A Dynamic Momentum Compaction Factor Lattice in the FERMILAB DEBUNCHER Ring

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I. Introduction

The primary purpose of the Fermilab Debuncher ring is twofold; to accept approximately 7μA/pulse of 8.9 GeV antiprotons ($\bar{p}$) downstream from the production target and to subsequently reduce the momentum spread, from $\Delta p/p \sim 4\%$ to $\sim .2\%$, and transverse emittance, from $\epsilon \sim 20\pi$ mm-mrad to $\sim 5\pi$ mm-mrad, for improved transfer and stacking performance in the Antiproton Accumulator ring. To accomplish this objective, rf- cavities are used to rotate and adiabatically debunch the beam on the time scale of ~ 40 msec, after which stochastic cooling systems, both transverse and longitudinal, are used to reduce the transverse emittance and longitudinal momentum spread throughout the remainder of the $\sim 2.4 sec\ \bar{p}$ production cycle.

In the initial and present design of the Debuncher ring, the momentum compaction factor (α), or equivalently the slip factor, $\eta = \alpha - 1/\gamma^2$, was chose to have a value which is a compromise between the two competing functions of the ring - that of accepting and debunching a large number of $\bar{p}$s/pulse and subsequently employing stochastic cooling feedback systems to precool before injection into the Accumulator. The goal of this experiment is to reconcile this compromise by changing η between two desired values during each $\bar{p}$ production cycle.

II. On Magnetic Optics Modifications to the FNAL Debuncher

The momentum compaction factor is the difference in the perimeter between the orbits of particles with differing momentum from that of the ideal design particle. Thus,

$$\frac{\Delta C}{C} = \alpha \frac{\Delta p}{p}$$

$$\alpha = \frac{1}{C} \oint \frac{D(s)}{\rho(s)} ds$$

These relations suggest the following simple fact, that in order to modify α of an existing lattice, it is sufficient to change the dispersion function.

The Debuncher ring has a circumference of 505 m and is composed of a sixfold symmetric seperated function optical lattice[1]. The basic arrangement of the ring consists of three long dispersion free straight sections together with arc sections consisting of 57 regular FODO achromats in total. Transition from the arcs to the straights is accomplished with a missing magnet scheme to produce a strictly zero horizontal dispersion in the straight sections for the purpose of locating rf- cavities and stochastic cooling devices. As a result of the dispersion killer chosen, each regular FODO cell has a phase advance of $\pi/3$. The ring operates on the positive side of transition with large dispersion in the arcs, thus limiting the momentum acceptance upon injection. The dispersion function in the arcs reaches a maximum value of 2.4 m and the maximum transverse beta functions are approximately 14 m with tunes typically operated at 9.79 horizontal, and 9.77 vertical.

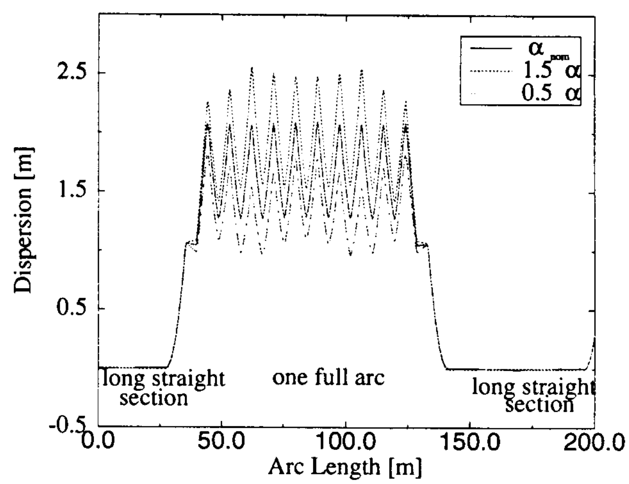

Figure. 1. Comparison of the dispersion function for different design cases to that of the nominal case with α(nominal)

To accomplish the task of uniformally changing the dispersion function in the arc sections, while maintaining a large number of practical constraints, interleaved localized dispersion waves were created by perturbing the strengths of judiciously chosen quadrupole pairs seperated by π in nominal betatron phase advance. Amongst some of the constraints were the requirements that: *(i)* perturbations to any quadrupole should not exceed $\approx \pm 20$ Amps due to the present power supply limitations, *(ii)* the tunes shifts in either plane may not exceed $\Delta\nu_{x,y} \sim \pm .005$ to avoid resonance crossing, *(iii)*the dispersion function remain strictly zero in the straight sections due to the location of stochastic cooling devices and rf- cavities, and *(iv)* the β functions not exceed 10% of their nominal values (except at the locations of the stochastic cooling pickup and kickers, for which one desires a large amplitude for increasing the sensitivity factor). Together with these constraints is the very important constraint that the phase difference between the pickup and kicker either remain the same or move closer to optimal for the correct phasing of the stochastic cooling system.

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A comparison of the dispersion functions in one sector for the nominal lattice and that for the lattice with a value of α 1.5 times larger is given in Figure 1. The lattice design for both large and small α maintain six- fold symmetry. Furthermore, the lattice designs for each case are symmetric in the required δI of each quadrupole from the nominal design lattice, i.e. it is merely a change of sign of each δI in the change from $.05\alpha \longrightarrow 1.5\alpha$.

A. Sensitivity to the Resonance Lines

A change in dispersion function in the arcs is accompanied with relatively large tune shifts, which must be removed through adjustments of the quadrupole fields in the zero dispersion straight sections. Thus it is required to understand the features of the resonance structure in the vicinity of the operating point. In particular, if the tunes are not properly corrected while differentially changing the lattice at each step with nonlinear ramps, relatively large tune excursions can occur during the ramping process for which beam loss may result due to resonance crossing. An experimental investigation was undertaken to map out the amplitude and widths of the resonances throughout a relatively large area of tune space ($\pm.1$ units in both planes) sufficient for the purposes of this project. The major resonance lines together with their relative width and strength are given in Table 1. Avoiding these resonances during the ramping process puts heavy restrictions on allowable tune excursions.

Resonance.	*% Beam Reduction*	*Width in tune units*
2/3	~ 100	$\pm.006$
3/4	~ 60	$\pm.003$
4th order sum	~ 45	$\pm.002$

III. On Improvements to the Stochastic Cooling of Particle Beams

With each revolution, the Schottky signal from a given group of particles is sampled, and that signal is applied (with a large gain) back upon the same sample of particles downstream from the pickup. If there is a large spread in revolution frequencies due to a large dispersion, then particles from adjacent samples will mix during subsequent turns through the ring. In the limit of complete mixing of particles between different samples, the cooling system would be most effective since it acts upon a statistically independent set of particles upon each revolution. This mixing of particles between adjacent samples is related to the optics of the storage ring through the momentum compaction factor, or equivalently the slip factor ($\alpha = 1/\gamma_t^2$, $\eta = \alpha - 1/\gamma^2$).

A quantity which is useful in quantifying the particle sampling of the cooling system is the mixing factor, M. The mixing factor is the number of turns it takes to mix a sample of particles. Presently, the numerical value of M in the Debuncher ring is approximately 12 at the end of the cooling cycle. An expression for the mixing factor assuming a gaussian distribution function ψ representing the density of particles, momentum spread σ_p/p, and bandwidth W is

$$M = \frac{\psi_0}{2W|\eta|\sqrt{\pi}\sigma_p/p}$$

Increasing the machine η results in a decrease in M, which increases the cooling rate. M appears naturally in the theory of stochastic cooling [2][3][4].

Estimates for an increased transverse stochastic cooling rate, due to the larger α, can be obtained through an integration of the first moment integral of the Fokker Plank equation which describes the time evolution of the transverse density function.

$$\frac{d\epsilon}{dt} = \frac{\psi_0}{N}\sum_l \Re[g_l(\omega_l)T(\omega_l)] - \frac{\pi N}{\Omega}|g_l(\omega_l)T(\omega_l)|^2[M(\omega_l) + U(\omega_l)]$$

g_l is the gain factor, T_l is the signal suppression, M_l is the mixing factor, U_l is the noise to signal ratio, $\Omega = 2\pi f_0$ is the revolution frequency, and the sum is over the Schottky bands in the cooling bandwidth.

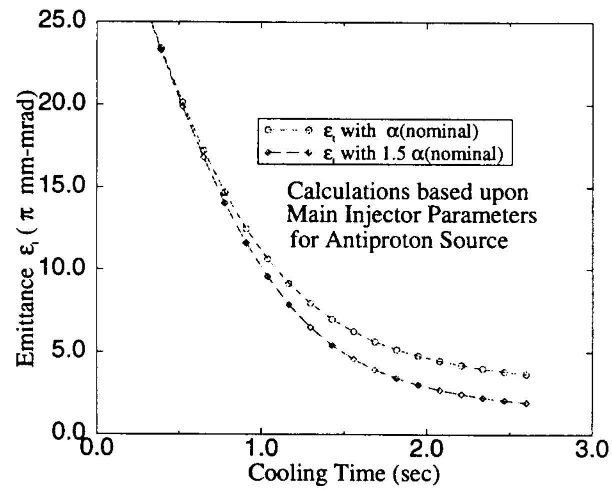

Figure. 2. Comparison of the transverse emittance ϵ_t as a function of time for the cases with and without a dynamic momentum compaction factor.

Using model fits to experimental data of the cooling system under the present conditions, Figure 2 compares two cases of emittance cooling with and without the application of a dynamic momentum compaction factor. The theoretical predictions are based upon future design upgrade parameters necessary for Main Injector operation. The model predicts an improvement to the transverse cooling rate of $\sim 15\%$. An improvement to the Debuncher to Accumulator (D/A) transfer efficiency is expected to be nearly commensurate on the basis of Figure 3, in which the increases in the D/A efficiency has been correlated to a smaller beam size prior to transfer. In that figure, quantities are plotted as a function of the amount of time that beam is allowed to remain in the Debuncher, or equivalently the production cycle time. Beyond the improvments in D/A transfer efficiency, there is also sufficient evidence suggesting the Accumulator $\bar{p}$ stacking rate is a strong function of the beam size upon injection.

IV. On a Larger Momentum Acceptance and Better Longitudinal Phase Space Rotation

A reduction of the momentum compaction factor from the nominal design value is expected to have two effects. First, a smaller value of η will allow for a larger momentum acceptance upon injection of antiprotons into the Debuncher. The second effect is to increase the rf- bucket.

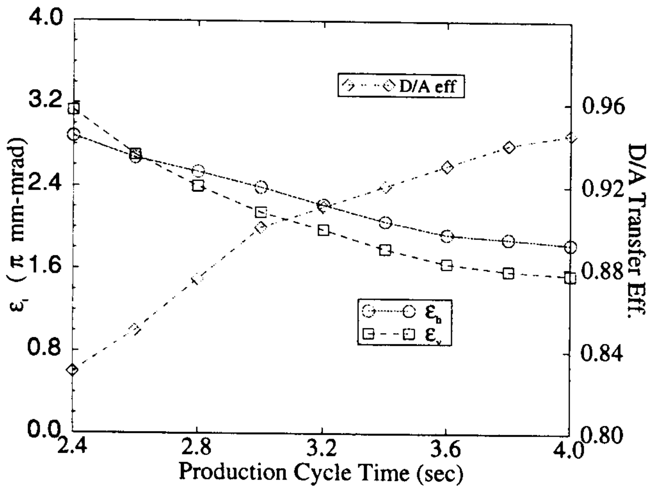

Figure. 3. Debuncher to Accumulator (D/A) transfer efficiency as a function of the duration of the Production cycle. Also plotted are the transverse emittance in the Debuncher illustrating the correlation of improved D/A efficiency with decreased beam size.

With regard to the first effect, Figure 4 is a comparison between the beam size for the nominal lattice (α_{nom}) design and that for a design with a $.48 \times \alpha_{nom}$ with an emittance of 20π mm-mrad and $\Delta p/p = 4\%$. Given the reduction in beam size for the small α lattice, a 25% increase in momentum acceptance of antiprotons upon injection is possible.

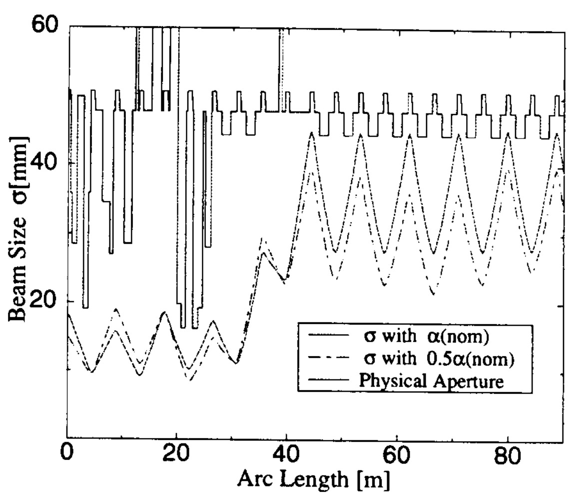

Figure. 4. Comparison of the beam size σ between the present Debucher lattice design having α and that with 0.5α. $\epsilon_t = 20\pi$ mm-mrad and $\Delta p/p = 4\%$

The narrow time structure of the antiprotons inherited by the the proton beam after targeting from the Main Ring is exchanged for a narrow energy spread through longitudinal rf- bunch rotation through 1/4 of a synchrotron period inn the Debuncher ring.

The longitudinal dynamics can be described by the differential equation

$$\frac{d^2\phi}{dt} + \frac{eVh\eta\omega_o^2\cos\phi_s}{2\pi E_o\beta^2\gamma} b(\phi - \phi_s) = 0$$

where the synchronous frequency is given by

$$\Omega_s^2 = \frac{eVh\eta\omega_o^2\cos\phi_s}{2\pi E_o\beta^2\gamma}$$

For the Debuncher $V = 5$MV, $f_o = 0.590035$MHz, and $h = 90$. From these equations, it can be shown that the phase space area of the oscillatory orbits is inversely proportional to $\sqrt{\eta}$. A larger phase space area is beneficial for the rf bunch rotation process to avoid filamentation, which lead to lower debunching efficiencies.

V. Conclusions

Commissioning of the project described in this paper is underway. At the present time, we are implementing only the case for $\alpha \longrightarrow 1.5\alpha$. The largest gains of this project, in general, both through an 15% increase in the cooling rate and a large increase ($\sim 25 - 30\%$) in momentum acceptance, are expected to come to fruition in the Main Injector era.

References

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