THE RESISTIVE-WALL KLYSTRON AS A HIGH-POWER MICROWAVE SOURCE

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A novel high-power high-frequency klystron is presented in which a relativistic electron beam is modulated at the first cavity and propagates downstream through a resistive wall. Because of the self-excitation of the space charge waves by the resistive-wall instability, a highly nonlinear current modulation is accomplished. Due to the relatively large growth rate of the instability, the required tube length of the klystron is short for most applications.

I. INTRODUCTION

Recent experiments with the relativistic klystron amplifier (RKA) indicate that the frequency and efficiency of current modulation are monochromatic and almost 100 percent in appropriate system parameters[1]. One of the main issues in the RKA is enhancement of power at high frequency. The size and opening of cavities in the RKA should be reduced, to increase the excitation frequency. Therefore, a high-power high-frequency klystron amplifier has inherent problems due to reduced cavity size, including electron emission and ac beam loading at the gap opening of cavities. To minimize these problems, we propose to eliminate the intermediate cavities and to modulate beam current by means of the resistive-wall instability. In the conventional klystron, the beam energy is modulated at the first cavity and this modulation is reinforced in the intermediate cavities before the beam segments arrive in the extraction cavity.[1] In the resistive-wall klystron, the energy modulation at the first cavity initiates self-excitation of the space charge waves in the resistive medium of the wall. Because of the self-excitation of these waves, a relatively low-level modulation is needed at the first cavity. A highly nonlinear current modulation of the electron beam is accomplished as the beam propagates through the resistive tube. The frequency of the resistive-wall instability is lower than the cut-off frequency in the waveguide and the microwaves cannot propagate through the drift section by themselves. Thus, these self-excited space charge waves move together with beam segments, further modulating their current. The resistive-wall instability has been interesting subject to other areas, including circular high energy particle accelerators and heavy ion inertial fusion.

II. RESISTIVE-WALL INSTABILITY

The system configuration of the resistive-wall klystron consists of a relativistic electron beam with radius R_b propagating through a resistive tube. A strong, externally applied magnetic field is needed to confine the beam electrons radially.

The radius of the grounded resistive tube in the klystron is denoted by R_w. This drift tube is wrapped by a cylindrical conductor with radius R_c. The conductivity of the resistive medium in the range of $R_w < r < R_c$ is denoted by σ. The skin depth δ of the resistive medium is assumed to be much less than its thickness ΔR, i.e., $\delta = c/(2\pi\mu\sigma\omega)^{1/2} << \Delta R = R_c - R_w$, where μ is the permeability of the resistive medium, ω is the oscillation frequency of the electric field, and c is the speed of light in vacuum. The first cavity in the klystron is excited by input microwaves with frequency ω. We also assume the initial condition that the beam segment labeled by $\theta = \omega\, t_0$ passes through the opening of the first cavity at time $t = t_0$. Note that the first cavity is located at z = 0. Then the energy gain $\Delta\gamma mc^2$ of the segment θ is expressed as

$$\Delta\gamma = -e\,(\Delta\phi/mc^2)\,\sin\theta \quad , \tag{1}$$

where Δθ is the maximum voltage at the cavity, and -e and m are the charge and rest mass of electrons, respectively. The energy gain or loss at the first cavity introduces the initial perturbation needed in the resistive-wall instability.

Any charge and current deviations from their equilibrium values will generate the axial component $E_z(r,z,t)$ of the perturbed electric field downstream. This perturbed electric field is obtained from the Maxwell equations. After a straightforward algebraic manipulation, we obtain

$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\mu}\frac{\partial}{\partial r}E_z\right)+\frac{1}{\mu}\frac{\partial^2}{\partial z^2}E_z-\frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}E_z \\ &-\frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}E_z=4\Pi\left(\frac{1}{c^2}\frac{\partial}{\partial t}J_z+\frac{1}{\mu\in}\frac{\partial}{\partial z}\rho\right), \end{aligned} \tag{2}$$

where J_z is the axial component of the beam current density.

In order to make the subsequent calculation analytically simple, we assume that $\mu\in\beta^2>>1$, where μ and ∈ are the permeability and dielectric constant, respectively, of the resistive medium, and βc is the instantaneous beam velocity. This inequality can easily be satisfied for a broad range of system parameters. In addition, the resistive medium is conductive rather than dielectric, satisfying $c/R_w\omega>>(\delta/R_w)^2(\mu\in)^{1/2}$. Thus, the term proportional to $\in(\partial^2/\partial t^2)$ in Eq. (2) is negligibly small in comparison with the term proportional to 4πσ. The charge and current densities in the right-hand side of Eq. (3) are approximated by

$$\frac{\partial J_z}{\partial t}=\omega\left(\frac{\partial J_z}{\partial\theta}\right)_z,\quad \frac{\partial\rho}{\partial z}=-\frac{\omega}{\beta c}\left(\frac{\partial\rho}{\partial\theta}\right)_z, \tag{3}$$

where use has been made of the definition $\theta=\omega t_0$. Because the phase velocity of the perturbations in a specified beam

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segment is very close to the beam velocity, the charge density is approximately related to the current density by $J_z \simeq \beta c \rho$. Assuming that the radial variation of the axial electric field dominates its axial variation, which is common in the relativistic klystron application, we can neglect the terms proportional to $\left(\partial^2 / \partial z^2\right)$ in Eq. (2). Making use of all these assumptions, Eq. (2) is simplified to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) - \frac{4\Pi\sigma\mu}{c^2}\frac{\partial}{\partial t}E_z = \frac{4\Pi\mu}{c^2}\frac{\partial}{\partial t}J_z \ , \qquad (4)$$

for the electric field in the range of $R_w < r < R_c$ and

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}E_z\right) = -\frac{4\Pi}{\gamma^2\beta^2 c^2}\frac{\partial}{\partial t}J_z \ , \qquad (5)$$

for the electric field in the range of $r < R_w$. In Eq. (5), the relativistic mass factor γ is defined by $\gamma^2 = (1-\beta^2)^{-1}$.

Whenever the beam current changes, the induced electric field $E_z(r,z,t)$ calculated from Eqs. (4) and (5) appears in the system. In terms of the Maxwell equation, the induced electric field is contributed by the change of the radial electric field E_r and the azimuthal magnetic field B_θ. However, we note in the resistive medium that the induced electric field due to the radial electric field is negligible in comparison with that due to the azimuthal magnetic field. In this context, the axial electric field E_z in the resistive medium is related to the azimuthal magnetic field B_θ by $(\partial/\partial r)E_z = (1/c)(\partial/\partial t)B_\theta$. Differentiating both sides of Eq. (4) by r, we obtain

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\theta\right)\right] - \frac{4\Pi\sigma\mu}{c^2}\frac{\partial}{\partial t}B_\theta = \frac{4\Pi\mu}{c}\frac{\partial}{\partial r}J_z \ , \qquad (6)$$

for the azimuthal magnetic field B_θ inside the resistive medium ($R_w < r < R_c$). The electric field penetrates a few skin depths into the resistive medium. The electric field E_z in the resistive medium is proportional to $\exp[-(r-R_w)/\delta]$. From a practical point of view, the radius ratio R_c/R_w is infinite if the thickness ΔR of the resistive medium is more than a few times the skin depth. The error associated with a finite value of R_c is on the order of $\exp(-\Delta R/\delta)$ or less. In the evaluation of the axial electric field at the inner surface of the drift tube ($r = R_w$), we thus assume that the radius R_c of the wrapping conductor is much larger than the inner radius R_w, i.e., $R_c / R_w \to \infty$.

One of the most important issues in the resistive-wall instability is the evaluation of the axial electric field at the inner surface ($r = R_w$) of the resistive medium. In the case when the skin depth of the resistive medium is much less than the wall radius R_w($\sigma << R_w$), Eq. (6) is represented by a planar approximation and the axial electric field at the inner surface of the resistive medium is given by[7]

$$E_z\left(R_w,z,t\right) = -\sqrt{\frac{2}{\Pi}}\frac{\mu\omega}{c^2}\frac{\delta}{R_w}\int_{\theta_h}^{\theta}\frac{d\theta'}{\sqrt{\theta-\theta'}}\frac{\partial}{\partial\theta'}I(z,\theta'), \qquad (7)$$

which is employed for the subsequent analysis of the resistive-wall instability. Here $I(z,\theta)$ is the instantaneous beam current. In Eq. (7), the normalized time $\theta_h = \omega t_h$ represents the time $t = t_h$ at which the head of the electron beam passes through the first cavity.

In order to find the axial electric field $E_z(r,z,\theta)$ in the range of r satisfying $r < R_w$, we solve Eq. (5) and make use of the boundary value $E_z(R_w,z,\theta)$ in Eq. (7). After carrying out straightforward algebra, we find the average axial electric field

$$E(z,\theta) = 2G\left(R_b\right)\frac{\omega}{\beta^2\gamma^2c^2}\left(\frac{\partial I}{\partial\theta}\right)_z + E_z\left(R_w,z,\theta\right), \qquad (8)$$

which acts on the beam electrons in the segment labeled by θ. In Eq. (8), $G(R_b)$ is the geometric factor of the configuration, i.e.,

$$G\left(R_b\right) = \begin{array}{ll} \ln\left(R_w/R_b\right) + 0.25, & \textit{solid beam}, \\ \ln\left(R_w/R_b\right), & \textit{hollow beam}. \end{array} \qquad (9)$$

Velocity modulation of the beam segment labeled by t_0 is obtained from

$$mc^2\frac{d}{dz}\gamma = -eE, \qquad (10)$$

with the boundary condition

$$\gamma_0(\theta) = \gamma_b - \left(e\Delta\phi / mc^2\right)f(\theta), \qquad (11)$$

where E is the average axial electric field in Eq. (8) and γ_b is the initial relativistic mass factor of electrons before they pass through the cavity opening. For convenience in the subsequent analysis, we define the normalized current $F(\zeta,\theta)$ by

$$F(\zeta,\theta) = I(\zeta,\theta)/I_b, \qquad (12)$$

where the normalized propagation distance $\zeta = \omega z/\beta_b c$ and I_b is the injection current of the beam before it passes through the first cavity. It is also useful in the subsequent analysis to introduce the Budker's parameter υ of the beam defined by $\nu = -eI_b / m\beta_b c^3$.

Substituting Eqs. (8) and (9) into Eq. (11), we obtain the relativistic mass factor $\gamma(\zeta,\theta)$,

$$\frac{\gamma}{\gamma_b} = 1 - \frac{e\Delta\phi}{\gamma_b mc^2}\sin\theta + \int_0^{\zeta}d\zeta'[\frac{2G\nu}{\gamma_b{}^3}\left(\frac{\partial F}{\partial\theta}\right)_{\zeta'} - \sqrt{\frac{2}{\Pi}}\frac{\nu\beta_b^2}{\gamma_b}\frac{\delta}{R_w}\int_{\theta_h}^{\theta}\frac{d\theta'}{\sqrt{\theta-\theta'}}\left(\frac{\partial F}{\partial\theta'}\right)_{\zeta'}], \qquad (13)$$

where use has been made of the assumption that the instantaneous velocity βc of the beam segment θ is close to the beam injection velocity $\beta_b c$. The instantaneous velocity $\beta(\zeta,\theta)c$ of the beam segment θ is expressed as

$$\beta_b / \beta = 1 + (\gamma_b - \gamma) / \gamma_b (\gamma_b^2 - 1). \tag{14}$$

Making use of the velocity definition dz/dt = βc and the definition φ = ωt, we obtain the relation

$$\varphi - \theta = \int_0^{\zeta} \frac{\beta_b}{\beta} d\zeta, \tag{15}$$

where the normalized propagation distance $\zeta = \omega z/\beta_b c$.

The beam current at the injection point is a constant value of I_b. The beam segment t_0 passes the injection point at time $t = t_0$. When this segment arrives at z in time t, it is stretched by a factor of dt/dt_0. Thus, the beam current of the segment t_0 at z is proportional to dθ/dφ. In this regard, the normalized current ratio F(ζ,θ) in Eq. (12) is expressed as

$$F(\zeta,\theta) = \frac{N(\zeta)}{|d\varphi / d\theta|}, \tag{16}$$

where the normalization constant N(ζ) is defined by

$$\frac{2\Pi}{N(\zeta)} = \int_0^{2\Pi} d\theta |d\theta / d\varphi|. \tag{17}$$

The normalization constant N(ζ) ensures the charge conservation. Substituting Eq. (15) into Eq. (16) gives[4]

$$\frac{N(\zeta)}{F(\zeta,\theta)} = |1 + \in \zeta \frac{df}{d\theta} - \int_0^{\zeta} d\zeta' \int_0^{\zeta'} d\zeta'' [h \frac{\partial^2}{\partial\theta^2} F(\zeta'',\theta) - K \frac{\partial}{\partial\theta} W(\zeta'',\theta)]|, \tag{18}$$

where the phase delay function W(ζ,θ) is defined by

$$W(\zeta,\theta) = \sqrt{\frac{2}{\Pi} \int_{\theta_h}^{\theta} \frac{d\theta'}{\sqrt{\theta - \theta'}} \left(\frac{\partial F}{\partial \theta'}\right) \zeta}, \tag{19}$$

and the initial energy gain ε, the self-field effects h, and the resistive-wall effects κ are defined by

$$\in = \frac{1}{\gamma_b(\lambda_b^2 - 1)} \frac{e\Delta\phi}{mc^2}, \quad h = \frac{2G\upsilon}{\gamma_b^3(\gamma_b^2 - 1)}, \quad \mathrm{K} = \frac{\upsilon\mu}{\gamma_b^3} \frac{\delta}{R_w} \ . \tag{20}$$

The initial condition of Eq. (18) is F(ζ,θ) = 1 at ζ = 0. For specified values of the physical parameters ε, h and κ, this integrodifferential equation can be solved by a numerical method. Once the normalized current F(ζ,θ) in Eq. (18) is determined in terms of the time θ and the propagation distance ζ, the energy modulation in Eq. (13) is calculated from the current modulation for a specified injection energy γ_b.

Numerical calculation of Eq. (18) has been carried out, neglecting the transient behavior of the current modulation caused by the beam head $(\theta_h = -\infty)$. A typical example of the current modulation calculated from Eqs. (18) and (19) for ε = 0.02, h = 0.02, κ = 0.02, the injection energy γ_b = 1.5 and the propagation distance ζ = 21. These physical parameters are easily attainable under the present experimental conditions. The current profile is very different from a sinusoidal wave form although the initial energy modulation at the first cavity is a sine function. The current peak occurs near the time at which d /d has a local maximum. The later segment with more energy speeds up and the previous segment with less energy slows down, piling up the beam current at this point. In order to investigate nonlinear mode evolution in the current profile systematically, we Fourier-decompose the current modulation in Eq. (19) with harmonic mode number ℓ. The mode strengths grow exponentially with respect to the propagation distance ζ except at the beginning and near to the location, where the peak modulation occurs. The strength c_1 of the fundamental mode is proportional to exp(0.102ζ) in the propagation range of the exponential growth, which agrees with its analytical estimation of exp(0.1ζ) from the linear theory of the resistive-wall instability. The peak modulation occurs at $\zeta = \zeta_m$ = 22 for the parameters mentioned above. The normalized mode strength c_t of order unity can easily be attainable in the resistive-wall klystron. We also note from numerical calculation that the mode structure at the peak modulation exhibits a broad spectrum.

The propagation distance $\zeta = \zeta_m$ of the peak current modulation determines the length of the resistive-wall klystron. The oscillatory wave of the current modulation beyond the propagation distance of $\zeta = \zeta_m$ breaks down to many wavelets. We thus call the propagation distance $\zeta = \zeta_m$ the wave breaking point. The main nonlinear saturation mechanism for the resistive-wall instability is the wave breaking phenomenon and multi-mode coupling. The tube length longer than ζ_m may not be advantageous for the fundamental mode (ℓ = 1) klystron. However, a long tube may be useful for the high-harmonic klystron. As an example, we consider the parameters mentioned above. The resistive-wall effect of κ = 0.02 corresponds to the beam current of I_b = 4 kA for γ_b = 1.5, μ = 2.5 and the ratio δ/R_w = 0.1. For the microwave frequency of 10 GHz, the conductivity of the resistive medium is given by 100 siemens/m, which is the conductivity of typical ferrite. The propagation distance ζ_m = 22 of the peak current modulation corresponds to the tube length of z_m = 10 cm for ω = 10 Ghz. These parameters are easily attainable in the present experimental conditions. There is a broad range of system parameters which the present technology allows. Obviously, the resistive-wall klystron has great potential for a high-power high-frequency microwave device.

III. REFERENCES

1. M. Friedman, J. Krall, Y. Y. Lau, and V. Serlin, J. Appl. Phys. **64**, 3353, (1988) and the references therein.