COMPARISON Of MEASURED AND CALCULATED DYNAMIC APERTURE

F. Willeke
Deutsches Elektronen-Synchrotron DESY Notkestraße 85,22607 Hamburg, Germany

Abstract

Dynamic aperture experiments at the Fermilab Main Ring, the TEVATRON, the SPS and HERA are reviewed and compared with simulations. The agreement between experiments with additional strong nonlinearities and corresponding simulations is fairly good, if the relavant parameters are sufficiently well known and included in the model. Direct comparison is more problematic if machines under normal operating conditions with relatively weak non-linear fields are considered. Nevertheless, the agreement is reasonable and discrepancies can be accounted for by known imperfections of the model.

I. INTRODUCTION

Non-linear dynamics in circular accelerators has been studied at many laboratories. The crucial issue for future machines is how well the dynamic aperture can be predicted by calculations. This is important in order to be able to specify the field quality and optimize the design of the magnets.

Particle motion in non-linear fields in accelerators can be described and understood in principle.

The subtle question of predicting the dynamic aperture and long term stability is difficult however.

The greatest problem is the limited knowledge of the system. Accelerators are built as linearly as possible. Non-linearities, apart from the chromatic sextupoles, occur as small imperfections with many sources. It is quite difficult to obtain precise knowledge on these errors. The measurement of magnetic fields with a precision of $\delta B/B \simeq 10^{-4}$ is at the technological limit. In superconducting magnets there are; in addition, persistent currents with slow decay, eddy currents and magnetic coupling between different coils. Furthermore there are the finite permeability of the vacuum components, saturation and remanence effects, and stray fields from injection elements or nearby accelerators. The beam optics is also not perfect,while closed orbit distortions, β-beats, linear coupling and chromaticity modify the dynamics in non-linear fields. Furthermore, the conditions in a real machine are never quite constant. Power supply ripple, ground motion, and vibrations excited by vacuum pumps or coolants cause fast oscillation of the fields seen by the beam. Slow changes are induced by thermal effects or by the variation of the load of the power system.

Accelerators are operated at optimized working points far from major resonances. Then, the effects which determine the dynamic aperture are quite subtle. The border of stability is determined by chaotic motion. Thus the motion becomes unpredictable. Simulation of the motion becomes difficult. Usually one needs to simulate a large number of turns before one can decide upon stability.

Observations involving real beams are multi-particle effects whereas tracking can be performed only with a few particles. The results of measurements depend on the particle distribution. This is impossible to model exactly. While the structure of the whole phase space is relevant for the stability of a particle beam, tracking can explore only a small fraction. This makes it very difficult to compare numbers directly.

In a real accelerator, additional effects influence the results of measurements. These are tune modulation, collective effects due to space charge, wakefields, beam-ion interaction and intra beam scattering, scattering from the rest gas, interaction with vacuum pumps, noise from rf cavities or damper systems and power supplies.

It is very hard to put all these effects simultaneously into a model.

The experimental strategy under these circumstances has been the following: One started with very simple experiments near strong resonances, which resemble the slow extraction scenario to test experimental equipment and procedures. The next step consisted in the investigation of the dynamic aperture of a machine with controlled strong non-linearities which dominate the non-linear fields. Such experiments have been performed in the FNAL-TEVATRON (1987-1990) and in the CERN-SPS (1886-1988). The importance of power supply ripple and tune modulation was recognized early. These effects have been investigated in the SPS 1988-1993 and in the TEVATRON 1988-1990. The ultimate goal is to study and to understand the real machine as it is operated. This has been addressed in TEVATRON Main Ring studies (1988-1989) and in experiments in the HERA Proton Ring (1994).

II. EXPERIMENTS WITH STRONG, CONTROLLED NON-LINEARITIES

Strong sextupole and octupole fields are turned on, so that the dynamic aperture limit occurs at small amplitudes at which the "natural" small nonlinearities can be neglected. Thus the relevant non-linearities are well under control. In such an experimental set up one has the same qualitative features of non-linear dynamics as in a normal machine, namely amplitude dependent tunes and non-linear resonances for all rational values of the tunes.

Experiments have been performed at the TEVATRON and at the SPS. In the TEVATRON, 16 strong sextupole magnets have been powered to drive the third integer resonance $3Q_x = 97$, whereas in the SPS experiment 8 strong sextupoles are driven such that this resonance is intrinsically canceled.

An important characteristic of a nonlinear system is the tune shift with amplitude. In both experiments the measurements have been compared with simulations. The measured tune shifts with amplitude agree with the simulations within the precision of the measurements [2] [5] [6] [8]. See also Fig. 1.

The comparison of experimental dynamic aperture values with simulations is satisfactory in both studies.

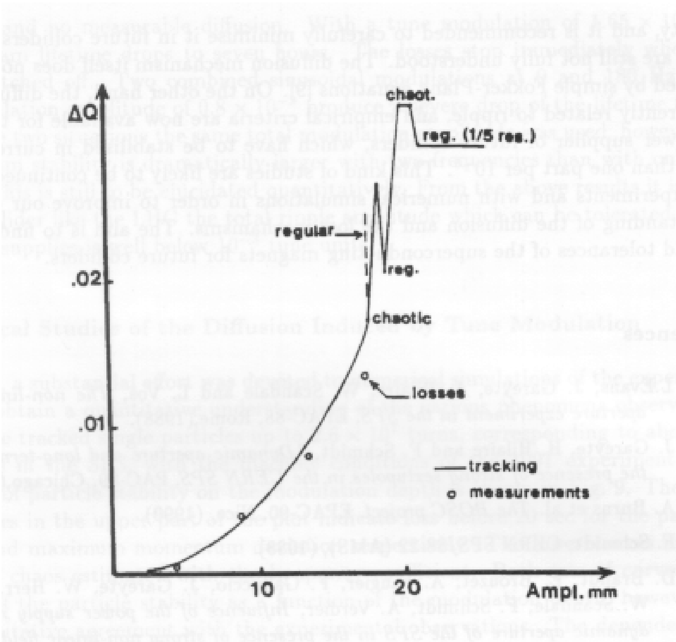

Figure 1. Detuning curves of the SPS Aperture Experiment with strong Sextupoles from [2] [5] (line: tracking, dots: experimental data. Indicated are also the amplitude limits for which losses are observed and the amplitude limit which gives chaotic motion in tracking calculations. They are in good agreement.

In the TEVATRON experiment at $150GeV$, the beam has been heated by applying noise to the beam via the horizontal damper. The available dynamic aperture was filled by this method. The coupling resonance was compensated and the tunes were kept away from the main coupling resonance to avoid the blow up of the emittance in the other plane. The horizontal beam profile was recorded with a wire scanner. Once the available aperture was filled, the beam emittance did not grow any more and the foot width of the profile is identified as the dynamic aperture. The experimental values for various sextupole strengths amount to 63% to 75% of the values obtained from 1000 turn tracking calculations.

In the SPS experiment [2], [5], [12], a bunched pencil beam at $120GeV$ was produced by scraping and this beam was kicked horizontally. Once the edge of the beam hit the dynamic aperture, beam losses occurred. The dynamic aperture has been reconstructed from the transverse distribution of the beam and the kick amplitude. The working point was kept away from major resonances ($Q_x = 26.53, Q_y = 27.63$). The results compare well with tracking calculations. In the tracking calculations, a vanishing Lyapunov exponent (see [16]) was considered as the criterion for stability. The values from tracking and experiment agree quite well.

III. EFFECT OF TUNE MODULATION

We know from the theory of nonlinear dynamics that tune modulation creates a side band structure of nonlinear resonances. These sideband resonances may reduce the dynamic aperture. Sidebands may also increase the probability for particles to escape from regions which otherwise would be inside the border of stability. A slow growth of the amplitudes of the particles can be observed, which resembles a diffusion process. The border of stability transforms under these circumstances into a soft transition from quasi-stable motion to fast instability. In simulations, this effect can be made visible by plotting the escape time of a particle versus its initial amplitude (survival plot).

Tune modulation may be caused by power supply ripple, by non-corrected chromaticity or by mechanical vibration of the magnets. It is very difficult to suppress tune modulations below a critical level of $\Delta Q \leq 10^{-4} - 10^{-3}$.

Some experiments have been devoted to exploring this quasi-diffusive behaviour and comparisons have been made with predictions from theory and simulation. Experiments have been carried out in the TEVATRON [11] and in the CERN-SPS [5] [9] [10] [13] [15]. As in the experiments described above, strong controlled nonlinearities have been added to the accelerator fields.

In the TEVATRON experiment, 14 sextupoles have been powered with an intrinsically canceled third integer resonance. The working point was near the resonance 5Qx=97. The beam was bunched. The phase of the driving term was adjusted to populate an island in the chain of the 5th order resonance by a horizontal kick. The fraction of the beam trapped in the resonant island gives raise to persistent coherent betatron oscillations, which can be easily detected by a beam position pick-up. The destruction of the regular phase space topology of these islands by tune modulation has been studied. Strong, overlapping side bands cause the trajectories which form island chains to be resolved into a band of chaotic trajectories. This can be observed experimentally by the decay of the coherent betatron oscillation signal, when the particles escape from the island and eventually are uniformly distributed in betatron phase. This behaviour was examined as a function of tune modulation depth and frequency and was compared with an analytical model and with simulations. The experiment confirmed the dependence of the process on tune modulation depth and frequency which was predicted by an analytical model. Strong decay of the coherent signal is observed for parameters for which chaos is predicted. The loss of coherence has also been simulated. The simulations are in excellent agreement with the experimental data.This experiment is a demonstration that the available concepts, methods, and tools are adequate. If the relevant parameters are under control, even subtle effects can be described quantitatively.

In the SPS , diffusion-like behaviour has been discovered in the 1988 sextupole experiments. Beam losses as a function of time have been observed after a scraper has been moved into the beam and has subsequently been retracted by a small amount. The slow losses which developed after the scraper was retracted indicated a diffusion-like growth of oscillation amplitudes. More recently, the experimental studies focused on the role of tune modulation in this process. The same configuration of non-linear fields as in the previously described experiments has been used. In addition, the detuning has been controlled by means of octupole magnets. The experiments have been carried out with a coasting beam. A strong influence on the long term dynamic aperture could be found by applying a tune modulation with a frequency of 9Hz and with a depth of $1.8 \cdot 10^{-3}$.

Quasi-diffusion constants have been extracted from observation of beam life-time after scraping and subsequent small retraction of the scrapers. The diffusion occurred mainly in the horizontal plane. The diffusion coefficient was enhanced by a factor of $5 - 10$ due to this additional tune modulation. In this experiment, the short time dynamic aperture was not affected by the tune modulation. This can also be seen in a simulation study. The tune modulation starts to affect the stability border only if 10^6 or more turns ($t \geq 20sec$) are considered.

An attempt has been made to simulate the diffusion experiment by a computer model [13]. The SPS with its natural and additional strong nonlinearities has been carefully modeled. The corresponding detuning reproduces the experimental values. The measured horizontal distribution of the kicked beam was described by 180 super-particles. The distribution in the vertical and in the longitudinal direction was neglected. The modeled distribution and the detuning can be used to calculate the tune spectrum which agrees well with the measured spectra. If the scraper measurement is simulated using this model, the qualitative features of the beam loss curve versus time are reproduced well. However the losses in the experiment are about three times greater than the simulated ones.

This discrepancy is beyond the range of uncertainty due to imperfect knowledge of parameters such as longitudinal and transverse distribution, kick strength or position of the scraper. Nevertheless the loss mechanism in this experiments could be clarified. According to the simulations, particles in the kicked beam either diffuse outward until they reach the vicinity of a 7th order resonance after which they will be lost quickly or alternatively they will be attracted by an 8th order resonance which is somewhat further inwards and from which no escape could be observed. Beam profile measurements indeed confirm, that particles are diffusing inwards which supports the tracking analysis.

IV. EXPERIMENTS UNDER NOMINAL OPERATING CONDITIONS

The ultimate question is how well one can model an accelerator under normal operating conditions. It has been addressed by two experiments. One of them is the FERMILAB Main Ring study performed in 1988 and 1989 [3] [8]. The other example is the superconducting HERA proton ring with its low injection energy and corresponding large persistent current field errors. Dedicated experiments have been performed recently in 1994 [14].

Here only the short term dynamic aperture is relevant. The machine has a short cycle time of 4sec. The dynamic aperture limitations are only important at $8GeV$, an energy at which the beam life-time is dominated by multiple scattering at the rest gas ($\tau = 40sec$). In this way the dynamic aperture is quickly filled with beam. Under the assumption that the phase space is not strongly distorted at the dynamic aperture and that particles beyond the dynamic aperture will be lost quickly, the beam distribution is well defined. The dynamic aperture has been studied by injecting beams of different sizes. If the beam size is too large, it will shrink until all the particles outside the dynamic aperture are lost. If the size is too small, it will grow due to gas scattering until a quasi stationary distribution is reached which extends up to the dynamic aperture.

Another type of measurement was performed by kicking the beam and observing subsequent beam losses. Since the distribution is well defined, the edge of the beam can be reconstructed from the fractional beam loss. The horizontal dynamic aperture obtained in this way is 16.4mm and the value for the vertical one is $13.3mm$.

In order to obtain a good model, the field of the dipole magnets has been remeasured carefully. The tracking calculations extended over a period of 35000 turns (the storage time at 8GeV). The horizontal and vertical start amplitudes of the particles have been chosen to be the same. The dynamic aperture values from long term tracking agree with an analysis of Lyapunov exponents. The particles with amplitudes up to about $16mm$ are stable. Beyond that, chaos is observed in the tracking.

The most recent experiment is the dynamic aperture study in the superconducting HERA proton ring. First crude estimates indicated that the dynamic aperture of the machine was about two times smaller than the values obtained from a very detailed tracking model which took into account the measured non-linear field components of each individual superconducting magnet [17]). In the recent studies, dedicated experiments have been performed for the first time. The dynamic aperture has been measured by two different methods. The beam has been scraped to produce a pencil beam. It was then kicked in the horizontal plane with different strengths. The part of the beam outside the dynamic aperture gave rise to intensity losses from which the dynamic aperture could be reconstructed. In the second type of experiment, the beam was kicked horizontally and the edge of the beam was measured by a restgas ionization beam profile monitor(see Fig.2). The dynamic aperture obtained by these methods agreed within the errors and amounted to $13mm - 15mm$. In Fig.3, the experimental results are summarized in form of survival plots. In some of the experiments, the working point has been chosen near the main diagonal close to the seventh order resonance ($Q_x = Q_y = .287$). For these values, the dynamic aperture is found significantly lower than for well separated tunes ($Q_x = .285, Q_y = .305$).

In the model calculations the persistent current decay has been taken for each individual magnet in addition to the steady state individual magnet errors. The persistent current decay causes an increase of the spread of the sextupole strength from magnet to magnet. It also changes the global distribution of sextupole strength around the machine since there are two types of magnets which differ in persistent current decay. The knowledge about the exact distribution of sextupole strength is imperfect since only two reference magnets are available from which the sextupole component can be monitored. The model has been optimized by adjusting the detuning. The distribution of sextupole strength has been varied slightly so as to obtain agreement with the measured values. Due to these uncertainties in the sextupole strengths, the calculated dynamic aperture values range from $16.6mm$ to $18.6mm$. The tracking results are shown in a survival plot in Fig.4.

There is thus a moderate discrepancy between experiment and tracking in the order of $(10 - 40)\%$. An imperfection of the model is that the space charge induced tune shift with its strong amplitude dependence has been omitted. This contribution causes a 20% change to the values due to the sextupoles, but

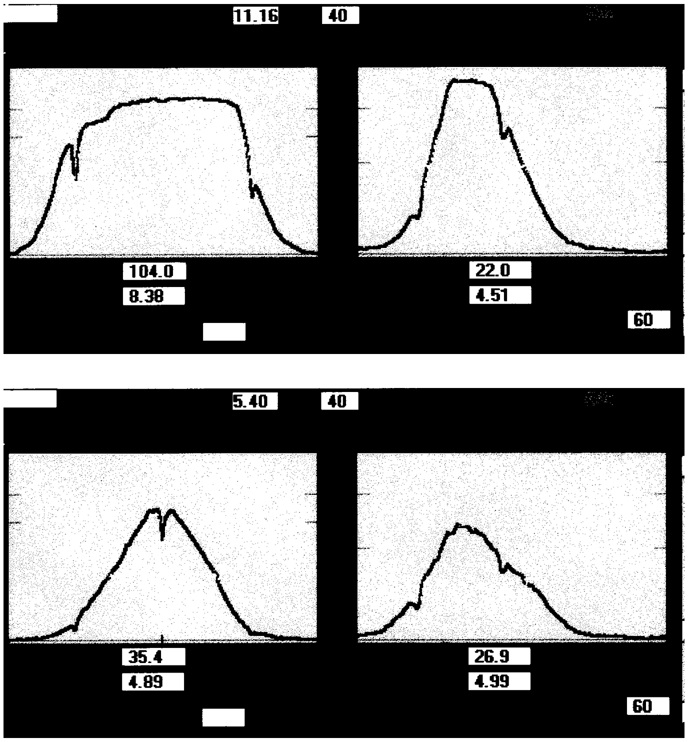

Figure 2. Evolution of Beam Size in HERA after the Beam has been kicked horizontally. The foot width of the beam profiles is interpreted as the projection of the dynamic aperture onto the horizontal and vertical planes respectively. The first profiles are recorded immediately after the kick. The second one is recorded after 8 minutes. The full scale is $36mm$.

it cannot be observed by a coherent oscillation of the beam. This additional tune shift with amplitude is strong enough to explain the remaining discrepancies.

What is more striking is the result from a tune modulation experiment. The compensation of existing tune modulation in the order of 10^{-4} by an external source was a big success in beam-beam operation of HERA p at large energies [18]. Background rates could be reduced by a factor of two. Tracking calculations as well as analytic calculations [17] predict that the dynamic aperture is strongly influenced by a tune modulation of depth 10^{-2} with frequencies of $50Hz$ and its multiples. The experimental result is that there is no influence of either adding a tune modulation of 10^{-3} or compensating the existing tune shift of $3 \cdot 10^{-4}$. A possible reason for this result are the 600 chopper power supplies of HERA which were operated at frequencies around 13kHz very close to the betatron tunes. This produced a large level of coherent excitation and emittance growth which overlayed the nonlinear quasi diffusion. After the chopper frequencies have been changed to 16kHz, the level of coherent excitation was reduced by 20db and the emittance growth is reduced by a factor of at least two. Further measurements are planned to clarify the open questions.

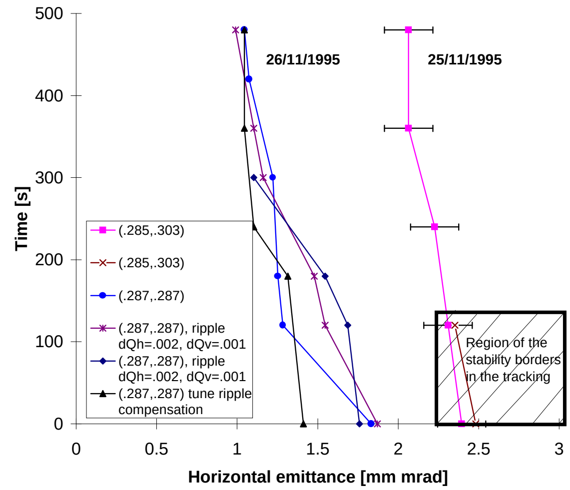

Figure 3. Horizontal Dynamic Aperture of HERA at Injection Energy Derived from Beam Profile Measurements. The box indicates the stability borders obtained by tracking calculations for small variations of closed orbit, ripple and strength of persistent current sextupoles.

V. CONCLUSION

The short term dynamic aperture in experiments with strong, controlled non-linearities is usually well explained by tracking. Good progress has been made in explaining long term effects such as quasi-diffusion and beam life time qualitatively. There are examples for quantitative agreement between experiment and calculations. The dynamic aperture in accelerators under normal conditions agrees reasonably well with predictions if the model is prepared carefully and the experimental conditions are well enough known. There are no results which force the assumption of any yet unknown physics. Discrepancies occur if the modeling of the accelerator and the relevant physics are imperfect.

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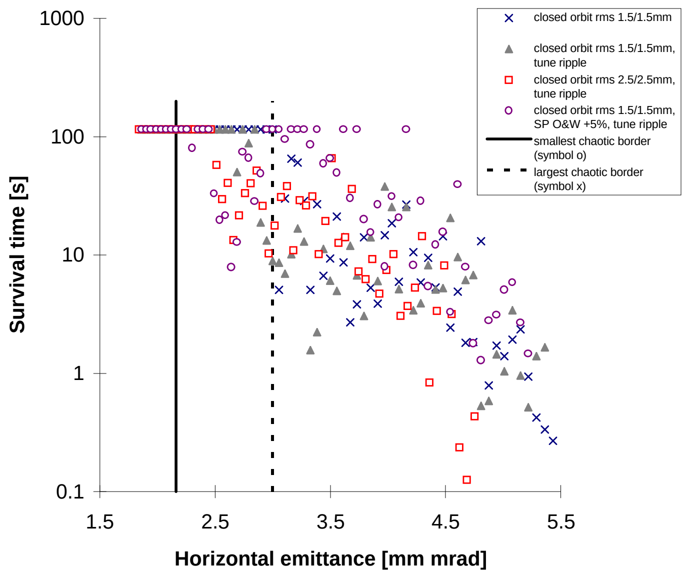

Figure 4. Tracking results for the HERA dynamic aperture studies for small variations of closed orbit, strength of persistent currents and tune modulation. Shown are the survival times in seconds as a function of initial amplitudes. Maximum stable amplitudes for long term tracking agree well with the stability border determined using the Lyapunov method. Tune modulations and variation of sextupole strength have a noticible influence on the dynamic aperture whereas closed orbit errors are not as important.

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