# ION CORE PARAMETERS IN THE BENDING MAGNETS OF ELECTRON STORAGE RINGS

E. Bulyak, National Science Centre "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

# Abstract

Effective m ethods of decreasing the density of ions neutralizing the space charge of electron beams in storage rings are improving the vacuum conditions and application of the dearing electrodes. There are presented the results of both theoretical and numerical studies of the relation of ion density with the density and composition of the residual gas in the vacuum chamber. Steady state of the ion core was investigated for the case of the dipole magnetic field and electrostatic c learing electrodes. Analytical expressions describing the longitudinal, the vertical and the horizontal shapes of the core density are presented. It is shown that the core density and ionizing rates of ions is determined by the gas-to-beam d ensity ratio. Transverse density of ions is much s harper then the beam density, and the ion induced force is sufficiently nonlinear.

## 1. INTRODUCTION

Positive i ons captured by electron beams cause limitations in performance of the storage rings. The most effective method of extracting ions from the beam is insertion of the dearing electrostatic electrodes into the vacuum chamber of a storage ring. This dearing is effective if there exists the longitudinal (along the beam orbit) stream of ions, because the dearing plates cover a small part of the ring c ircumference. In this paper we present results from both t heoretical estimations and digital calculations of the ion core parameters in dipole magnet of the electron storage ring provided with dearing electrodes. Mainly we deal with the number of ions in the beam and its transverse density shape.

#### 2. MODEL

Let us assume the electron beam to be round in the cross section of radius *a* and have homogeneous density  $n_b(x,y,z)=n_b=\text{const}$ ,  $x^2+z^2<a^2$ . The frequency of bunches passing is much greater than the frequency of ion oscillations, so the beam is coasting for the ions. Thus,  $n_b$  denotes the average beam density.

This beam passes through a dipole magnet with a field strength of  $B_z=B=$ const, and the orbit length in the magnet is Y. At the magnet ends there are installed the electrostatic clearing electrodes. The vacuum chamber of the storage ring contains n eutral gas with a density of  $n_0$ , consisting of molecules of mass M and the ionization cross section  $\sigma_0$ . The positive i ons produced in collision of circulating electrons with the residual gas molecules are trapped by the electric field of the beam. These ions oscillate vertically (along the magnetic field lines) and circulate around ellipses

in the xy plane. The ellipses move along the beam axis (socalled drift in the crossed fields) and finally reach the clearing e lectrodes. The outer ellipses  $(x_c>0)$  d rift downstream the electron beam, the inner ellipses  $(x_c<0)$  -upstream. During t his drift the ions impart additional focusing to the beam particles.

We l imit our consideration to the case of low neutralization when the density of the ion core is negligibly lower than that of the beam, e.g. we consider the case of effective dearing.

Considering the ellipses as new objects we find the density of ellipse centers (EC)  $n_c$  at the distance y downstream the clearing station [1], which reads as:

$$n_c = \frac{cn_0\sigma_0 B(1+D)}{\pi e} y,$$

$$D \equiv n_b \frac{2\pi M c^2}{B^2} .$$
(1)

Integrating (1) over the beam cross section we obtain the total number of ions per unit orbit length:

$$N_{ion} = \frac{n_0 \sigma_0 Y B a (1+D)}{\pi e} \left\{ \log \left[ \frac{1+\sqrt{1-\chi^2}}{\chi} \right] - \sqrt{1-\chi^2} \right\}$$
(2)

 $\chi \equiv \frac{x_m}{a} (1+D)$ 

Here  $x_m$  is the minimal distance of the EC from the beam axis when the ion is considered to be drifting, i.e., reaching the dearing station.

The key point in our analysis is the determination of  $x_m$  value because the ion density in the beam is fully relevant on t hat parameter. The basic hypothesis of this paper consists in considering the ion core separated into 3 parts: the central rest part  $x_m < x < x_m$  and two streams mentioned above. The density of the central part is supposed equal to the average beam density (full neutralization of the beam charge). The density of both streams is matched to the central part at  $x=\pm x_m$ .

## **3. FINITE TEMPERATURE**

It is essential to take into account the initial ion temperature *T* to eliminate divergence in the vertical density shape of the core. This temperature does not eliminate divergence in the radial shape but decreases the core density at  $x_m < x < < a$ .

Finite-temperature taking into account does not change the distribution of EC (1). The transverse core density related to the EC distribution (1) is:

$$n_i(x,z) \sim F_x(x) \cdot F_z(x,z), \tag{3}$$

$$F_{x}(x) = \frac{1}{2} \int_{x_{m}/t_{x}}^{a/(1+D)t_{x}} \frac{B_{i}(y) d y}{\left(x - \left(x^{2} - (1 - D^{2})\sqrt{1 + y^{2}}\right)\right)\sqrt{x^{2} - (1 - D^{2})(1 + y^{2})}$$

$$F_{z}(x, z) = \frac{t_{z}}{a} \int_{-z^{2}/t_{z}}^{(a^{2} - z^{2})/t_{z}^{2}} \frac{B_{i}(y) dy}{\sqrt{y + z^{2}/a^{2}}}$$

$$B_{i}(y) = \frac{4}{\sqrt{\pi}} \exp(y) \int_{\sqrt{\max(y, 0)}}^{\infty} (u^{2} - y) \exp(-u^{2}) du$$

$$t_x = \frac{1}{eB} \sqrt{\frac{2kTMc^2}{3(1+D)}} \qquad t_z = \sqrt{\frac{kT}{3\pi e^2 n_b}}$$

The function  $B_i$  defined here describes the 'track' of the thermalised ion assemble produced at the point *u* with a 'zero temperature' amplitude  $\alpha$  and the center of oscillation  $u_0$ 

$$y = \frac{\alpha^2 - (u - u_0)^2}{t^2}$$

Examples of vertical and horizontal shapes are depicted in Fig.1.



Fig.1. Transverse ion core shapes for the case: beam current I = 50 mA, radius a = 0.05 mm, field strength B = 1 Tesla.

## 4. SURVEY OF THE RESULTS

Based on the approach proposed and expressions (1) and (2), the computer code has been developed which computes the longitudinal density (the neutralization factor  $\eta$ ) and the transverse density shape of the ion core confined by the electron beam in the dipole magnet of the storage ring with the clearing electrodes.

The results of computing are as follows:

(i) For the typical beam parameters in the synchrotron storage ring (a=0.05 cm, Y=1 meter, B=1 Tesla, residual gas pressure 1 nTorr of Nitrogen) ion clearing keeps the neutralization factor below the 5% limit (Fig.2). Therefore our assumption of low neutralization of the beam space charge is proved valid.



Fig.2. Neutralization factor  $\eta$  and density of ions  $n_i$  (in arbitrary units) versus beam current I (in Amperes), a = 0.05 mm, B = 1 Tesla, Y = 1 m.

(ii) The transverse core density is essentially nonlinear because the main part of ions is confined in the near-to-axis region (Fig.3);



Fig. 3. Relative ion core density in the beam central quarter. I = 50 mA, a = 0.05 mm, B = 1 Tesla, Nitrogen molecules.

(iii) The density of ions increases very little when the beam current is increased up. So, the neutralization factor decreases as it is depicted in Fig.2;

(iv) Response of the ion density to increase in the residual gas pressure (as well as the distance between the clearing stations Y) is less than proportional, see Fig. 4.



Fig. 4. Neutralization factor versus residual gas pressure (Nitrogen). I = 50 mA, a = 0.05 mm, B = 1 Tesla, Y = 1 m.

(v) The core density decreases when the magnetic field strength is increased, Fig. 5.



Fig.5. Neutralization factor versus magnetic field strength. I = 100 mA, a = 0.05 mm, P = 2 nTorr, Y = 1 m.

Thus, considering production of ions and their drift in the dipole magnets of the electron storage rings and supposing full neutralization of the beam charge without clearing we come to the conclusion that the clearing electrodes can de a help in keeping the ion density in the beam within the required limits of a few percents of the average beam density. Residual ions are located in the near-to-orbit region adding sufficiently nonlinear focusing forces to the beam particles [2]. Process of neutralization in the near-to-axis region of the beam needs thorough studying both analytically and experimentally.

## REFERENCES

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[2] Bulyak E. Ion driven effects in the intence electron beam circulating in storage rings Proc. of PAC95 (Dallas TX, May 1995)