

The slope of the curves, for a well-sampled distribution (more than one sample per σ) also makes sense. As the sampling frequency is doubled, more samples are obtained near the peak of the Gaussian. One is effectively getting twice as much signal for the same amount of noise, so the error should drop by a factor of $2^{1/2}$. Thus the slope on a log-log plot should be 1/2, as is observed.

For a more sparsely-sampled distribution (less than one sample per σ), the slope is greater than this. This is because the sampling is so sparse that sometimes there are NO sample points near the peak of the Gaussian, leading to even larger errors.

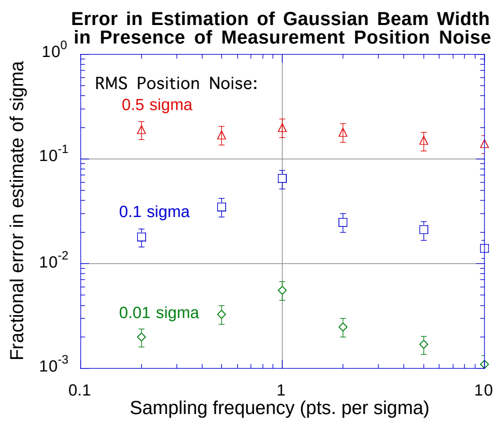

Figure 2: Simulation results of adding noise to sample positions of a sampled Gaussian distribution.

For the case of noise in sample positions (Fig. 2), the slope for a well-sampled distribution should again be 1/2, for reasons similar to those given above, which is roughly what is observed. As the sampling frequency is reduced, the estimate of σ is seen to improve. This may be because more of the data points lie in the tails, where noise in sample position has relatively little effect. Thus, even though the signal level is small, the noise is also small and a good estimate results. By this reasoning, noise in sample position will have the most serious effect where the slope of the Gaussian is largest, i.e. at an offset of σ from the peak. Thus it is reasonable that the greatest error in estimating σ occurs at a sampling frequency of around one sample per σ.

V. CONCLUSIONS

It is advisable to sample a Gaussian with at least one sample per σ to keep amplitude noise under control, and preferably with at least two samples per σ to reduce sensitivity to position noise. The rule of thumb of two or three samples per σ seems to be a good one.

For the Medium Energy Booster (MEB) at the SSC, where it was necessary to measure σ to at least a 7% precision, it would have been sufficient to sample the distribution with at least two points per σ, while maintaining a S/N of at least 30dB and a $\pm 0.1\ \sigma$ tolerance on sample positions. Specifications for flying wires and beam profile monitors were to be based on this data.

VI. REFERENCES

[1] Robert Webber, Fermi National Accelerator Laboratory, private communication.

[2] Particle Data Group, "Review of Particle Properties", *Phys. Rev.* **D50**, 1 Aug. 1994, Part 1, §17, pp. 1275ff.