

# Low-Dispersion $\gamma_t$ Jump for the Main Injector

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## Abstract

A bipolar  $\gamma_t$ -jump design is reported for the Fermilab Main Injector (Lattice MI-17). The total amount of jump is 1.3 units. Both the betatron and dispersion waves are confined, while the betatron tunes remain nearly unchanged.

## I. Introduction

The Fermilab Main Injector has been designed to overcome some of the unfavorable effects on the particle motion around transition energy. Unlike the Main Ring, the Main Injector has a very large aperture, so that beam loss due to scraping can be avoided. The bunch area at transition will be less than 0.1 eV-sec, so that the nonlinear Johnson effect [1] can be avoided. Also rapid ramping across transition is possible, so that the nonadiabatic time can be reduced. Nevertheless, negative-mass instability will develop when the bunch intensity is high enough. The only way to avoid this instability is to incorporate a  $\gamma_t$  jump.

During a  $\gamma_t$  jump, it is difficult to confine the betatron waves and dispersion wave, and at the same time preserve the betatron tunes. The existing  $\gamma_t$  jumps at the CERN PS and the Fermilab Booster have been performed at the expense of creating unfavorable increases in dispersions. In the scheme to be presented below, such unpleasant dispersion increase has been avoided by utilizing the dispersion-free regions in the Main Injector. Similar scheme had been considered by Bogacz et al and Peggs et al. [2],[3]

## II. Review of Theory

An off-momentum particle will pass through the special quads for  $\gamma_t$  jump off-centered and acquire a kick, thus changing its path length around the ring. The change in  $\gamma_t$  has been given by Risselada: [4]

$$C_0 \Delta(\gamma_t^{-2}) = - \sum_{i=1}^N K_i (1 + M + M^2 + \dots) D_i^2, \quad (1)$$

with the matrix  $M$  defined as

$$M_{ij} = - \frac{K_i \sqrt{\beta_{xi} \beta_{xj}}}{2 \sin \pi \nu_x} \cos(\pi \nu_x - |\phi_i - \phi_j|_x). \quad (2)$$

In the above,  $C_0$  is the circumferential length of the ring,  $K_i$  is the strength of the special quad at location  $s_i$ , while  $\beta_{xi}$ ,  $\phi_{xi}$ , and  $D_i$  are the horizontal beta function, phase advance, and dispersion at  $s_i$ , and  $\nu_x$  the horizontal betatron tune, all before the pulsing

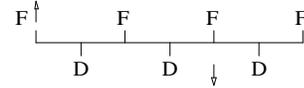
of the special quads. Thus, Eq. (1) is an expansion in terms of  $\mathcal{O}(K\beta)$ .

During the jump, naturally we would like little or no change in the betatron tunes  $\nu_x$  and,  $\nu_y$ , while keeping the dispersion and the horizontal and vertical beta functions below reasonable values.

For small quad strength  $K_i$ 's, the changes in tunes are

$$\Delta \nu_{x,y} = \pm \frac{1}{4\pi} \sum_{i=1}^N K_i (\beta_i)_{x,y}. \quad (3)$$

To keep  $\Delta \nu_x = 0$  and  $\Delta \nu_y = 0$ , we can go with doublets, each having  $\beta_1 K_1 + \beta_2 K_2 = 0$ . The Main Injector has  $90^\circ$  cells. So we can put one special quad of strength  $K$  at the F quad of one cell and another of strength  $-K$  at the F quad of some later identical cell as follows:



F quads are used because  $\beta_x$  and  $D$  are usually at a maximum there; so  $\Delta \gamma_t$  will be maximized.

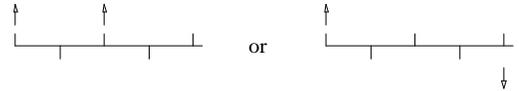
For a special quad of strength  $K_i$ , the betatron waves downstream are

$$\Delta \beta_{x,y}(s) = \mp (\beta(s) \beta_i)_{x,y} K_i \sin[2(\phi(s) - \phi_i)_{x,y}], \quad (4)$$

whereas the dispersion wave downstream is

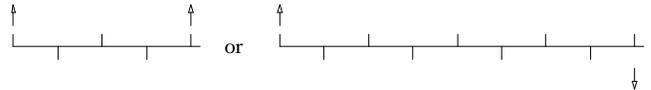
$$\Delta D(s) = -\sqrt{\beta_x(s) \beta_{xi}} D_i K_i \sin[\phi(s) - \phi_i]_x. \quad (5)$$

To confine betatron waves in  $90^\circ$  cells, we can place a doublet of special quads of the *same* sign at successive identical cells, or place one at F of one cell and the other of *opposite* sign at F of the 3rd cell that is  $180^\circ$  downstream:

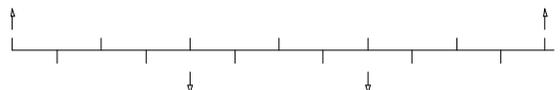


The latter is preferred since  $\nu_x$  and  $\nu_y$  are preserved.

On the other hand, dispersion wave can only be confined between two special quads of the *same* sign placed at  $90^\circ$  cells  $180^\circ$  apart, or of *opposite* sign placed  $360^\circ$  apart:



One possible way to accommodate all these restrictions is



But, unfortunately, these 4 quads give  $\Delta \gamma_t = 0$ . Thus, it appears that there is no way to satisfy all the restrictions.

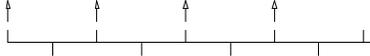
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For the  $\gamma_t$  jumps in the CERN PS and the Fermilab Booster, doublets of special quads are used to null the tune changes and confine betatron wave but *not* the dispersion wave. This scheme gives  $M^{-2} = 0$  and simplifies Eq. (1):

$$\Delta\gamma_t^{-2} = \frac{1}{2C_0 \sin \pi \nu_x} \sum_{i=1}^N D_i^2 \beta_{xj} K_i K_j \cos(\pi \nu_x - |\phi_i - \phi_j|_x),$$

so that the change in  $\gamma_t$  is second order in  $M$  or  $(K\beta)^2$ . In order to have a bipolar jump, two families of doublets must be needed. This scheme had also been considered by Holmes for the Main Injector. [5]

For machines with dispersion-free regions, there is another scheme. [2] Here, groups of 4 quads are used to confine both the betatron and dispersion waves, but changes in betatron tunes are ignored for the time being:



Next, we place in dispersion-free regions another family of special quad doublets at F's  $90^\circ$  apart. Here, betatron waves are again confined. But no dispersion wave will be created and  $\Delta\gamma_t$  will be unaffected. The strength of this second family is adjusted to null out  $\Delta\nu_x$  and  $\Delta\nu_y$ . In this scheme, all orders of  $M$  or  $K\beta$  contribute. But the first order usually dominates. This scheme is termed *matched* by Bogacz et al, and the first one *unmatched*. [2]

### III. Application to Main Injector

The Main Injector will be ramped at  $\dot{\gamma} = 163 \text{ s}^{-1}$  when crossing transition. The nonadiabatic time is  $t_c = 1.96 \text{ ms}$ . For a bunch of emittance 0.1 eV-sec, the nonlinear time is  $t_n = 1.06 \text{ ms}$ , assuming that the nonlinear momentum compaction  $\alpha_1 = \frac{1}{2}$ . Thus, we need a  $\gamma_t$  jump of at least  $\Delta\gamma_t \approx 2\dot{\gamma}_t(t_c + t_n) = 1.0$ . Numerical simulations show that a much cleaner crossing will result if the jump is  $\sim 1.3$ . In this bipolar application, our aim is therefore  $\Delta\gamma_t \approx \pm 0.65$ .

The Main Injector lattice MI-17 is two-fold symmetric, so we need only to study one half. We start at location MI52. There are 19  $90^\circ$  cells in a row in the arc; we put in 20 special quads (5 sets of 4's). After a dispersion-free neutrino-extraction region at MI40, there are 6  $90^\circ$  cells in a row; we put in 4 special quads (1 set of 4's). The second family of doublets is placed in the dispersion-free regions: 2 quads at MI40 ( $\nu$  extraction), 2 quads at MI32 (opposite to kicker), 4 quads at MI30 (rf region), and 2 quads at MI22 (opposite to kicker). The results are listed in Table 1. The lattice functions  $\beta_x^{1/2}$  and  $D$  before the jump, at jump-up, and jump-down are plotted in Fig. 1(a), (b), and (c), respectively. We see that a  $\gamma_t$  jump of  $+0.683$  and  $-0.625$  has been achieved with dispersion well-confined within 2.2 m and  $\beta_x$  within 78 m. Both betatron tunes are matched up to  $< 0.002$ . In fact, the MI-17 lattice is not well-matched; the dispersion-free regions are not exactly at  $D = 0$ , and the betatron functions do not repeat themselves exactly for every FODO cell. In addition, the cells are not exactly  $90^\circ$ . If these were corrected, the confinements of the betatron and dispersion waves could have been very much improved. Nevertheless, these results are much

Table 1: A bipolar  $\gamma_t$  jump scheme for MI-17

	MI-17	up	down
$\nu_x$	26.40748	26.40606	26.40940
$\nu_y$	25.40998	25.40836	24.41182
$\beta_x$ Max (m)	59.86643	78.04456	68.88534
$\beta_y$ Max (m)	63.17612	64.12093	65.25474
$D$ Max (m)	1.97820	2.04025	2.20696
$\gamma_t$	21.58789	+0.6826	-0.6254
$\int B'dl$ (1)		+0.1822 T	-0.1586 T
$\int B'dl$ (2)		-0.4305 T	+0.3724 T

better than those obtained in Ref. 2 for the MI-15 lattice. There, the dispersion reaches a maximum of 2.64 m and the  $\beta_x$  reaches 97.5 m.

### IV. Jump Rate

During the process of the  $\gamma_t$  jump, the special quads are first pulsed at the roughly the machine ramping rate so that  $\gamma_t$  is raised slowly by 0.68 unit. Near the transition energy, the current in the special quads are reversed suddenly, so that  $\gamma_t$  drops to a value which is 0.63 unit below. After that, the pulse current is slowly reduced to zero. The rate of jump is limited only by the allowable flux change in the laminations of the quads, which is roughly 1.5 T at 100 Hz, or  $\dot{B} \approx 942 \text{ T/s}$ , provided that good silicon steel is used. From Table 1, we see that the largest integrated change in flux gradient is 0.803 T in the second family. If we assume the quads are of length 1 m and the distance to pole tips is 2 in, the change in flux at pole tips is  $\Delta B = 0.041 \text{ T}$ . Therefore, the fastest jump or reversal of current can be made in 43  $\mu\text{s}$  or about 4 turns.

For the design of Holmes [5], the dispersion increases by 4.8 m per unit  $\Delta\gamma_t$ . In order that the horizontal emittance does not increase appreciably, one needs to limit the rate of jump. However, since the change in dispersion is so small in the present design, this restriction no longer applies.

### References

- [1] K. Johnsen, Symposium on High Energy Accel., CERN, 1956, Vol. 1, p.106
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- [3] S. Peggs, S. Tipikian, and D. Trbojevic, Proc. PAC 93, Washington, 1993, p.168
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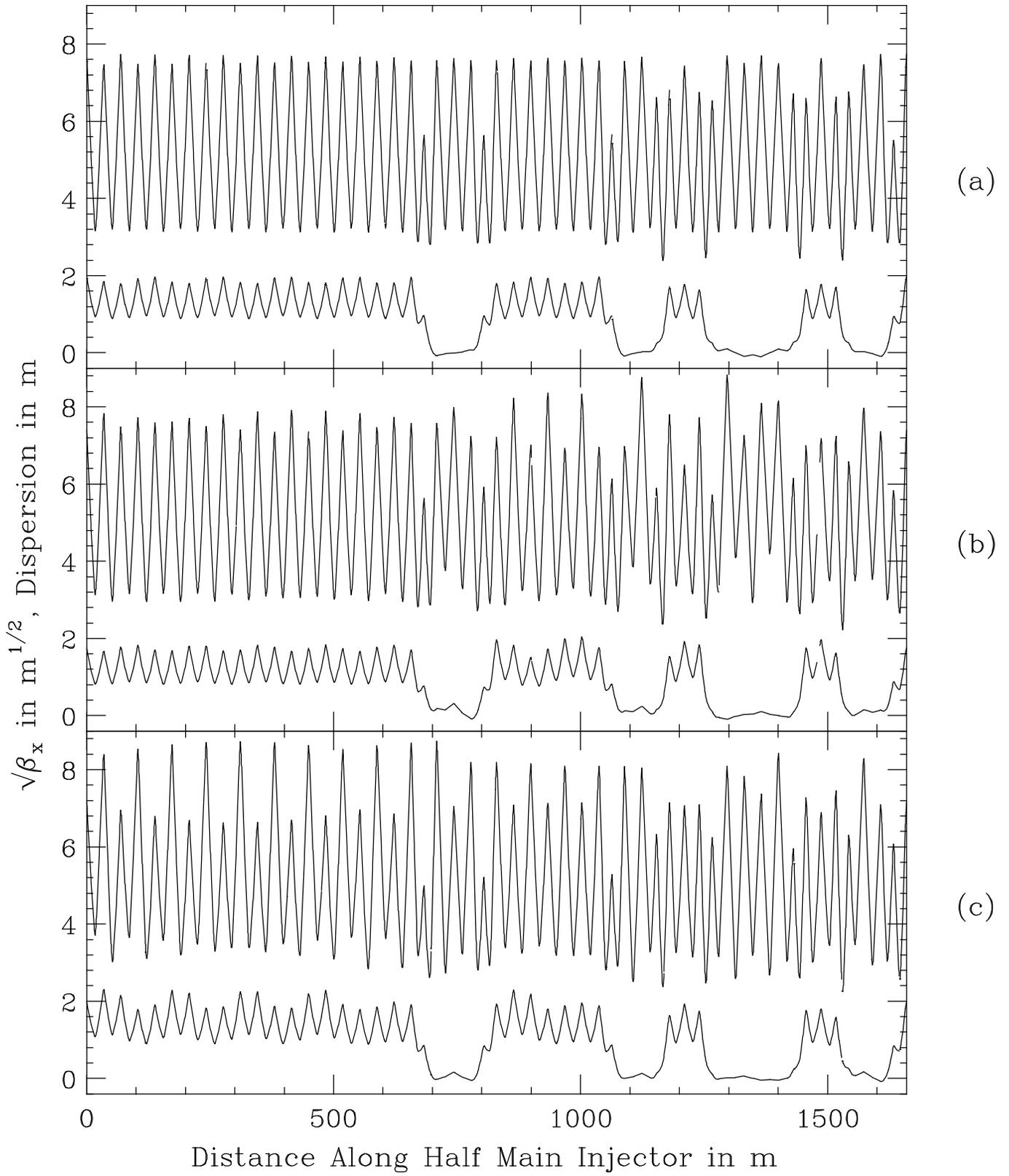


Fig. 1.  $\sqrt{\beta_x}$  and  $D$  of the Main injector M-17 (a) before transition jump, (b) at jump-up, and (c) at jump-down.