MODE ANALYSIS OF SYNCHROTRON MAGNET STRINGS

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Abstract

Synchrotron magnet strings is able to be regarded as a 6 terminal ladder circuit. It can be shown that the voltage and current has two different modes of resonant property. this mode is able to be decoupled by the transformation. In presence of symmetric configuration, it is further shown that the mode are reduced to two orthogonal mode of normal and common mode which enables us to find the analytical solution.

I. INTRODUCTION

In a synchrotron where a current of a magnet string is excited trapezoidally, a reproducibility, a stability and a tolerance of a ripple content of the excitation current is stringent due to a slow beam resonant extraction property of the synchrotron. Among others, the relative ripple content is one of the most important and required to be a ppm level or less. Most of the synchrotron utilizes a thyristor controlled power supply generating voltage of logical ripple which is a multiple of number of thyristor firing pulses. There is also an illogical ripple which is due to imbalance among the phase and amplitude of primary line voltage or noise from the sensor such as DCCT where the frequencies are integer fractions of the fundamental harmonic of 50 Hz. Furthermore a spike voltage is induced across each thyristor when the thyristor is turned on and off. The spike frequency ranges between a few kHz to a tens of kHz depending upon magnitude of capcitances and other parameters of a relevant circuit. This thyristor spike has long been one of the major causes limiting the performance of the synchrotron power supply. The spike is regarded as source of a high frequency effective ripple component. A close look of the spike reveals that amplitude of a train of spikes is modulated by lower illogical frequencies and considered to be other source of illogical ripple.

In conventional research and development of a power supply illogical ripple have been suppresssed by improving an imbalance among phases of primary AC voltages and equalizing a timings of firing pulses of each thyristor or ba an active filter and a band-pass filter.

The spikes and logical ripples are suppressed by low pass static filter. In spite of those efforts, achieving the ripple content of ppm level has been difficult.

Reviewing the existing lower limit of the ripple performance of the conventional technology, we proposed a model which includes a stray capacitance of the coil to the ground [1,2,3,4]. In it, a magnet string is modeled as a ladder circuit, which is a repeated circuit of lumped element of L, C and R. The excitation coils of the magnet are divided into the upper coil and the lower coil. The upper coil is connected to neighboring coil in series instead of connecting to its lower coil and again connected to the separated upper coil of next neighboring coil and so on. So does the lower coil. the magnet core is regarded to have ground potential and all the magnet core are connected in series by the erth line. The model circuit of an unit cell is shown in Fig. 1.

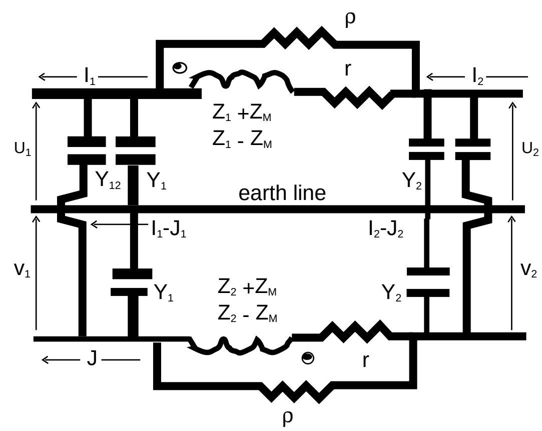

Fig. 1: Unit cell of the six terminal magnet string. ρ represents the bridge resistor as well as the ac loss of the magnet.

In conventional synchrotron magnet strings, although the earth line is not physically set up, the capacitance between the excitation coil and the magnet yoke can not be neglected and a similar six terminal circuit as Fig. 1 may be applied in analyzing the magnet string.

II. ANALYSIS

Let us designate the relevant parameter of the voltage and the current of the upper coil U and I and the lower coil V and J. The suffix of the input and output voltage and current is designated as 1 and 2. We then write the transfer matrix of the voltage and the currrent in general as

$$\begin{pmatrix} U_1 \\ I_1 \\ V_1 \\ J_1 \end{pmatrix} = \begin{pmatrix} 1+Z_1Y_1 & Z_1 & Z_{M2}Y_2 & Z_{M2} \\ Y_1(2+Z_1Y_1) & 1+Z_1Y_1 & Z_{M2}Y_1Y_2 & Z_{M2}Y_1 \\ Z_{M1}Y_1 & Z_{M1} & 1+Z_2Y_2 & Z_2 \\ Z_{M1}Y_1Y_2 & Z_{M1}Y_2 & Y_2(2+Z_2Y_2) & 1+Z_2Y_2 \end{pmatrix} \begin{pmatrix} U_2 \\ I_2 \\ V_2 \\ J_2 \end{pmatrix} \quad (1)$$

with

$Z_M = Z_{M1} = sM_{12} = Z_{M1} = sM_{21}$.

Each component of the transfer matrix of eq. (1) is non-zero and have finite value. the pair of the voltage and current of (U, I) and (V, J) is considered to be a representation of a pair of two different mode. Non-zero component of the matrix signifies that the two mode is coupled each other.

By intuition, one is able to find the equation for the sum component and difference component of the voltage and current which is written as,

$$\begin{pmatrix} U_1+V_1 \\ I_1+J_1 \\ U_1-V_1 \\ I_1-J_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left\{\begin{matrix} 2+(Z_1+Z_M)Y_1+ \\ (Z_2+Z_M)Y_2 \end{matrix}\right\} & \frac{1}{2}(Z_1+Z_2+2Z_M) & \frac{1}{2}\left\{\begin{matrix} (Z_1+Z_M)Y_1- \\ (Z_2+Z_M)Y_2 \end{matrix}\right\} & \frac{1}{2}(Z_1-Z_2) \\ \frac{1}{2}\left\{\begin{matrix} Y_1(2+Z_1Y_1 \\ +Z_MY_2)+ \\ Y_2(2+Z_2Y_2 \\ +Z_MY_1) \end{matrix}\right\} & \frac{1}{2}\begin{pmatrix} 2+ \\ (Z_1+Z_M)Y_1+ \\ (Z_2+Z_M)Y_2 \end{pmatrix} & \frac{1}{2}\left\{\begin{matrix} Y_1(2+Z_1Y_1 \\ +Z_MY_2)- \\ Y_2(2+Z_2Y_2 \\ +Z_MY_1) \end{matrix}\right\} & \frac{1}{2}\left\{\begin{matrix} (Z_1-Z_M)Y_1- \\ (Z_2-Z_M)Y_2 \end{matrix}\right\} \\ \frac{1}{2}\left\{\begin{matrix} (Z_1-Z_{M1})Y_1- \\ (Z_2-Z_{M2})Y_2 \end{matrix}\right\} & \frac{1}{2}(Z_1-Z_2) & \frac{1}{2}\left\{\begin{matrix} 2+ \\ (Z_1-Z_M)Y_1+ \\ (Z_2-Z_M)Y_2 \end{matrix}\right\} & \frac{1}{2}(Z_1+Z_2-2Z_M) \\ \frac{1}{2}\left\{\begin{matrix} Y_1(2+Z_1Y_1-Z_MY_2)- \\ Y_2(2-Z_2Y_1+Z_MY_2) \end{matrix}\right\} & \frac{1}{2}\begin{pmatrix} Z_1Y_1-Z_2Y_2- \\ Z_M(Y_2-Y_1) \end{pmatrix} & \frac{1}{2}\left\{\begin{matrix} Y_1(2+Z_1Y_1-Z_MY_2)+ \\ Y_2(2+Z_2Y_2-Z_MY_1) \end{matrix}\right\} & \frac{1}{2}\left\{\begin{matrix} 2+ \\ (Z_1-Z_M)Y_1+ \\ (Z_2-Z_M)Y_2 \end{matrix}\right\} \end{pmatrix} \cdot \begin{pmatrix} U_2+V_2 \\ I_2+J_2 \\ U_2-V_2 \\ I_2-J_2 \end{pmatrix} \quad (2)$$

It is easily shown from the equation (2) that when the magnitude of the element of the upper coil and lower coil is identical, the pair of the sum of (U+V) and (I+J) and the difference of (U-V) and (I-J) are linearly independent. The former sum component is defined as the normal mode and the latter difference component is called the common mode. The decoupled equation is treated as an four terminal circuit and it can be shown that analytical form of the solution is available [5]. In HIMAC, we have constructed not only the magnet string but the power supply in a symmetrical fashion such that normal mode and common mode are decoupled each other and successfully achieved the power supply system of the unpredecented ripple level, below ppm [5]. In conventional configuration of the magnet string, the elements of the magnet string of the coils are not separately wound and the earth line is obscure. The symmetricity is not assured and one may have to solve the coupled eq. (2) or (3).

We found the coupled equation (2) is able to be decoupled by an eigenvalue method. We start from the equation known as the telegram equation or the transmission line equation. The transfer matrix obtained from the equation still holds for that of the ladder circuit.

Let us consider the ladder circuit with the capacitance between the coil and the grounded magnet yoke, a magnitude of an outgoing current I_0 flowing into the load and that of incoming current J_0 returning to a power supply are not necessary to be identical. They could be bypassed via capacitance and flow back to the power supply side through a ground line. Furthermore voltages developed between a positive input U_0 and a negative input V_0 also may differ.

This voltage difference may be due to a flip-flop nature of a thyristor firing timing or difference of an asymmetric configuration with respect to the ground line of the magnet string.

Equations for voltage of U and V are written as,

$$\frac{d^2U}{d^2z} = Z_{11}'\,U + Z_{12}'\,V \quad (3)$$

$$\frac{d^2V}{d^2z} = Z_{21}'\,UZ_{22}'\,V\,(4) \quad (4)$$

where z_{11}', z_{12}', z_{21}',and z_{22}' are

$$Z_{11}' = Z_1'\left(Y_1' + Y_{12}'\right) + Z_M'\,Y_{12}' \quad (5)$$

$$Z_{12}' = Z_1'\,Y_{12}' + Z_M'\left(Y_2' + Y_{12}'\right) \quad (6)$$

$$Z_{21}' = Z_2'\,Y_{12}' + Z_M'\left(Y_1' + Y_{12}'\right) \quad (7)$$

$$Z_{21}' = Z_2'\left(Y_2' + Y_{12}'\right) + Z_M'\,Y_{12}' \quad (8)$$

and the ' denotes the impedance per unit length (m). This equation is valid both for the transmission line model and also for the ladder circuit model; z is regarded as a coordinate along a propagation direction of the voltage wave for the formercase and a discrete coordinate of the magnet number for the ladder circuit. Similar equation holds for the current I and J.

Two set of equations designate the present of two mode of voltage and current in the magnet string. Indeed this formulation could be extended to N mode equations of a system (N-1) signal lines and a single ground line. As is shown by the equations, voltage U and V are coupled. These coupled voltage propagates down along the magnet strings. The resonance characteristic also shows the coupled property. In conventional synchrotron magnet strings where the common mode ripple is not suppressed, there is a possibility the normal mode ripple is mixed with common mode ripple and its performance is not improved as expected.

In general, it is tedious to solve two simultaneous second order differential equations. There is an orthodox method to solve this problem known as an Eigenvalue problem. In Eigenvalue problem, mode-decoupling is done by finding a proper transformation matrix.

We need to transform a matrix of,

$$M = \begin{pmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{pmatrix} \quad (9)$$

to

$$M' = \begin{pmatrix} Z_{11}'' & 0 \\ 0 & Z_{22}'' \end{pmatrix} \quad (10)$$

i.d., the diagonalization is required.

After some algebra, one can find, multiplication of a following matrix P from right and P^{-1} from left to M:

$$M'' = P^{-1} M' P \quad (11)$$

with

$$P = \begin{pmatrix} 1 & \dfrac{q}{Z_{21}'} \\ \dfrac{q}{Z_{12}'} & -1 \end{pmatrix} \quad (12)$$

where q is expressed as,

$$q = \frac{1}{2}\left(Z_{11}' - Z'_{22} \pm \sqrt{\left(Z_{11}' - Z_{22}'\right)^2 + 4Z'_{12} Z_{21}'} \right) \quad (13)$$

The above discussion shows that in general case of asymmetric 6 terminal circuit, the mode separation is possible by the transformation given by eq. (12). this transformation enables to find the analytical expression in a closed form.

In HIMAC, the magnitude and location of every possible elements are set to be equal with respect to the earth line which is defined as “symmetry.” In this case, considerable simplification is possible.

In symmetric case, where Z_1' and Z_2' are equal, one obtains for M',

$$M'' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (14)$$

Equation (14) indicates two mode of coupled voltage is reduced to the decoupled normal mode and common mode voltage.

III. ACKNOWLEDGEMENT

The author like to thank to the guidance and discussion to Prof. K. Sato of University of Osaka. He also gratefully acknowledges the support of all the other members of the division of accelerator physics and engineering, research center of heavy charged particle therapy, NIRS.

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