Operating Conditions of High-Power Relativistic Klystron

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An electron beam pre-modulated at the first cavity in a klystron enters the second cavity opening, exciting it. Induced voltage at the second cavity in a high-power klystron forms a virtual cathode momentarily, sending back a part of the beam toward the first cavity. The relationship between the induced voltage and the return current at the first cavity is investigated. The boundary between the amplifier and oscillator operation regions is described in the parameter space defined by the return current strength and inter-cavity distance.

I. INTRODUCTION

There is a growing body of literature on theoretical and experimental studies of relativistic klystron amplifiers driven by modulated intense relativistic electron beams. The relativistic klystron amplifier (RKA) exploits the strong self-electric field, which effectively modulates the beam current, thereby enhancing electron bunching and amplifier efficiency. The frequency and efficiency of the current modulation in a RKA are monochromatic and almost 100 percent in appropriate system parameters. One of the main issues in present RKA development is the enhancement of power and frequency simultaneously. The size and opening of the cavities in RKA should be reduced, to increase the excitation frequency. Therefore, a high-power high-frequency klystron amplifier has inherent problems due to reduced cavity size, including electron emission and ac beam loading at cavity gap opening. However, if the induced voltage at the second cavity is high enough, it forms a virtual cathode and reflects part of the electron beam back to the first cavity. The return beam from the second cavity enters the first cavity opening and excites it further if the return current modulation is in phase with first cavity excitation. The in-phase return-current modulation may reduce the ac beam loading at the first cavity, significantly improving the klystron performance. As a proof-of-principle experiment, Serlin and Friedman[1] built the two-beam klystron, where two annular electron beams propagate through a grounded tube. These beams are pre-modulated at the first cavity by input microwaves. Because the inner beam energy is considerably less than the outer beam energy, part of the inner beam is reflected by the virtual cathode formed at the second cavity and further excites the first cavity. Significant improvement of the current modulation has been reported from this experiment.[1] A theory describing the relationship between the induced voltage and the modulated return current at the first cavity opening is developed. Boundaries defining the amplifier and oscillator operation regions are also described in terms of the normalized return-current strength h and the inter-cavity distance represented by the phase angle α.

This work was supported by IR Fund at NSWC

II. TWO BEAM KLYSTRON

High-level stable excitation of the first cavity is very important for current modulation in a high-performance klystron. The first cavity is excited first by external input microwaves, which have enough pulse length, saturating the induced voltage to the steady-state value ϕ_w. Sometime during this microwave pulse, the electron beam is allowed to enter the klystron. The relationship between the induced voltage and the ac return current in the first cavity opening can be found from an equivalent circuit representation of the cavity impedance L_1, C_1 and R_1. The inductance L_1 and capacitance C_1 are related to the resonance frequency ω_1 of the cavity by $\omega_1 = (L_1C_1)^{-1/2}$ and the cavity Q-value is related to the resistance R_1 of the equivalent circuit by $Q_1 = \omega R_1 C_1$ (Ref.2). The resonance frequency ω_1 of the first cavity is assumed to be in resonance with the input microwave frequency ω, i.e., $\omega_1 = \omega$. The intensity of the return current is unknown. However, the level of the return current modulation increases as amplitude ϕ_2 of the induced gap voltage at the second cavity increases. Note that the amplitude ϕ_2 is proportional to the amplitude ϕ of the induced voltage at the first cavity.[3] In this regard, we assume that the return current modulation is proportional to the amplitude ϕ of the induced voltage at the first cavity.

Collecting all terms together, the induced gap voltage $V_1(t)$ at the first cavity can be calculated from[2]

$$\frac{d^2V_1}{dx^2} + \frac{1}{Q_1}\frac{dV_1}{dx} + V_1 = \frac{1}{Q_1}\phi_w \sin. \qquad (1)$$

$$+ f_s \phi(x) \sin[x - \Psi(x) + \alpha],$$

where f_s represents the intensity of the return current and other coupling mechanisms, the variable x is the normalized time defined by $x = \omega t$, and the phase angle α is related to the inter-cavity distance L by

$$\alpha = \Psi_3 - \frac{\omega L}{c}\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right) - \Psi_2. \qquad (2)$$

In Eq. (2), $\beta_1 c$ and $\beta_2 c$ represent velocities of the forward and backward beams, Ψ_2 is the phase shift of the induced voltage at the second cavity relative to the forward current modulation, and Ψ_3 is the phase shift of the return current due to reflection at the virtual cathode. In Eq. (2), the term proportional to ϕ_w represents the contribution from the input microwaves and the term proportional to f_s originates from the incoming return current. In obtaining Eq. (2), we have assumed that the induced gap voltage $V_1(t)$ is expressed as

$$V_1(t) = \phi(x)\sin[x - \Psi(x)], \qquad (3)$$

where $\phi(x)$ and $\Psi(x)$ are amplitude and phase shift, respectively, of the induced voltage at the first cavity. They are slowly varying functions of time x. We assume that the input microwaves and modulated return current drive the excitation of the first cavity, which accommodates the driving signals by changing its amplitude and phase. Thus, selecting the time frame in which the phase is a non-zero value of $\Psi(x)$ as shown in Eq. (3) is quite appropriate in the subsequent theoretical analysis.

Substituting Eq. (3) into Eq. (1), and defining the normalized amplitude Y and normalized time y by

$$Y = \phi(y)/\phi_w, \quad y = x/2Q_1 = \omega t, \quad \textbf{(4)}$$

we find the equations which govern the phase Ψ and amplitude Y. They are

$$\frac{d\Psi}{dy} = \frac{\cos\Psi}{Y} + h\cos\alpha \quad \textbf{(5)}$$

and

$$\frac{dY}{dy} + (1 - h\sin\alpha)Y = \sin\Psi, \quad \textbf{(6)}$$

where the normalized return-current strength h is defined by $h = f_s Q_1$. Although the parameter f_s is a small number, the normalized return-current strength h can easily be on the order of unity because of a large cavity-Q value. Before solving Eqs. (5) and (6), we assume the initial condition that at time y = 0, the electron beams enter the system, thereby turning on the terms proportional to h in Eqs. (5) and (6). Otherwise, the cavity is saturated by the microwave input at y < 0 and the initial conditions for the phase and amplitude are given by $\cos[\Psi(0)] = 0$ and $Y(0) = \sin[\Psi(0)]$ at y = 0. These conditions are equivalently expressed as $\Psi = \pi/2$ and Y = 1 at y = 0. After a careful examination of Eqs. (5) and (6), we note the functional properties of $Y(\pi-\alpha) = Y(\alpha)$ and $\Psi(\pi-\alpha) = \pi - \Psi(\alpha)$. Therefore, the amplitude Y and phase shift Ψ for $\alpha = \pi - \alpha_1$ can be expressed by those for $\alpha = \alpha_1$.

The homogeneous solution Y_h to Eq. (6) increases exponentially, provided $h\sin\alpha > 1$, which is called the self-excitation. On the other hand, when the phase angle α satisfies $h\sin\alpha < 1$, the solution Y to Eq. (6) is bounded and the klystron is the amplifier operation region. The boundary between the amplifier and oscillator regions in the (α,h) parameter space can be illustrated and the border line is obtained from $h\sin\alpha = 1$.

Amplifier Operation: In the amplifier operation region characterized by $h\sin\alpha < 1$, the solution Y to Eq. (6) is bounded, and the steady-state values of the amplitude Y and phase Ψ induced at the first cavity opening are determined by $d\Psi/dy = dY/dy = 0$ at the time $y = \infty$. Thus, after a straightforward calculation, we obtain

$$\begin{aligned} \cos\Psi_1 + \chi h\cos\alpha &= 0, \\ \sin\Psi_1 + \chi h\sin\alpha &= \chi, \end{aligned} \quad \textbf{(7)}$$

for amplifier operation from Eqs. (5) and (6). In Eq. (7), $\chi = \phi_1/\phi_w$ and Ψ_1 are steady-state values of the amplitude and phase shift. It is important to find in what parameter regime the steady-state value χ is larger than unity. The modulated return current amplifies the induced voltage only in this parameter regime. Otherwise, the return current dampens the induced voltage. To find boundary of the amplifying region, we substitute $\chi = 1$ into Eq. (7) and obtain

$$\begin{aligned} \cos\Psi_1 + h\cos\alpha &= 0, \\ \sin\Psi_1 + h\sin\alpha &= 1. \end{aligned} \quad \textbf{(8)}$$

We remind the reader that the phase shift Ψ_1 satisfies $\sin\Psi_1 > 0$ for $\chi > 0$. After a straightforward algebraic manipulation, Eq. (8) is expressed as $h = 2\sin\alpha$ for $0 < \alpha < \pi$. Note that the value of the parameter h in Eq. (8) at $\alpha = \pi/4$ or at $\alpha = 3\pi/4$ is $2^{1/2}$. The curves obtained from $h = 2\sin\alpha$ represent the boundary of the amplifying region in the (α,h) parameter space. For a specified value of the normalized return-current strength h, the amplifying region is defined by

$$\sin^{-1}\left(\frac{h}{2}\right) < \alpha < \pi - \sin^{-1}\left(\frac{h}{2}\right), \quad \textbf{(9)}$$

where h is less than $2^{1/2}$. To investigate transient behavior of the induced voltage $V_1(t)$, we solve the coupled differential equations (5) and (6) numerically. As expected, we find from the numerical calculation that the amplitude Y and phase shift Ψ approach their steady-state values as time goes by. The closer the steady-state amplitude to unity, the quicker the transient behavior dies out. In the limit of the angle $\alpha = \pi/2$, we note $d\Psi/dy = 0$ from Eq. (5), and Eq. (6) is simplified to

$$\frac{dY}{dy} + (1 - h)Y = 1, \quad \alpha = \frac{\pi}{2}. \quad \textbf{(10)}$$

Solution to Eq. (10) is given by

$$Y = \frac{1}{1-h}\{1 - h\exp[-(1-h)y]\}, \quad \textbf{(11)}$$

which eventually saturates to $Y = \chi = 1/(1-h)$. The maximum amplification of $1/(1-h)$ occurs at $\alpha = \pi/2$, which is called the in-phase condition. The steady-state amplitude at $\alpha = \pi/2$ increases to infinity as the strength h approaches unity. This observation may mislead the outcome of practical present experiments. When $h \to 1$, Eq. (11) is further simplified to $Y = 1 + y$, which increases linearly in time. Therefore, amplification for $\alpha = \pi/2$ and h = 1 is limited by the electron beam pulse. In the out-of-phase case characterized by $\alpha = -\pi/2$, the solution to Eq. (6) is given by

$$Y = \frac{1}{1+h}\{1 + h\exp[-(1+h)y]\}, \quad \textbf{(12)}$$

where the return current dampens significantly the gap voltage induced by the microwaves.

Oscillator Operation: It is pointed out that Eq. (1) for the induced gap voltage at the first cavity is a linear equation, which is an excellent representation for an amplifier operation. However, mentioned earlier, the amplitude Y in Eq. (6) increases exponentially in the oscillator operation region satisfying $h\sin\alpha > 1$. In reality, the term proportional to the parameter f_s in Eq. (1), which represents the modulated return current, may stop to grow as the amplitude ϕ approaches saturation. For example, the location at which the maximum current modulation of the forward beam occurs, starts to shift toward the first cavity from the second cavity location, if the amplitude ϕ increases significantly.[3] Remember that the second cavity location was initially selected for a maximum forward current modulation of moderate amplitude ϕ. Once the maximum modulation location starts to shift, the term proportional to f_s in Eq. (1) does not increase linearly with ϕ; instead, it may start to saturate. We also observe that the modulated return current originates from reflection at the second cavity. As long as the return current is much less than the forward beam current, it may be proportional to the excitation level of the second cavity, which is also proportional to the first cavity excitation. This assures linearity in Eq. (1). If the return current is a substantial fraction of the forward current, due to lack of a sufficient amount of the forward current, it may start to saturate as the cavity excitation increases. There may be other saturation mechanisms for the modulated return current as the amplitude ϕ of the induced voltage grows. In this regard, Eqs. (5) and (6) for the oscillator operation are modified to

$$\frac{d\Psi}{dy} = \frac{\cos\Psi}{y} + h(1-\epsilon Y^2)\cos\alpha,$$
$$\frac{Y}{y} - [h(1-\epsilon Y^2)\sin\alpha - 1]Y = \sin \qquad \textbf{(13)}$$

where the nonlinear saturation coefficient ϵ is much less than unity in a typical klystron. Equation (13) is a typical van der Pol equation for a forced oscillator. Obviously, the terms proportional to ϵY^2 in Eq. (13) provide a saturation of the amplitude. The normalized saturation amplitude Y_s is obtained from Eq. (13) and is given by

$$Y_s = \sqrt{\frac{h\sin\alpha - 1}{\epsilon h\sin\alpha}}, \qquad \textbf{(14)}$$

which is typically much larger than unity, i.e., $Y_s >> 1$. As expected from Eq. (11), the maximum amplitude of the induced voltage appearing on the first cavity occurs at the in-phase angle $\alpha = \pi/2$ for either the amplifier or the oscillator operation.

From a numerical calculation of Eq. (13), we find that the amplitude Y for an oscillator grows exponentially at the beginning and then executes a small oscillation about the saturation value Y_s. Meanwhile, the phase shift Ψ increases almost linearly in time. For a large-amplitude operation typical of the klystron oscillator, the phase shift equation is approximated by $d\Psi/dy = \cot\alpha$ from Eq. (13) and the frequency shift at the saturation is expressed as

$$\delta\omega = \omega\cot\alpha/2Q_1. \qquad \textbf{(15)}$$

The frequency shift $\delta\omega$ of the oscillator in Eq. (15) is determined in terms of the phase angle α and the cavity Q-value. The in-phase condition of $\alpha = (0.5 - 2n)\pi$, at which the modulated return current is in phase with the first cavity excitation, can be expressed in terms of the inter-cavity distance L, once the phase shifts Ψ_2 and Ψ_3 are known. Here, n is an integer. When segments of the forward beam arrive on the second cavity, a certain limited portion of the beam will be reflected at the cavity. The phase of the return current may be very close to the phase of the induced voltage at the second cavity. We thus approximate $\Psi_3 = \pi/2$. According to a previous study,[2] the optimum current modulation occurs at the phase shift satisfying $0 < \Psi_2 < \pi/2$. We assume $\Psi_2 = \pi/4$, which is the value corresponding to the middle in the allowable range of the phase Ψ_2. The error associated with this assumption is one-sixteenth of the wavelength or less in the klystron. Substituting these phase shifts into Eq. (2), the in-phase condition is simplified to

$$\frac{L\omega}{c}\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right) = \left(2n - \frac{1}{4}\right)\pi. \qquad \textbf{(16)}$$

The second cavity should be located where the forward current has a maximum modulation. The second cavity location is therefore determined in terms of the beam parameters and geometrical factor G. Maximum modulation location is given by[3]

$$z_o = \beta_1^2\gamma_1^2\frac{\pi c}{2\omega}\sqrt{\frac{\gamma_1}{2\nu G}}, \qquad \textbf{(17)}$$

where $\gamma_1^{\,2} = (1-\beta_1^{\,2})^{-1}$ and ν is Budker's parameter of the forward beam. As a numerical example, we consider the physical parameters of the two-beam klystron experiment[1] at the Naval Research Laboratory. The theoretical result from Eqs. (16) and (17) predicts the optimum inter-cavity distance L = 14 cm, which is close enough to the experimental observation of L = 14.1 cm. We also observe from this numerical example that a deviation of about 13 percent (ΔL = 2 cm) from 14 cm results in a significant reduction of the current modulation, which also agrees with the experimental observation.

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