Chaos, a source of Charge Redistribution and Halo Formation in Space-Charge Dominated Beams

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Abstract

A significant breakthrough in the understanding of the space-charge dominated beam dynamics has been achieved with the introduction of the Particle-Core Model at the previous PAC (O'Connell, Wangler, Mills & Crandall, May 1993). Using this model, we have shown the chaotic behaviour of the particles for mismatched beams in continuous focusing channels (August 1993 & August 1994) and for matched beams in FODO channels (December 1993 & June 1994). In these studies, it has been settled that the recent theories developed for the analysis of nonlinear dynamic systems ("chaotic dynamics") are very useful to understand charge redistribution and halo formation phenomena. They are now used to describe the behaviour of mismatched beams in FODO channels. It is shown that a mismatch adds perturbing forces which can deeply modify the dynamics. The particle diffusion far from the beam core is studied.

1. INTRODUCTION

In numerous nonlinear dynamical systems studied in various disciplines (fluid dynamics, celestial mechanics, chemistry, biology, economy, ecology...), chaotic (stochastic) motions are generated by the dynamics itself whereas no random force is present. The chaotic behaviour of the particle trajectories (already studied in the accelerator field to understand the beam-beam effect) has been observed from numerical experiments on space-charge dominated beams [1 – 4]. This significant breakthrough in the understanding of the mechamisms leading to charge redistribution and halo formation in the new generations of high-power linacs has been achieved thanks to the Particle-Core Model (PCM) [5].

The PCM is obviously not a self consistant model but, as it is usualy done to study *complex systems*, it is a simplified model which keeps the dominant properties of the real physical system and which allows an analysis of the basic phenomena. For the PCM :

- the influence of the halo on the beam core is neglected because halo particles are supposed to be a very low fraction of the total beam,
- the "breathing" mode which is the main oscillation mode exited by the quadrupoles in a real accelerator is the sole mode taken into account. All the other modes are neglected.

The first numerical experiments completed using the PCM [5, 1, 6, 2] dealt with the evolution of beams in a continuous focusing channel. As pointed out in ref.[1, 2], when the envelope of a mismatched beam oscillates, space charge can be assimilated to a *nonlinear periodic perturbing force* which excites resonances. The *resonance overlap mechanism* [7] can then lead to the formation of a halo area where the particle trajectories are stochastic. This chaotic behaviour has been clearely observed using the Poincaré surface of section technique. Sensitive dependance on initial conditions and intermittencies which characterize chaotic systems have been also shown.

In ref.[3, 4], the PCM is used to analyse the behaviour of a <u>matched beam</u> evolving in a FODO channel. Again, the test particles trajectories are studied using the Poincaré section technique to display the chaotic areas prominently. Here, the beam envelope oscillations are naturally *"excited"* by the quadrupoles then, as for a mismatched beam in a continuous focusing channel, several resonances can overlap and form a halo. Therefore, it seems important to emphasize that halos can be generated even if the beam is perfectly matched, not only via non-ideal conditions as found stated in some recent papers.

In a FODO channel, both order and number of the resonances which are present around the beam core are determined by the choice of the phase advances with (σ_t) and without (σ_{0t}) space charge. Actually, the tune of the test particles which travel near the core is close to $\upsilon = \sigma_t/2\pi$ and it tends towards $\upsilon = \sigma_{0t}/2\pi$ when the transverse energy is increased because the effect of space charge becomes more and more negligeable. For K-V or monotonically-decreasing distribution functions for which σ_t is the phase advance near the axis, the parametric resonances which can be excited are then in the range :

$$\boxed{\sigma t/2\pi \le \upsilon < \sigma_{0t}/2\pi}$$

The size of the stochastic areas, consequently the particle diffusion, is limited when the strong resonances $\upsilon = 1/4$ and $\upsilon = 1/5$ are avoided ($\sigma_{0t} < 72^o$) [3]. This can be done using modified octupoles to reduce the tune spread in the beam core vicinity [4] or, simply choosing $\sigma_{0t} \sim 61.9^o$ and $\sigma_t \sim 19.9^o$ for example. The resonances which surround the K-V beam core are then in the range $1/18 \le \upsilon \le 1/6$ with the $\upsilon = 1/18$ resonance ($\sigma = 20^o$) very close to the core and the $\upsilon =$ 1/6 resonance ($\sigma = 60^o$) far from it. Figure 1 gives a Poincaré section for uncoupled (x,x', y=y'=0) test particles showing the $\upsilon = 1/6$,1/7 and 1/8 resonances. A thin stochastic layer appears around each resonance (weak chaos) but they do not overlap, they stay isolated by KAM surfaces. The perturbation is too weak to form a large stochastic sea which could lie down up to the $\upsilon = 1/6$ resonance area.

In Ref.[3], it has been shown that to couple the RF defocusing effect to the radial motion leads to the formation of large stochastic seas. Therefore, the dynamics is deaply modified when "coupled oscillators" are added to the system. In the following sections, the effects of an additional source of perturbation : a beam core mismatch, will be studied.

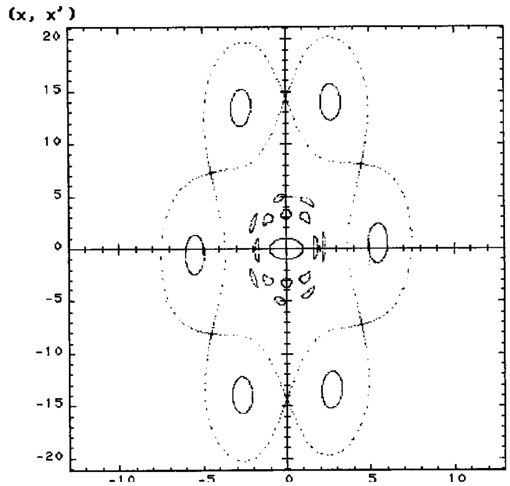

Fig. 1: Poincaré section for a matched beam.
(uncoupled particles with $\sigma_{0t} \sim 61.9^o$ and $\sigma_t \sim 19.9^o$)

2. MISMATCHED ENVELOPES

For weak mismatchs, the envelope equations in smooth approximation can be linearized. When $a_0 = b_0$ are the matched beam mean radii, the envelopes in the two transverse planes can be noted $a = a_0 + \delta a$ and $b = b_0 + \delta b$. The two mismatch terms ($\delta a << a_0$ and $\delta b << b_0$) are then determined by two coupled linear differential equations :

$$\delta a'' + \sigma_a^2\, \delta a + \sigma_{ab}^2 \delta b = 0 \quad \text{and} \quad \delta b'' + \sigma_b^2\, \delta b + \sigma_{ab}^2 \delta a = 0$$

with $\sigma_{ab}^2 = 2K / (a_0 + b_0)^2$

and $\sigma_a^2 = \sigma_b^2 = \sigma_{0t}^2 + (3\varepsilon^2 / a_0^4) + \sigma_{ab}^2$

where K is the generalized perveance and $\varepsilon = \varepsilon_x = \varepsilon_y$ is the beam emittance. Following I. Hofmann [8], the two eigen modes can be called even and odd, they are given by :

$$\sigma_e^2 = \sigma_a^2 + \sigma_{ab}^2 = 2(\sigma_t^2 + \sigma_{0t}^2) \quad \text{and} \quad \sigma_o^2 = \sigma_a^2 - \sigma_{ab}^2 = 3\sigma_t^2 + \sigma_{0t}^2$$

The K-V envelope equations have been numerically integrated (without smooth approximation) for a weak initial mismatch $\delta a = 0.1 a_0, \delta b = \delta a' = \delta b' = 0$ and a FODO channel with $\sigma_{0t} \sim 62^o$ and $\sigma_t \sim 20^o$. The Fourier spectrum of the envelopes (fig.2) shows that, even for this weak mismatch, the amplitudes of the odd and even modes are already large compared to the one of the main mode (f=1). The eigen frequencies are close to those calculated using the theoretical formulas but, for larger mismatchs, the envelope oscillations become rapidly chaotic leading to a more complicated frequency spectrum.

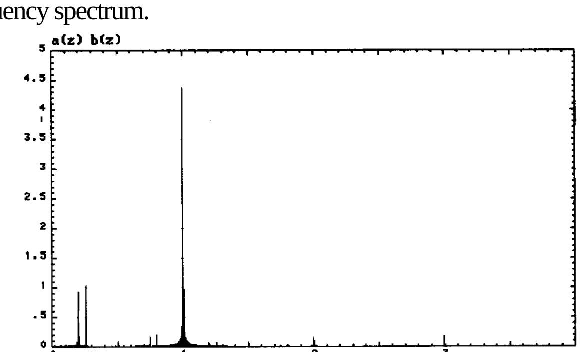

Fig. 2 : Fourier spectrum of a mismatched beam envelope.

3. RESONANCES EXCITED BY A MISMATCH

Above the resonances excited by the natural beam modulation due to the quadrupolar focusing system ($\sigma_t/2\pi \le \upsilon < \sigma_{0t}/2\pi$), new resonances are excited by the two eigen frequencies of the envelope oscillations when the beam is mismatched. These additional resonances are in the range $\sigma_t/\sigma_{e,o} \le \upsilon_{e,o} < \sigma_{0t}/\sigma_{e,o}$ then, with $\eta = \sigma_t/\sigma_{0t}$:

$$\eta / \sqrt{2(\eta^2+1)} \le \upsilon_e < 1/\sqrt{2(\eta^2+1)} \quad \text{for the even mode}$$

and $$\eta / \sqrt{3\eta^2+1} \le \upsilon_o < 1/\sqrt{3\eta^2+1} \quad \text{for the odd mode.}$$

Figure 3 shows the range of resonances which can be excited by the two modes as a function of the tune spread η . As in the case of a continuous focusing channel [2], the strong $\upsilon_{e,o} = 1/2$ resonance is definitively always present.

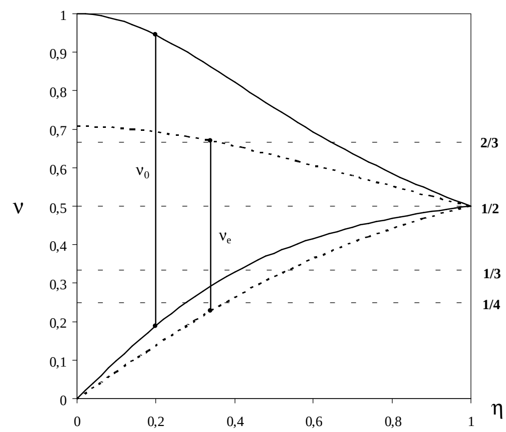

Fig. 3 : Range of excited resonances versus η .

4. CHAOS INDUCED BY A MISMATCH

The simplest way to analyse the test particle chaotic motions is to use the Poincaré section technique. For a mismatched beam in a FODO channel, it is therefore necessary to choose an initial mismatch such as only one mode (even or odd) is excited, and also to choose the channel parameters such as the beam core oscillation is periodic. An optimization code has been written to find these conditions, the case $\sigma_{0t} = 61.87659^o$ and $\sigma_t = 19.92206^o$ has been choosen for the numerical experiments. Using these parameters :

- the main resonances which are present around the K-V beam core are $1/18 \le \upsilon \le 1/6$ (see section 1 and figure 1),
- the beam core oscillation is exactly five FODO periods for an initial mismatch $\delta a = .1 a_0,\ \delta b = -.1 a_0,\ \delta a' = \delta b' = 0$.
- $\eta = 0.322$ and the main resonances excited by the odd mode are $\upsilon_o = 2/3$, 1/2 and 1/3 (see figure 3).

Using these parameters, a Fourier analysis of the envelopes shows that the amplitude of the odd mode is about half of those of the fundamental mode. In a Poincaré section (fig.4) a large stochastic area which is created by the overlaping of resonances appears around the beam core. The $\upsilon_o = 1/2$ (easily recognizable in figure 4) and the $\upsilon_o = 1/3$ resonances overlap with the low order resonances excited by the fundamental mode. For uncoupled particles, this chaotic sea stays isolated from the $\upsilon = 1/6$ resonance by KAM curves which are not destroyed by the weak mismatch .

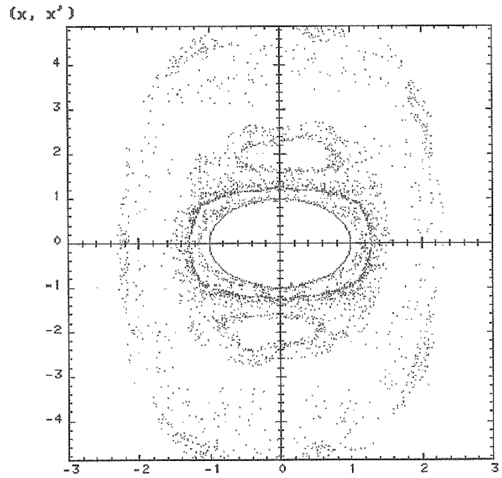

Fig. 4 : Poincaré section for a mismatched beam.
(uncoupled particle, σ_{0t} = 61.87659º and σ_t = 19.92206º)

5. SEARCH FOR ARNOL'D DIFFUSION AT LARGE AMPLITUDES

When there is more than two degrees of freedom, the KAM surfaces no longer isolate the stochastic areas which form a dense "Arnol'd web" [7, 4]. Diffusion of particles toward large amplitudes is then possible along this web with diffusion rates which increase when the size of the chaotic areas increases (due to a mismatch for example). This "Arnol'd diffusion" is a phenomenon which has been studied for numerous systems (see Ref.[4] for a matched beam in a FODO channel), but the random behaviour of the particle motions along the web highly complicates the calculation of low diffusion rates.

Regarding the preliminary study presented here, the beam core and FODO channel parameters are those of the previous section. To analyse the particle diffusion, 2500 test particles have been followed over 200 FODO periods in order to analyse the maximum radius (R_{max}) reached with a given initial radius (R_0). Figure 5 gives the values of $(R_{max}-R_0)/R_0$ for initial conditions (normalized with respect to the core envelope) in the range 0 < x or y < 6 and x' = y' = 0. The large stochastic area located near the beam core and the $\upsilon = 1/6$ resonance area can be clearely localized. But any diffusion from one area to the other can be observed. For the choosen parameters (weak mismatch), the diffusion rate seems therefore low and very difficult to estimate.

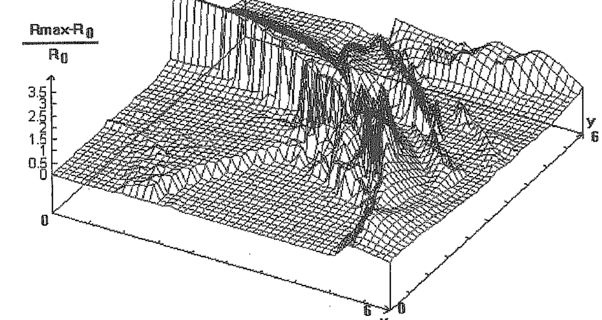

Fig. 5 : Maximum amplitudes reached by the test particles versus their initial positions (x, y, x' = y' = 0).

6. CONCLUSION

For the new generation of high-power accelerators, the most important aim is to limit the beam losses to an extremely low fraction of the total beam, the basic phenomena leading to emittance growth and halo formation must then be understood. In this paper as well as in ref. [1-4], we have tried to show that chaos induced by resonance overlap and Arnol'd diffusion can drive particles far from the beam core. Even if the studies are far from beeing exhaustive, they have shown that these phenomena become more and more significants when perturbations such as beam core modulations due to the quadrupolar focusing, synchrobetatron coupling or mismatchs are taken into account. Nevertheless, the random character of the particle motions will not facilitate the studies. As far as we know, analytic calculation of the diffusion rates will be practically impossible and several questions can be asked on the numerical experiments :

- What is the number of test particles requested to obtain a true evaluation of the diffusion rates? Can a judicious choice of the initial conditions reduce this number?
- Can the diffusion rates for some focusing periods be deduced from a tracking over a large number of periods?
- Do the round off errors allow an accurate evaluation of the diffusion rates?

It will be also very important to take into account the (nonlinear) longitudinal motions because Arnol'd diffusion can drive particles out of the separatrix, then leads to unbounded motions. This study is in progress.

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7. REFERENCES

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