

# Current Losses and Equilibrium in rf Linear Accelerators

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## ABSTRACT

In the conventional design of rf linear accelerators the charged particle bunches are not in thermal equilibrium. With high currents, space charge couples the transverse and longitudinal self forces, leading to emittance growth and halo formation as the beam relaxes toward an equipartitioned state. Particle losses to the walls can occur as a result of halo formation and also through the natural tail on the equilibrium distribution. Particle losses due to either a halo or a tail can cause radioactivity in the conducting channel, inhibiting routine maintenance. The properties of the beam are described in a new design for rf linacs in which the beam is kept in thermal equilibrium, and the current loss rate is found for the tail on the thermal equilibrium distribution.

## I. INTRODUCTION

Many advanced applications of rf linear accelerators, such as injector linacs for high-energy physics colliders, spallation neutron sources, transmutation of radioactive nuclear waste, heavy ion inertial fusion and free electron lasers, require high beam currents in which space charge forces play a dominant role in the particle motion. In the conventional design of such linacs the bunched beams are not in thermal equilibrium [1]. Space charge couples the longitudinal and transverse forces, driving the beam toward an equilibrium state, causing emittance growth and halo formation [2-4]. Halos are of particular concern in linacs with high average power, in which particle losses as low as 1 nA/m have been predicted to result in nuclear activation, preventing routine maintenance [5]. Simulations using on the order of  $10^5$  particles are of little use in predicting such a small halo or tail, which corresponds to fractional particle losses of around  $10^{-8}$  per meter. Halos have been observed in existing high-current linacs like LAMPF [6] and in experiments at the University of Maryland [7]. The beam parameters are described here for a linear accelerator in which the beam is kept in thermal equilibrium, minimizing emittance growth and halo formation. When space charge dominates, each bunch has a uniform density profile with a sharp boundary. When emittance is significant, the thermal equilibrium distribution has a tail which can result in current losses. Equations are derived and results are presented for the current losses from the thermal equilibrium distribution, in which the beam is axially centered in a cylindrical conducting pipe. This represents a best-case scenario, since deviations from equilibrium due to mismatch and misalignments add to the particle losses through emittance growth and halo formation. The particle losses from a beam which is kept in thermal equilibrium, however, will always be less than in the conventional linac design, in which equipartitioning adds to the emittance growth and halo formation.

## II. AN RF LINAC WITH A BEAM IN THERMAL EQUILIBRIUM

The transverse and longitudinal evolution of a bunched beam is described by the coupled envelope equations [1, 8]. An equipartitioned beam has equal transverse and longitudinal temperatures,  $T_{\perp} = T_{\parallel}$ . This is equivalent to  $\epsilon_{nx} \gamma z_m = \epsilon_{nz} r_m$ , in which  $\epsilon_{nx} = 5^{1/2} r_m (\gamma k_B T_{\perp} / mc^2)^{1/2}$  and  $\epsilon_{nz} = 5^{1/2} z_m (\gamma^3 k_B T_{\parallel} / mc^2)^{1/2}$  are the transverse and longitudinal normalized emittances,  $r_m$  is the peak radius and  $z_m$  is the half-length, both of the equivalent uniform ellipsoid. The stationary coupled envelope equations with the equipartitioning condition are [8]

$$k_{x0}^2 r_m^4 = \frac{3N r_c \epsilon_{nx}}{2\beta^2 \gamma^2 \epsilon_{nz}} \left( 1 - \frac{\epsilon_{nx}}{3\epsilon_{nz}} \right) r_m + \frac{\epsilon_{nx}^2}{\beta^2 \gamma^2}, \quad (1)$$

and

$$k_{z0}^2 z_m^4 = \frac{N r_c \epsilon_{nz}}{\beta^2 \gamma^5 \epsilon_{nx}} z_m + \frac{\epsilon_{nz}^2}{\beta^2 \gamma^6}, \quad (2)$$

where  $k_{x0}$  and  $k_{z0}$  are the transverse and longitudinal focusing wave numbers,  $N$  is the number of particles in the bunch,  $r_c = q^2 / (4\pi\epsilon_0 mc^2)$  is the classical particle radius and  $\beta$  and  $\gamma$  are the relativistic velocity and energy factors. It is assumed that the bunch aspect ratio in the beam frame,  $\gamma z_m / r_m$ , satisfies  $0.7 < \gamma z_m / r_m < 4$ . Equations (1) and (2) are both of the same form as a fourth order polynomial for which a simple approximate solution was found [9]. Using this solution, the stationary bunch radius and half-length are [8]

$$r_m \approx \left[ \frac{3N r_c \epsilon_{nx}}{2\beta^2 \gamma^2 \epsilon_{nz} k_{z0}^2} \left( 1 - \frac{\epsilon_{nx}}{3\epsilon_{nz}} \right) + \left( \frac{\epsilon_{nx}}{\beta \gamma k_{x0}} \right)^{3/2} \right]^{1/3} \quad (3)$$

and

$$z_m = \left[ \frac{N r_c \epsilon_{nz}}{\beta^2 \gamma^5 \epsilon_{nx} k_{z0}^2} + \left( \frac{\epsilon_{nz}}{\beta^2 \gamma^3 k_{z0}} \right)^{3/2} \right]^{1/3}. \quad (4)$$

With the equipartitioning condition substituted into the coupled envelope equations, the ratio between the transverse and longitudinal focusing wave constants becomes [8]

$$\frac{k_{x0}}{k_{z0}} = \left[ \frac{3\Gamma \epsilon_{nz} / \epsilon_{nx} - \Gamma + 2\epsilon_{nz}^2 / \epsilon_{nx}^2}{2(\Gamma + 1)} \right]^{1/2}, \quad (5)$$

where  $\Gamma = Nr_c r_m / \epsilon_{nx}^2$  is the intensity parameter. This relation between  $k_{x0}/k_{z0}$  and  $\epsilon_{nz}/\epsilon_{nx}$  is plotted in Figure 1 for several values of the intensity parameter. The traditional method of allowing  $k_{x0}/k_{z0}$  to increase in proportion to  $\beta^{1/2}\gamma^{3/2}$  increases the temperature anisotropy and the aspect ratio ( $\gamma z_m/r_m$ ) during acceleration and is responsible for the equipartitioning effect and the associated emittance growth in conventional high-current linacs [1, 8]. An example of a linac which uses these results has been given elsewhere [8].

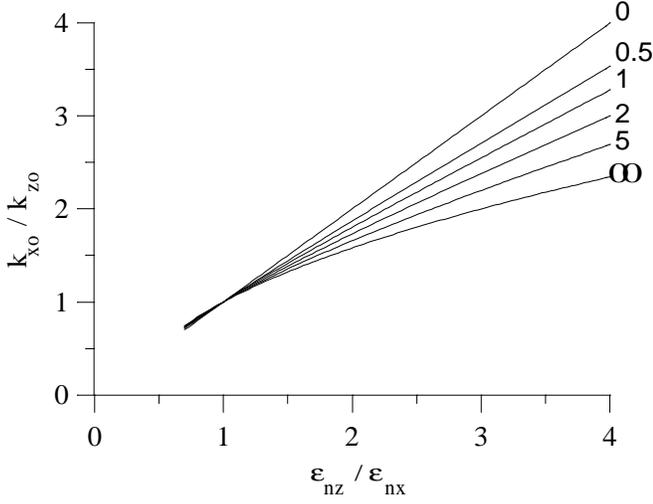


Figure 1: The ratio of the focusing wave numbers as a function of the ratio of the emittances from Equation (5) for several values of the intensity parameter ( $\Gamma = 0, 0.5, 1, 2, 5$  and  $\infty$ ).

### III. PARTICLE LOSSES

The fractional particle loss per unit length for bunched and continuous (unbunched) beams in equilibrium can be found by considering the flux across a cylindrical boundary due to thermal motion in the thermal equilibrium distribution. With a Maxwellian velocity distribution with temperature  $T$ , the flux of all particles across a cylindrical boundary with radius  $b$  is [10]

$$\Phi = \frac{\bar{n}(b)}{(2\pi)^{1/2}} \left( \frac{k_B T}{m} \right)^{1/2}. \quad (6)$$

For bunched beams,  $\bar{n}(b) = \int n(b,z) dz / \Delta z$  is the density at the pipe averaged over each bunch in the longitudinal direction, where  $\Delta z$  is the distance between bunches. For unbunched beams,  $\bar{n}(b) = n(b)$  is the density at the pipe, which is constant along  $z$ .

The number of particles lost per unit length per unit time along the channel is found by multiplying the flux by the circumference ( $2\pi b$ ). Multiplying the result by the particle charge  $q$  gives the current lost per unit length, so the fractional loss per unit length along the channel is

$$f = \frac{q(2\pi)^{1/2} b \bar{n}(b)}{I_{ave}} \left( \frac{k_B T}{m} \right)^{1/2}, \quad (7)$$

where  $I_{ave}$  is the average beam current ( $I_{ave} = I$ , the continuous current, for unbunched beams).

Equation (7) can be written in terms of the normalized transverse emittance and rms radius for bunched and unbunched beams. For unbunched beams the normalized transverse emittance is  $\epsilon_{nx} = 2^{1/2} a (\gamma k_B T_{\perp} / mc^2)^{1/2}$  and the current is  $I = q n_0 \pi a^2 v_0$ , where  $n_0$  is the density of the equivalent uniform beam, and  $a = 2^{1/2} \bar{r}$  is the radius of the equivalent uniform beam. The fractional particle loss per unit length along the channel for continuous (unbunched) beams is then

$$f = \left( \frac{1}{4\pi^{1/2} \beta \gamma} \right) \left( \frac{n(b)}{n_0} \right) \left( \frac{b}{\bar{r}} \right) \left( \frac{\epsilon_{nx}}{\bar{r}^2} \right). \quad (8)$$

For bunched beams the average current is  $I_{ave} = q N v_0 / \Delta z$ , where  $N$  is the total number of particles in each bunch. The resulting fractional loss per unit length is [10]

$$f = \left( \frac{6}{25\pi^{1/2} \beta \gamma} \right) \left( \frac{\bar{n}(b)}{\bar{n}_0} \right) \left( \frac{b}{\bar{r}} \right) \left( \frac{\epsilon_{nx}}{\bar{r}^2} \right). \quad (9)$$

where  $\bar{n}_0$  is the density on the axis of the equivalent uniform ellipsoid, averaged over the longitudinal direction. The terms containing the densities in Equations (8) and (9) are rapidly decreasing functions of  $b/\bar{r}$ , so that increasing the beam radius or decreasing the pipe radius causes an increase in the particle losses, despite the appearance of  $b/\bar{r}$  and  $\epsilon_{nx}/\bar{r}^2$  as multiplying factors.

Since a continuous beam does not have any image fields from the conducting pipe, the effect of the pipe is only to truncate the thermal distribution. The density as a function of radius required for Equation (8) can be found from previous results [11] for any pipe radius. For bunched beams, thermal equilibria have been found numerically [10, 12] for aspect ratios of 1, 5 and 20; pipe radii of 2, 3 and 5 times  $r_m$ ; and transverse space charge tune depressions ( $k_x/k_{x0}$ ) of 0.2, 0.3, 0.4, 0.5, 0.65, 0.8 and 0.95. The transverse space charge tune depression is calculated from the envelope equations as [8]

$$\frac{k_x}{k_{x0}} = \left( 1 - \frac{3Nr_c(1-r_m/3\gamma z_m)}{2\beta^2\gamma^3 r_m^2 z_m k_{x0}^2} \right)^{1/2}. \quad (10)$$

The extent of the thermal tail in the radial direction was found to be independent of the pipe radius with constant  $k_x/k_{x0}$ , for all aspect ratios and pipe radii which were tested, just as in the case of unbunched beams.

The extent of the tail was found to have a much greater dependence on temperature than on bunch length. The results were also found, for all aspect ratios tested, to be similar to those for unbunched beams. Figures 2 and 3 show the average

radial density profiles with density on a logarithmic scale (with log base 10), which emphasizes the differences between the tails of the distributions. The solid lines are for unbunched beams, obtained by the same method as in Reference 11. The dashed lines are for spherical bunches ( $\gamma z_m/r_m = 1$ ) in Figure 2 and for profiles with aspect ratio  $\gamma z_m/r_m = 20$  in Figure 3. The profile numbers from 1 through 8 correspond to transverse space charge tune depressions of, respectively, 0.95, 0.8, 0.65, 0.5, 0.4, 0.3, 0.2 and 0.0. Radii are in units of the rms radius, which is  $2^{-1/2}$  a for unbunched beams and  $(5/2)^{-1/2} r_m$  for bunched beams. An example which uses these results has been given elsewhere [10].

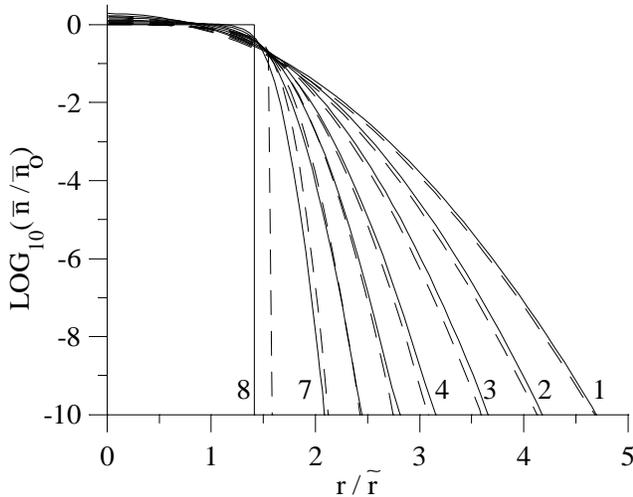


Figure 2: Average radial density profiles on a logarithmic scale, for unbunched beams (solid lines), and bunched beams with an aspect ratio of  $\gamma z_m/r_m = 1$  (dashed lines).

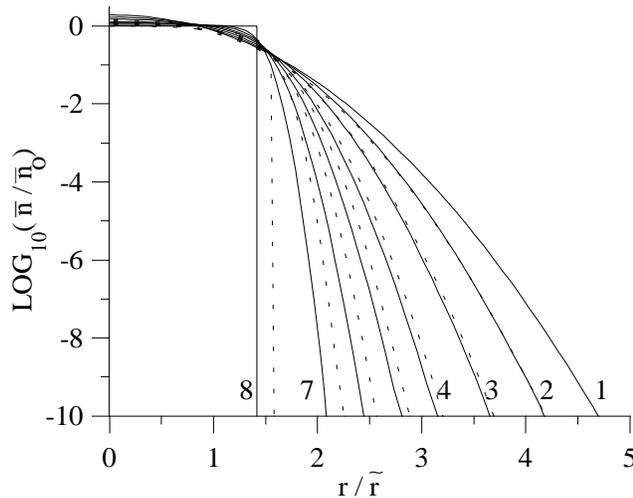


Figure 3: Average radial density profiles on a logarithmic scale, for unbunched beams (solid lines), and bunched beams with an aspect ratio of  $\gamma z_m/r_m = 20$  (dashed lines).

## CONCLUSION

The properties of the beam in a proposed rf linear accelerator have been described, in which the ratio of the transverse and longitudinal focusing wave numbers is adjusted to keep the beam in thermal equilibrium in order to minimize emittance growth and halo formation. When space charge dominates over emittance, the bunch has a uniform density profile with a sharp boundary. When emittance is significant, the thermal equilibrium density profile has a natural tail. Equations have been derived and results presented graphically for the fractional current loss due to this tail in the thermal equilibrium distribution. In practice there will always be deviations from the equilibrium state due to mismatch and misalignments, but the resulting emittance growth will always be less than in the conventional design in which equipartitioning adds to the emittance increase and halo formation.

## REFERENCES

- [1] M. Reiser, *Theory and Design of Charged Particle Beams* (John Wiley & Sons, New York, 1994), Appendix 4 and Section 5.4.11.
- [2] R. A. Jameson, IEEE Trans. Nucl. Sci. **NS-28**, 2408 (1981).
- [3] I. Hofmann, IEEE Trans. Nucl. Sci. **NS-28**, 2399 (1981).
- [4] R. A. Jameson, in *Advanced Accelerator Concepts*, edited by J. S. Wurtele, AIP Conf. Proc. 279 (AIP, New York, 1993).
- [5] J. M. Lagniel, Nucl. Inst. Meth. Phys. Res. **A345**, 46 (1994).
- [6] R. A. Jameson (private communication).
- [7] D. Kehne, M. Reiser and H. Rudd, in Proceedings of the 1993 Particle Accelerator Conference **93-CH3279-7**, p65; see also Reference 1, Section 6.2.2 and M. Reiser, J. Appl. Phys. **70**, 1919 (1991).
- [8] M. Reiser and N. Brown, Phys. Rev. Lett. **74**, 7, 1111 (1995).
- [9] N. Brown and M. Reiser, Part. Accel. **43**, 231 (1994).
- [10] N. Brown and M. Reiser, Phys. Plasmas **2**, 3, 965 (1995).
- [11] M. Reiser and N. Brown, Phys. Rev. Lett. **71**, 18, 2911 (1993).
- [12] N. Brown, PhD Dissertation, University of Maryland, College Park, 1995.

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