Printed-Circuit Quadrupole Design*

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I. INTRODUCTION

A printed-circuit quadrupole has been designed for low-energy electron beam research in the Institute for Plasma Research at the University of Maryland. Previously, solenoid focusing has been employed. Quadrupoles will allow more realistic beam studies to be performed in straight beams, and are necessary for a ring geometry currently under consideration. Applications include high-current, high-energy accelerators and heavy-ion inertial fusion.

Based on the quadrupole design, a scaled-up prototype has been built and the field measured. In this report we describe the design procedure and the measurements. The conductor geometry is similar to that described by Avery, Lambertson and Pike for printed-circuit steering coils [1]. Our goal was a very short quadrupole (length = diameter) for low fields, <30 G, but with good precision and linearity.

For such short quadrupoles the fringe fields dominate the behavior. Integrals of the field parallel to the z-axis must be used to check the linearity rather than the field itself. Also, attention must be paid to the external connections.

Three desktop computer programs were used in the design: (1) a spreadsheet to set up formulas which determine conductor location based on a few key input parameters and an algorithm; (2) a 3-D magnetics program to study the linearity of the magnetic field integrated along lines parallel to the beam axis at various radii, and (3) a CAD program to provide the file required by the printed-circuit fabricator. To save time in searching for optimum parameters, the spreadsheet program is designed to automatically generate the appropriate file needed for the magnetics program. It also generates the file required by the CAD program.

All of the conductors lie on the surface of a cylindrical mount concentric with the beam pipe. The printed circuit is made flexible to conform to the cylindrical mount.

II. DESIGN

The widely known method--using a current distribution that follows a cos(nθ) distribution, where n=2 for a quadrupole and θ is the azimuthal coordinate--does not apply here. It applies when the active conductors have equal length. For a short, compact quadrupole the inactive return conductors should lie in planes perpendicular to the beam axis. Thus a practical quadrupole must employ gradually shorter active conductors. This leads to a small increase in current density with increasing angle rather than the normal cosine-like decrease, in order to compensate for the short conductors. This was recognized by Avery *et al* [1]. Their results encouraged us to apply the same ideas to quadrupoles.

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A detailed analytical approach for conductors which require smooth bends at the coil ends (e.g., superconducting wire) has been given by Laslett, Caspi and Helm [2]. Our approach is to adopt a simple algorithm for our square-cornered geometry and search for a solution with the 3-D magnetics code. We chose twenty conductors for each 45° segment, sufficient for good linearity yet small enough for practical fabrication.

Coil Geometry

Figure 1 shows our final geometry, as printed by the CAD program. Four coils are shown, two on top and two on the underside of a thin, flexible insulating base. These represent half of the quadrupole, i.e., 0-180°. The other half, 180°-360°, is identical. Each coil has 20 turns and 40 active conductors--parallel to the axis--and 40 return, arc-shaped conductors in planes perpendicular to the axis. There are 640 conductors, all in series. The eight coils employ four different connections, arranged for minimum stray field due to the external leads and interconnections. This can only be done using both sides of the plastic base. Also, use of both sides doubles the magnetic field.

We adopted equal spacing for the return conductors. Linearity is optimized using the algorithm described below to locate the active conductors.

Computer Programs

The spreadsheet program has several major functions. Input parameters are the coil radius, *R*, axial length, *L*, the return conductor spacing, ΔL, and the spacing between the first two active conductors, $\Delta\theta$. The remaining active conductors are located using

$$\theta_{n+1} - \theta_n = \Delta\theta/(1 + \theta_n^2/k) \qquad (1)$$

where n =1 to 20, $\theta_1 = \Delta\theta/2$, and *k* is an adjustable constant.

We studied the magnetic fields using the 3-D, iron-free magnetics program MAG-PC, a product of the Infolytica Corporation. The required input ASCII file which gives the location of all of the conductors is generated automatically in the spreadsheet. An additional parameter is required: the number which the magnetics program uses to divide up the arcs into straight segments for computation. We used 10 segments for trial runs and 16 segments for final runs.

It should be noted that, while Fig. 1 shows the connections required for a continuous coil, the field calculations are based on a series of interlocking rectangles. To obtain the CAD drawing one corner of every rectangle is shortened so that it connects to the adjacent inner rectangle. We expect the error introduced in this way to be negligible.

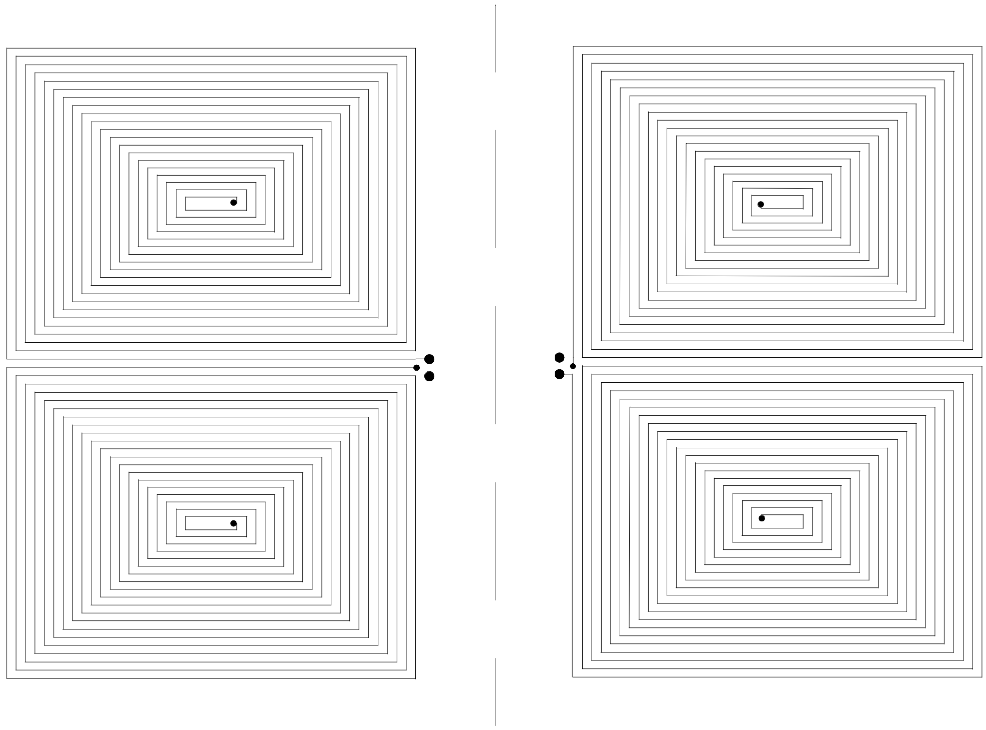

Fig. 1. Upper side (left) and underside (right) of the double-sided printed circuit. The beam axis is horizontal. One-half (0 to 180°) of the circuit is shown. The other half (180° to 360°) is identical. The dark spots are feedthru connections. The external twisted pair is connected to the two largest spots.

The magnetics program computes the Biot-Savart law to calculate B_x, B_y and B_z for each line and segment of arc. The output B_y is summed to obtain the z-scan integral.

The final program is a CAD program, AutoSketch for Windows, from Autodesk, Inc., to generate the file--known as a .DXF file--given to the printed circuit fabricator. The entire file is programmed into the spreadsheet and generated automatically based on the input parameters. Separate files are required for the two sides because of the different end connections.

III. RESULTS

Table I shows the field integral results for the parameters found to yield the best linearity for a scaled-up quadrupole with $L = 2R$. = 100 cm. Specifically, $k = 3.0$, $\Delta L/L = 0.023$, and $\Delta\theta = 2.34^{\circ}$. The scans are taken along lines parallel to the z-axis, at five radii and three polar angles.

Table I -- Deviations from Linearity for Scaled Coil
(100-pt. z-scan, 16-segment arcs, 3-digit output file)

Polar Angle	Relative Gradient	-------Deviations in % for r/R = -------- 0.2	0.4	0.6	0.7	0.8
0	558.1	-0.11	-0.10	+0.21	+0.58	+1.49
30°	557.4	+0.03	+0.08	-0.12	-0.33	-0.58
45°	557.3	+0.03	+0.02	-0.05	+0.17	+0.70

The relative field gradient at each angle (corrected for the angle) is the average of the gradients for the first three radii, i.e., r/R = 0.2, 0.4, and 0.6. Deviations from the average gradient at each angle are given in percent.

Computational accuracy in MAG-PC was checked by increasing the number of scan points, the number of segments used for the return arcs, and the number of decimals used in the output files.

The results indicate a deviation from linearity <0.6% for radii up to r/R = 0.7. Runs with slightly different parameters indicate the sensitivity. For example, scans performed with k = 2.7 and 3.3 gave deviations up to 1.4% for r/R<0.6 and 3.3% for r/R=0.8. Similarly, runs varying $\Delta\theta$ and $\Delta L/L$ indicate sensitivity of about 0.03° and 0.01, respectively.

Prototype Quadrupole

A scaled model with radius R = 12.8 cm was built to check the calculations. A Hall probe was used to make z-axis scans at four different radii, r/R = 0, 0.2, 0.4 and 0.6. The vertical component of the earth's field was balanced with large Helmholtz coils and corrections were made for the small x-component. Care was taken in probe positioning and zero drift of the gaussmeter A typical z-axis scan is shown in Fig. 2. The agreement between MAG-PC calculations and measurements is excellent.

Measurements indicated that the mechanical axis and the magnetic axis were coincident within about 1 mm. No

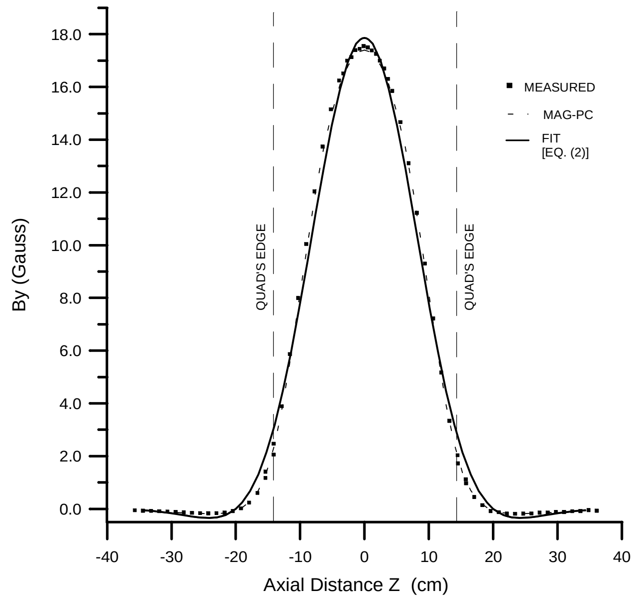

Fig. 2. Axial scan of prototype magnetic field B_y at r=7.62 cm (r/R=0.60), θ=0 and I=10 A.

evidence was found of significant differences between the magnetic fields produced by the spiral geometry of Fig. 1 and the concentric-rectangle geometry used in MAG-PC.

Finally, the calculated transverse field $B_y(z)$ was fit to an analytical expression whose parameters were adjusted for optimum linearity of the axially-integrated magnetic field at four radii. The resulting empirical function is

$$B_y(r,z,\theta) = (B_o r/a)\exp(-z^2/2\sigma^2)\cos(2\pi z/\lambda)\cos\theta \qquad (2)$$

where B_o/a = 2.34 Gauss/cm, σ = 10.2 cm, and λ = 80 cm for the prototype. When these quantities are scaled to the printed-circuit quadrupole (quad radius/prototype radius = 0.172) we get B_o/a = 7.93 Gauss/cm, σ=1.75, λ=13.75 cm, and I=1 A. Eq. (2) is particularly useful for future particle-in-cell code simulations. Eq. (2) can be integrated using the expression below.

$$\int_{-\infty}^{\infty} \exp(-z^2/2\sigma^2)\cos(2\pi z/\lambda)dz = \sigma(2\pi)^{1/2}\exp[-2\pi^2(\sigma/\lambda)^2]$$

Lattice Design

For the final quadrupole we chose $L = 2R$ = 4.4 cm to conform to our normal aperture. The integrated field from MAG-PC is 11.1 G-cm per Ampere at r = 0.44 cm, equivalent to a gradient of 7.93 G/cm for a 3.19-cm effective length, square-edged quadrupole. These values were then compared with solutions of the envelope equations for a space-charge dominated beam in a FODO lattice. For this purpose we used a convenient program developed by one of us for studying heavy-ion recirculating induction linacs [3].

For a moderately high beam current of 100 mA at 10 keV (generalized perveance = 1.5×10^{-3}), a normalized emittance of 8 mm-mrad, and a zero-current phase advance of 76°, a solution of the envelope equations is obtained with a

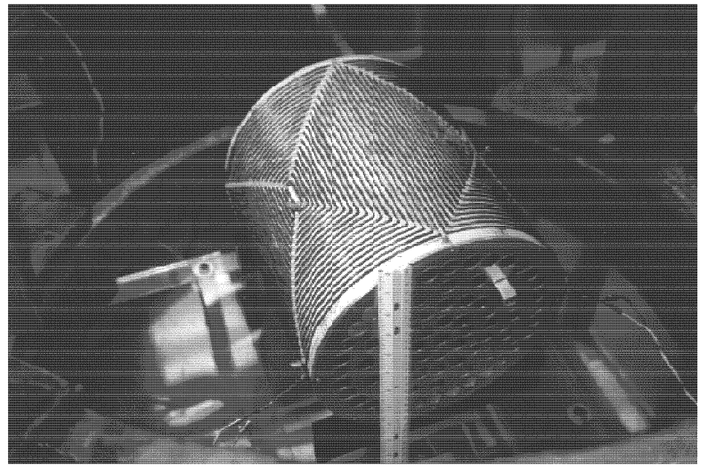

Fig. 3. Photograph of 25.6-cm diam prototype.

mean beam radius of 0.9 cm, depressed phase advance of 9°, and a half-lattice period of 15 cm. The quadrupoles occupy 29% of the axial space for this lattice. The beam clearance factor (aperture radius/mean beam radius) is 2.1. The solution is a compromise between the desire for a long lattice length and the desire to have a somewhat larger depressed phase advance.

Fawley *et al* studied space-charge dominated transport in large-aperture-ratio quadrupoles [4]. They found very little emittance growth for a well-matched beam when the beam radius/half-lattice-period < 0.1. Our ratio, 0.06, satisfies this criterion.

The required gradient for the above solution is 9.4 G/cm, which means that the quadrupoles should be operated at 1.2 A circuit current, taking into account the factor of two from the double-sided circuit. From tables supplied by the printed-circuit fabricators, this will produce a temperature rise of about 15° C in the circuit for "2-ounce" copper plating. If necessary, the quadrupole can be pulsed.

A separate, general issue that must be faced is the use of low magnetic fields in the presence of stray fields and the earth's field. A combination of cancellation coils, magnetic shielding and precise steering corrections should be sufficient.

We thank A. Faltens for pointing out the 1971 LBL reference, L.K. Len for assistance with the computer programs, and J.G. Wang and W.M. Fawley for discussions.

[1] Robert P. Avery, Glen R. Lambertson and Chester D. Pike, Proc. 1971 Part. Accel. Conf., p 885.
[2] L. Jackson Laslett, S. Caspi and M. Helm, Particle Accelerators 22, 1 (1987).
[3] T.F. Godlove, Particle Accelerators 37-38, 439 (1992).
[4] W.M. Fawley, L.J. Laslett, C.M. Celata, A. Faltens and I. Haber, IEEE Cat. 93CH3279-7, 724 (1993).