CHROMATICITY COMPENSATION – BOOSTER SEXTUPOLES

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Abstract

Current Booster lattice is studied in the context of full chromaticity compensation in the presence of the sextupole fields generated by the combined function magnets. The sextupole excitation at various energies, found from chromaticity measurements and Booster lattice analysis, was compared with magnetostatic multipole calculations. Both results agree very well and they are consistent with the original design specifications. Two families of correcting sextupole magnets are employed to compensate the sextupole excitations and to adjust the chromaticity (in both planes) to a desired value, which is set by head-tail stability consideration. Analysis of the required correcting sextupole strengths is carried out along the momentum ramp with the measured sextupole excitations of the combined function magnets. The results of our calculation give quantitative insight into the requisite performance of the sextupole magnets. It calls for much stronger sextupole strengths – at the level which can no longer be supported by the present correcting sextupole magnet design.

I. INTRODUCTION – BOOSTER LATTICE

The Booster lattice is made up of 24 identical FDOODFO cells: horizontally focusing magnet – short drift space – horizontally focusing magnet – horizontally defocusing magnet – long drift space – horizontally defocusing magnet, provides room for the RF cavities within the standard cells. Since the lattice half-cell is not symmetric, the beam size is different in each magnet and, consequently, the focusing strengths of F and D combined function magnets are different. The magnets are assembled in 48 modules . Apart from the F and D magnets, each module consists of a choke, a capacitor bank, an ion pump, a set of correction magnets and a beam position monitor. Two trim correction magnet packages are placed in each period. Each package contains a horizontal dipole, a vertical dipole, a quadrupole and a skew quadrupole. There are also two families of correcting sextupoles, but they are not considered a part of the correction packages. All Booster corrections elements are air core magnets.

The nominal betatron frequencies in the horizontal and vertical planes are ν_h = 6.7 and ν_v = 6.8, respectively. Therefore, there are no second or third order structure resonances adjacent to the working diamond. The lattice tunes are set by the quadrupole strengths of the combined function magnets (focusing and defocusing).

As we will show in this study, more sextupole field is needed to compensate the net chromaticity to the desired level set by head-tail instability present in the Booster. This calls for either stronger sextupole magnets, or for larger number of correcting sextupoles. Both options are explored here. Possible performance improvement of the present air core sextupole magnet (enhanced sextupole strength) can be achieved by surrounding a sextupole magnet with an iron shell (to decrease the reluctance of the exterior magnetic path). The maximum enhancement level is estimated using magnetostatic calculation assuming an infinitely thick shell. The second option – putting additional sextupole correctors at various new locations, which have recently opened was also examined. Both options of stronger sextupole compensation were studied from the point of their impact on the dynamic aperture. No significant second order distortions effects were found, which supports our claim that one can safely add more sextupole field.

II. BOOSTER CHROMATICITY

The integrated sextupole strength, g, of an individual sextupole magnet of length L, in Tesla/m is introduced as follows

$$\Delta x' = -\left(\frac{1}{B_o\rho} \int_0^L \frac{1}{2} B''(l)\, dl \right) x^2 = -\frac{1}{B_o\rho}\, g\, x^2 , \quad (1)$$

where x, x' are generic coordinates of a transverse phase-space. Here $B_o\rho$ is the magnetic rigidity and B'' is the second derivative of the vertical magnetic field with respect to x.

Apart from two families of correcting sextupoles, there are also additional sextupole fields contributed by the 96 combined function magnets (F and D). The sextupole contribution from a combined function magnet is due primarily to pole geometry and remanent magnetization. Detailed numerical modeling of the multipole content of the F and D magnet geometries is presented in the next section.

For the purpose of our model, the sextupole content of each F and D magnet can be accounted for in the Booster lattice by inserting identical zero-length sextupoles at five equally spaced locations along each magnet. Significant variation of the horizontal and vertical beta functions along the F or D magnet calls for distributed sextupole contribution, rather than a lumped sextupole inserted at the middle of the magnet.

The goal of the two families of sextupoles (h and v) is to compensate the natural chromaticity, $\underline{\chi}^o$, in the presence of the F and D magnet sextupole excitations, S_F and S_D, to some desired value, $\underline{\chi}$. Assuming that the net chromaticity (in both planes) depends linearly on four independent sextupole sources (S_h, S_V, S_F, S_D) one can quite generally write down chromaticity in terms of eight sensitivity coefficients. Using matrix multiplication this relationship assumes the following compact form

$$\underline{\chi} = \underline{\chi}^o + \mathbf{M}\begin{pmatrix} S_h \\ S_v \end{pmatrix} + \mathbf{D}\begin{pmatrix} S_F \\ S_D \end{pmatrix} \quad . \qquad (2)$$

Here, the underlined symbols denote 2-dim column vectors (their components correspond to the horizontal and vertical planes). The bold face characters, **M** and **D**, represent two-by-two matrices – one can easily identify the eight sensitivity coefficients with the elements of the two matrices. One can notice in passing, that both **M** and **D** depend exclusively on the lattice properties. A generic sensitivity coefficient can be expressed in terms of the Twiss functions according to the following relationship

$$M_{\mu\nu} = \frac{1}{2\pi} \sum_{i(\nu)} \beta^{\mu}{}_i \; D^{\mu}{}_i \quad . \qquad (3)$$

Here, the summation $i(\nu)$ goes over locations of all sextupoles of a given family (ν), where $\beta^{\mu}{}_i$ and $D^{\mu}{}_i$ are values of the beta function and dispersion at those locations (μ indicates either horizontal or vertical Twiss functions).

Solving Eq.(3) with respect to the correcting sextupole strengths $\underline{g}$ (in Tesla/m) yields the following formula

$$\underline{g} = (B_o\rho)\,\mathbf{M}^{-1}\left[\underline{\chi} - \underline{\chi}^o - \mathbf{D}\begin{pmatrix} S_F \\ S_D \end{pmatrix}\right] . \qquad (4)$$

The above expression will be used to analyze the required sextupole strength as a function of changing momentum along the Booster ramp. The sensitivity coefficients for all four families of sextupoles, **M** and **D**, are simulated for the Booster lattice using MAD tracking code [1].

To complete the sextupole strength analysis, outlined by Eq.(4), one has to gain some insight into the sextupole excitations of the combined function magnets (F and D) and their variation with the B-field. This will be discussed in detail in the next section via magnetostatic simulation for both geometries of the F and D magnets.

Another independent way of obtaining information about the sextupole excitation of the combined function magnets comes from the beam measurement. Using available chromaticity measurement [2] with both families of correcting sextupoles (h and v) turned off, one can calculate the sextupole excitations of the F and D magnets at various energies along the ramp. This information could be recovered by solving Eq.(2) with respect to S_F and S_D. The corresponding expression is given below:

$$\begin{pmatrix} S_F \\ S_D \end{pmatrix} = \mathbf{D}^{-1}\left[\underline{\chi} - \underline{\chi}^o - \frac{1}{B_o\rho}\,\mathbf{M}\,\underline{g}\right] \qquad (5)$$

III. SEXTUPOLE CONTENT

The bending guide field in the Booster synchrotron is provided by 96 combined function magnets, each approximately 3 m long. The magnetic field varies from approximately 500 Gauss at injection up to 7000 Gauss at extraction. The magnets are powered in a resonant circuit by a 15 Hz sinusoidal waveform resulting in a field of the form [3]

$$B(t) = B_{min} + \frac{1}{2}\left(B_{max} - B_{min}\right)\left[1 - \cos(\omega t)\right] \quad . \qquad (6)$$

Calculations were performed using a standard finite element code (PE2D). The results confirmed the design dipole and focusing strength. Furthermore, higher multipole values (up to the 24-pole) were also calculated for both F and D magnets. The multipoles are normalized values at 1 inch. For the D magnet, the calculated dipole field was 6.65239×10^2 Gauss with an excitation of 1518 Ampere-turn and 6.18884×10^3 Gauss for an excitation of 14145 Ampere-turn. For the F magnet, 8.31474×10^2 Gauss for 1384 Ampere-turn and 7.68313×10^3 Gauss for 12900 Ampere-turn.

The Booster magnets are operated well below saturation and not surprisingly, the calculations show that there is no significant dependence of the field harmonics on the excitation current. The magnitudes of the dipole and quadrupole components of the field are in excellent agreement with the design values. As explained before, the sextupole component of the bending magnet magnetic field can be extracted from a beam-based chromaticity measurement. The beam-based (measured) [2] values of b_2 in $[m^{-2}]$ are listed as: 2.0×10^{-5} for the F magnet and -6.9×10^{-5} for the D magnet. One can see that the calculated sextupole components at 8 GeV are in good agreement with the values inferred from chromaticity measurements. The distinctive characteristic of the remanent magnetization contribution is that it tends to be relatively independent from the excitation. Therefore, when normalized with respect to the main field, the relative contributions from the remanent magnetization to the magnetic field are expected to gradually be reduced to zero as the excitation current is increased from its minimum to its maximum value.

IV. POTENTIAL IMPROVEMENTS TO AIR CORE SEXTUPOLES

It has been suggested to increase the strength of the existing air-core chromaticity correction sextupole magnets by introducing an external iron shell. The field enhancement effect due to an iron shell can be estimated by using the following result: for a filament of current located at (ρ, ϕ) inside a circular hole of radius R carved into a medium of relative permeability μ the complex coefficients $C_n e^{i\alpha_n}$ – the multipole expansion of the field are given below [4]

$$\frac{C_n e^{i\alpha_n}}{R^{n-1}} = -\frac{\mu_0}{2\pi} \int \frac{dI}{\rho^n}\left[1 + \frac{\mu - 1}{\mu + 1}\,\frac{\rho^{2n}}{R^{2n}}\right] e^{in\phi} \quad . \qquad (7)$$

For a pure sextupole current layer of inner radius a and outer radius b, with a uniform current density given by

$$J_3(\rho,\phi) = J_3 \cos 3\phi \qquad (8)$$

one can easily carry out the integration in the right hand side of Eq.(7), which reduces to the following simple expression

$$\frac{C_3}{R^2} = -\frac{\mu_0 J_3}{2}\left[\frac{b - a}{ab} + \frac{\mu - 1}{\mu + 1}\,\frac{b^5 - a^5}{5R^6}\right] . \qquad (9)$$

Under ideal conditions, i.e. $\mu = \infty$ (μ is about 100 for iron) and a = b = R, the sextupole field could be doubled. More realistically, let $a \approx b < R$ and $t = b - a << R$. The above expression becomes

$$\frac{C_3}{R^2} = -\frac{\mu_0 J_3\, t}{2a^2}\left[1 + \frac{\mu - 1}{\mu + 1}\,\frac{a^6}{R^6}\right] . \qquad (10)$$

It is probably unrealistic to expect an enhancement factor larger than 1.5 in a real device. We note that because the exterior field decays faster as the pole number increases, field enhancement with an iron shell is less effective for a sextupole than for a dipole magnet.

V. HEAD-TAIL INSTABILITY LIMITS

Following Sacherer's argument [5] the inverse growth-time as a function of chromaticity was evaluated for different slow head-tail modes (l = 0, 1, 2, 3) The l = 0 mode appears to be unstable above transition for small negative chromaticities and might lead to significant enhancement of coherent betatron motion. The obvious cure to stabilize the dipole mode [6] is to maintain appropriate sign (positive) of the net chromaticity. Otherwise, this potentially offending mode can be effectively suppressed by the active damper system. This efficient cure for the l = 0 mode obviously does not work in case of the higher modes, since its feedback system picks up only the transverse position of a bunch centroid, which remains zero due to the symmetry of the higher modes. Another possible cure especially effective for the $l \geq 1$ modes would involve the Landau damping, e. g. through the octupole-induced betatron tune spread. Increasing betatron amplitude of initially unstable mode causes increase of the tune spread, which will eventually self-stabilize development of this mode. Therefore, presented head-tail stability analysis suggests adjusting the net chromaticity at −7 (+7) units below (above) transition energy.

VI. CONCLUSIONS – SEXTUPOLE STRENGTH

Our analysis of the required correcting sextupole strengths, carried out along the momentum ramp with the measured and simulated sextupole excitations of the combined function magnets, concludes that maintaining the net chromaticity at the level set by head-tail instability limits requires much stronger sextupoles. The required sextupole strength is at the level, which can no longer be supported by the present correcting sextupole magnet design. One has to consider either a new iron core sextupole magnet design, or the upgraded air core magnets placed at all accessible high beta locations – the 'enhanced' sextupole layout, which is proposed in this paper. Quantitative assessment of the effect of the stronger compensating sextupoles on the dynamic aperture, carried out in terms of the distortion functions shows that the requisite sextupole configuration would not significantly enhance the third order resonance stop-band – the dynamic aperture remains at acceptable level.

References

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