EFFECT OF QUADRUPOLE NOISE ON THE EMITTANCE GROWTH OF PROTONS IN HERA

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Abstract

The emittance growth in HERA at luminosity is about 1πmm-mrad/hr. Intra-beam scattering and the beam-beam interaction coupled with tune modulation account for only part of the growth. Here we discuss the contribution of random tune fluctuations in the presence of non-linear fields to the emittance growth of protons. At injection energy, a recent experiment with noise deliberately injected into a chain of correction quadrupoles showed no significant effect on the emittance growth and loss rates except at high noise voltages. The results of this experiment are compared with a tracking study. We also present the results of a study done of the emittance growth due to the beam-beam interaction in the presence of tune noise. A future experiment to determine the effects of tune noise on the proton emittance growth rate with colliding beams is planned.

I. DYNAMIC APERTURE AND DIFFUSION AT INJECTION ENERGY

Particles are tracked through the model of the HERA lattice in SIXTRACK with multipolar field errors up to order 10 and without tune modulation. A complete description of the model and tracking procedure can be found in [1]. The chaotic boundary is estimated by examining the evolution of the distance in phase space between two particles which initially differed only in their x coordinate by 10^{-8}mm. A random kick is fed into 8 quadrupoles distributed uniformly around the ring. The amplitude of the kick Δk_0 is chosen so that a constant kick of this magnitude would lead to a tune shift of about 0.0001. We simulate the random kick by an Ornstein-Uhlenbeck process with a well-defined correlation time. The auto-correlation function of the random kick decays exponentially with time constant τ_c and the spectral density of the Ornstein-Uhlenbeck process decreases with the inverse of the frequency squared. The power spectrum drops to half its maximum value at a frequency $f_{1/2} = f_{rev}/(2\pi\tau_c)$, f_{rev} the revolution frequency of HERA is 47.317kHz.

In addition to the onset of chaotic motion, we also look at the diffusion rates of the particles. The diffusion coefficient is calculated by the method of Chirikov [2]. The idea is to calculate an averaged diffusion coefficient over two different time scales which differ by an order of magnitude. For bounded oscillatory motion, the two coefficients so calculated will differ by two or three orders of magnitude while for true diffusive motion, the two coefficients will be quite close, reflecting the underlying scale invariance of diffusive motion.

Without any tune modulation or noise, tracking over 10^5 turns indicates that the chaotic border is at an emittance of 2.90 πmm-mrad. This value is within 10% of that observed in the recent

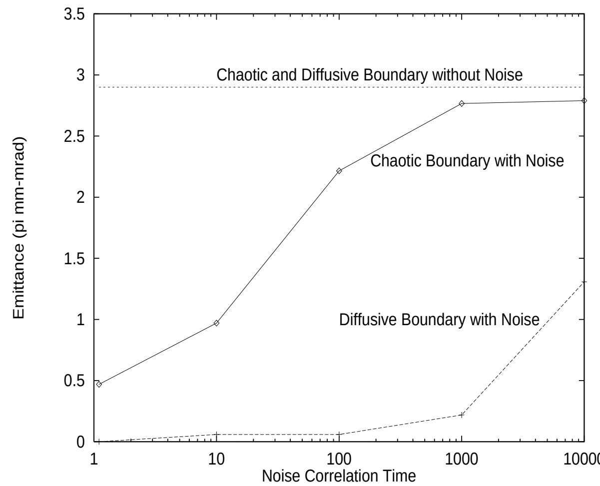

Figure. 1. The chaotic and diffusive boundaries as a function of the noise correlation time.

measurements [1]. The two coefficients D_1 and D_2 averaged over 1000 and 10000 turns respectively, differ by two orders of magnitude for particles launched with an an initial emittance of less than 2.90 πmm-mrad but are close for greater initial emittances. The chaotic border and the diffusive border are thus practically identical in this case. The addition of quadrupole noise separates these two boundaries. With a correlation time $\tau_c = 1.1$ or $f_{1/2} = 6.85$kHz, the chaotic border is at 0.47πmm-mrad while the the two diffusion coefficients are nearly the same for all particles implying that all particles are diffusing. This would indicate that there is no absolutely stable region in the beam and that given enough time all particles would be lost. The chaotic boundary increases with the noise correlation time and eventually at $\tau_c = 10,000$ or $f_{1/2} = 0.75Hz$, the chaotic boundary is at 2.71πmm-mrad which is close to the value obtained without any noise. However in all cases with noise, the boundary where diffusion begins is considerably within the chaotic boundary. The diffusive boundary ranges from 0.0πmm-mrad at $\tau_c = 1.1$ to 1.3πmm-mrad at $\tau_c = 10000$. The dependence of these boundaries on the correlation time can be seen in Figure 1.

II. EXPERIMENTAL RESULTS AT INJECTION

An experiment was performed at the injection energy of 40 GeV where we introduced noise into a chain of correction quadrupoles using a noise source which produced a nearly flat Gaussian noise spectrum with a bandwidth of 10MHz. The closed orbit deviation through these quads was minimized to the extent possible at 2mm rms to avoid dipole noise excitation of the

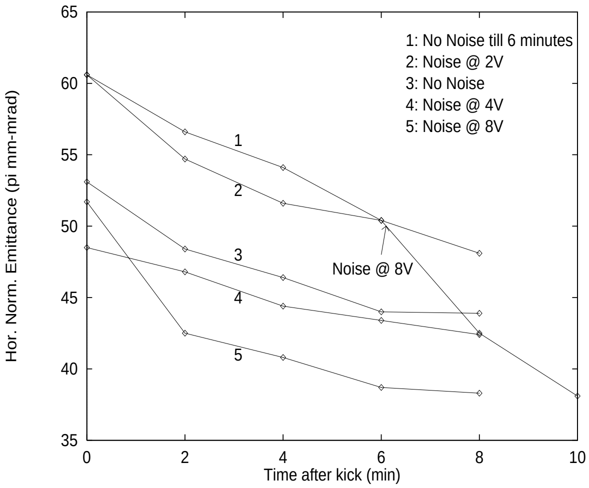

Figure. 2. Evolution of the horizontal emittance after a kick to measure the dynamic aperture

beam. The aims were to examine the dependence of the following quantities on the noise voltage: a) the background loss rates, b) dynamic aperture and c) diffusion rates in the beam halo.

Without any external noise and a beam lifetime between 5 to 8 hours, the background rates at two collimators were about 170 and 700 Hz respectively. A noise voltage of 8V peak to peak was applied across the quadrupoles, and this increased the rates at the collimators to about 290 and 1200 Hz respectively. These observations were reproduced three times. We repeated this at a lower noise voltage of 2V but with a new beam. The lifetime of this beam was considerably lower, around 1 hour, and the applied noise made no significant difference to the loss rates.

The dynamic aperture was measured by kicking the beam horizontally with an injection kicker and observing the beam profile with the gas monitor. The outermost particles were kicked to an amplitude larger than the dynamic aperture and are lost so the remaining particles fill up the available dynamic aperture. This was done with five beam fills, twice without noise, initially, and once each with noise voltages of 2V, 4V and 8V across the quadrupoles. The evolution of the normalized horizontal emittance is shown in Figure 2. Clearly noise applied at 8V has a significant impact on the dynamic aperture while the effects of the other noise voltages is somewhat marginal. The fact that the dynamic aperture was reduced shows that a strong noise source can cause the loss of particles in the beam halo.

In the last part of the experiment, scrapers were moved in and out of the beam halo and the loss rates recorded. When the jaws are moved into the beam, the loss rates go up sharply before relaxing down to their previous value and the reverse behaviour is observed when the jaws are moved away from the beam. In this case the application of noise even at 8V did not make a significant difference in the loss rates observed without noise. The beam lifetime for this part of the experiment was also low, between 1 to 2 hours. It turns out [3] that a quantitative estimation of the diffusion rate from these measurements is difficult at injection energy because the observed relaxation does not fit a simple diffusion model. Consequently this was not attempted. The fact that even a noise voltage of 8V did not change the loss rates at the collimator shows that noise has little effect on particles in the beam halo over short time scales. The dynamic aperture measurements showed however that, on a long time scale, strong noise does remove particles from the halo.

III. DIFFUSION AT COLLISION ENERGY

A study done by Brinkmann [4] showed that there was strong emittance growth due to tune noise when the tunes were close to a non-linear resonance. This growth was surprisingly not strongly dependent on the correlation time of the noise. One can calculate analytically the diffusion due to phase fluctuations in a region of phase space far from strong non-linear resonances. Assuming for simplicity a one dimensional round beam-beam kick, then using a method due to Stupakov [5], we find that the diffusion in action J is

$$\begin{aligned} D_J \;=\; & 64\pi^2\xi^2 J^2 e^{-\beta^* J/\sigma_e^2} \sum_{n=-\infty}^{\infty} K_\psi(n) \\ & \Bigg\{ [I_0(\frac{\beta^* J}{2\sigma_e^2}) + I_1(\frac{\beta^* J}{2\sigma_e^2})]^2 + \\ & \frac{1}{2}\sum_{k=1}^{\infty}[I_{k-1} + 2I_k + I_{k+1}]^2 \cos 4\pi k n \nu(J) \Bigg\} \end{aligned} \tag{1}$$

where ξ is the linear beam-beam tune spread, β^* is the beta function at the interaction point, σ_e is the rms beam size of the opposing beam, $K_\psi(n)$ is the correlation function of the phase fluctuations, the I_k's are the modified Bessel functions, all with the same argument, and $\nu(J)$ is the betatron tune at the action J. The crucial observation here is that the first term in curly braces is independent of the betatron tune and shows that even noise of very low frequency can lead to a diffusion in action, one of Brinkmann's observations. The second term is large whenever the noise spectrum is appreciable at sidebands of twice the betatron frequency. At small J, $D_J \sim J^2$ while at large J we find, using the asymptotic expansion for $I_k(J)$, that $D_J \sim J$. However for J sufficiently large, the density of non-linear resonances is large enough that the assumptions of this calculation are not satisfied.

We now discuss the results of a simulation where we studied the interaction of the machine non-linearities with the beam-beam non-linearity in the presence of quadrupole noise. The HERA lattice was modelled by a series of FODO maps with 6 sextupoles and 4 quadrupoles with random kicks distributed between them. The sextupoles were adjusted to give roughly the same tune shifts as observed in the recent measurements [1]. Even then the effect of the sextupoles is quite exaggerated in this model since the diffusive boundary (without any noise) is about half of that obtained with a more realistic model of HERA in section I. However, it serves to emphasize the effects of machine non-linearities on a shorter time scale. We use Brinkmann's rational approximation [4] for the complex error function to speed up the computation of the beam-beam kick.

We follow the evolution of 100 particles for a million turns and calculate the two diffusion coefficients mentioned in section I over intervals of 10^4 and 10^5 turns respectively. With only the

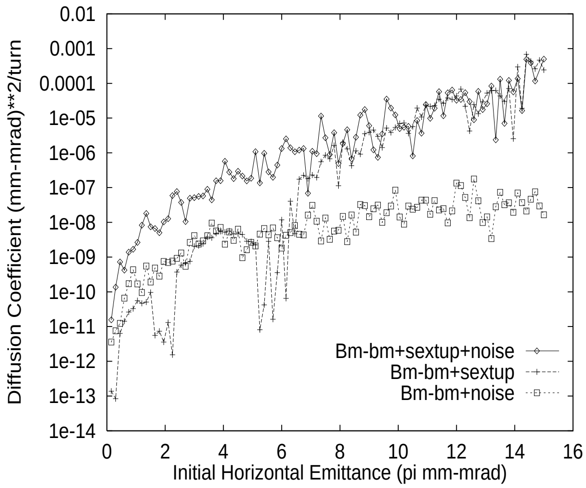

Figure. 3. Diffusion coefficient for the horizontal action calculated after a million turns for three cases. The noise correlation time τ_c=10,000.

linear lattice and the beam-beam potential, the two diffusion coefficients differ by two or three orders of magnitude, except in a region between 2.5πmm-mrad to 3.5πmm-mrad, up to an initial emittance of 15πmm-mrad. There is a dramatic change when we add the sextupoles to the beam-beam kick in the map. There is diffusive motion at all amplitudes. The addition of quadrupole noise to the beam-beam potential also leads to diffusive motion at all amplitudes but with a different amplitude dependence. Finally when both sextupoles and quadrupole noise are included, the diffusion is larger than with either of them alone. These results are shown in Figure 3. To see the effects of very low frequency noise, the correlation time was set to 10,000.

With the beam-beam potential and quadrupole noise the diffusion coefficient is larger at small amplitudes and then appears to saturate. With the beam-beam potential and sextupoles, the diffusion coefficient is smaller (or comparable in some regions) but after an emittance of around 6.0πmm-mrad, it is larger by orders of magnitude. Thus the sextupoles are the dominant source of diffusion in the tails. With noise of shorter correlation times, the diffusion coefficient increases at all amplitudes.

IV. DISCUSSION

Quadrupole noise can effect the beam in three ways. Firstly there is a linear resonance which occurs if there is noise in the quadrupoles at twice the betatron frequency, i.e. at about 28kHz for the HERA proton beam. It is unlikely that frequencies above 1kHz penetrate into the beam since the inductance and capacitance of the quadrupoles filter out high frequencies and there is additional shielding due to the copper layers in the beam pipe. So the linear resonance is not expected to play a role. Quadrupole noise also causes the phase advance to have a fluctuating component and hence successive non-linear kicks (due to magnets or the opposing beam) occur between partially de-correlated phases. Greater the degree of de-correlation or magnitude of the fluctuating phase, larger will be the diffusive growth. This effect will be dominant at low amplitudes. Finally, the (partially) random nature of the kicks can also transport particles into stochastic layers and enhance the rates at which resonances are crossed. This effect will be dominant where the density of non-linear resonances is greatest, i.e. at intermediate amplitudes for the beam-beam kick and large amplitudes for magnetic multipole kicks. This enhancement due to noise could be swamped by other effects such as tune modulation. Bruning [6] has studied diffusion in the tails due to tune modulation. Our results at collision energy show that machine non-linearities also contribute strongly to the diffusion in the tails of the beam. In the core of the beam we find that in addition to intra-beam scattering, quadrupole noise also contributes to diffusion.

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