Cosmic Particle Acceleration at Very High Energies*

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I. High Energy Cosmic Particles and Fields

In very strong electromagnetic fields, near to the magnetic poles of a neutron star, for example, where the magnetic field strength may well exceed 10^{12}G, and thus the corresponding mass equivalent of field energy be higher than 200 g cm^{-3}, mechanisms can evolve, that are interesting for a number of reasons:

Something may be learnt about electromagnetism at extremely high energy densities,

Astrophysicists want to understand the structure and dynamics of pulsar magnetospheres in terms of underlying physics,

Rotating cosmic magnets, rotation-powered radio pulsars, for example, are possible candidates for high energy cosmic particle accelerators [1],

The physics at work may be helpful for the designing of new types of man-made accelerators.

The present paper is directed to a better understanding of the classical equation of motion for particles in very strong electromagnetic fields at its possible rôle in the generation of gamma ray bursts and in the formation of plasma jets.

II. Self-Consistent Electrodynamics

According to the conventional interpretation of Maxwell theory, electromagnetic radiation is 'generated' through shear acceleration of an electromagnetically interacting particle, irrespective of the nature of accelerating forces.

Consequently, Larmor's radiation formula

$$P^{RAD} = m\tau_0 (d\mathbf{v}_{MRS}/dt)^2 = m\tau_0 ||d\underline{u}/d\tau||^2 \qquad (1)$$

then contains the (four-) vector of 'kinematical' acceleration, $d\underline{u}/d\tau$. $d\mathbf{v}_{MRS}/dt$ is the corresponding (three-) vector in the momentary rest frame, MRS, of that particle. m is its mass and $\tau_0 = 2e^2/3mc^3$ is the radiation constant.

Also in agreement with this interpretation, the equation of motion

$$du_j/d\tau = \eta_0 F^{EXT}_{jk} u^k + \tau_0 G_{jk} u^k, \qquad (2)$$

contains a radiation force tensor

$$\begin{aligned} G_{jk} &= G^{L-D}_{jk} \\ &:= ([d^2u_j/d\tau^2]u_k - u_j[d^2u_k/d\tau^2])/c^2 \end{aligned} \qquad (3)$$

which is governed by (this time the second) kinematical acceleration[1]. F^{EXT}_{jk} is the tensor of the external field[2] in an arbitrary inertial frame of reference (IS) and $\eta_0 = e/mc$. If acceleration is due to external *electromagnetic* fields, Larmor's radiation formula (1) specializes to

$$P^{RAD} = m\tau_0 (c\eta_0 \mathbf{E}^{EXT}_{MRS})^2 = m\tau_0 ||\underline{u}^L||^2, \qquad (4)$$

where $u^L_j := \eta_0 \mathrm{F}^{EXT}_{jk} u^k$ is used as an abbreviation for the 'Lorentz acceleration'. $\mathbf{E}^{EXT}_{MRS}$ is the electric field vector of the external field, in the MRS.

Accordingly, the radiation force tensor (3) specializes to

$$G_{jk} = G^{EXT}_{jk} := \eta_0 u^l \partial_l F^{EXT}_{jk} + G^T_{jk}, \qquad (5)$$

with

$$G^T_{jk} := (u^{LL}_j u_k - u_j u^{LL}_k)/c^2 \qquad (6)$$

and $u^{LL}_j := \eta_0^2 F^{EXT}_{jk} F^{EXT\,kl} u_l$ as an abbreviation for the 'second Lorentz acceleration'.

In what follows I shall, tentatively, suggest that (I) *radiation is generated and radiation reaction may be felt only if and insofar as acceleration is due to external electromagnetic fields* [2], [5], [6].

As a consequence of this suggestion, (4) is expected to be the correct form of Lamor's radiation formula, (at least in the lowest order of the interaction constant) and (5) is expected to be the correct form of the radiation force tensor.

When the external field is a *wave* field, radiation from a charged particle is often looked upon as the *scattered* wave field ('synchro-Compton radiation') and the fourth-order component of radiation reaction force in this special case is seen as the *knock-on* force due to many-photon Thomson- (or Compton-) scattering[3],

$$\mathbf{K}^T_{MRS} = (\sigma^T/c)\mathbf{S}^{EXT}_{MRS}, \qquad (7)$$

where $\sigma^T = 8\pi e^4/3m^2c^4$ is the Thomson cross-section and $\mathbf{S}$ is the Poynting vector.

Here it will be shown how this interpretation can be extended also to electromagnetic fields of *arbitrary* shape, namely that (II)

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[1] Equation (2) with (3) sometimes is called Lorentz-Dirac equation (L-D equation) or Abraham-Lorentz equation. Equation (2) with (5) sometimes is referred to as Lorentz-Dirac-Landau equation (L-D-L equation). A review is given in: [3].

[2] The external field is understood as the field due to all other electromagnetically interacting particles around.

[3] As one might expect, (6) can be deduced from (7) by Lorentz transformation [4].

radiation emitted from an electromagnetically interacting particle in general can be understood as the result of the scattering of an external electromagnetic field.

Suggestions (I) and (II) will be integrated in a Schrödinger picture of the photon and referred to as *self-consistent electrodynamics*.

III. Wave Mechanics of the Photon

In Maxwell's equations

$$\begin{aligned}
[\nabla, \mathbf{E}] &= -(1/c)\partial_t \mathbf{H} - (4\pi/c)\mathbf{j}^{(m)}, \\
[\nabla, \mathbf{H}] &= (1/c)\partial_t \mathbf{E} + (4\pi/c)\mathbf{j}^{(e)}, \\
(\nabla, \mathbf{E}) &= 4\pi\varrho^{(e)}, \\
(\nabla, \mathbf{H}) &= 4\pi\varrho^{(m)},
\end{aligned}$$

the electromagnetic field is represented by the electric and magnetic vectors $\mathbf{E}$ and $\mathbf{H}$, respectively. $\mathbf{j}^{(e)}$ and $\mathbf{j}^{(m)}$ are the electric and magnetic current vectors, and $\varrho^{(e)}$ and $\varrho^{(m)}$ are the electric and magnetic charge densities, respectively.

As is well known, these classical field equations may be written[4]

$$i\hbar\partial_t\psi = \mathcal{H}^+\psi - 4\pi i\hbar\zeta, \tag{8}$$

where $(\nabla, \psi) = 4\pi\eta$. $\psi := C(\mathbf{E} + i\mathbf{H})$ is the wave function of the photon and $\psi^\dagger = C^*(\mathbf{E}^T - i\mathbf{H}^T)$ is its adjoint. Correspondingly, the source terms[5] are defined through the electric and magnetic charges $\eta = C\{\varrho^{(e)} + i\varrho^{(m)}\}$ and the respective currents $\zeta = C\{\mathbf{j}^{(e)} + i\mathbf{j}^{(m)}\}$, respectively[6]. $\mathbf{s} := ((s_\lambda)_{\mu\nu}) := -i\hbar(\varepsilon_{\lambda\mu\nu})$ is the spin and $\mathbf{p} := ((p_\lambda)_{\mu\nu}) := -i\hbar((\partial_\lambda)\delta_{\mu\nu})$ is the momentum of the photon. $\mathcal{H}^+ := (c/\hbar)(\mathbf{s}, \mathbf{p})$.

In a source-free region, Schrödingers's equations is

$$i\hbar\partial_t\psi = \mathcal{H}^+\psi. \tag{9}$$

Alternatively, one might have written

$$i\hbar\partial_t\check{\psi} = \mathcal{H}^-\check{\psi} \tag{10}$$

with the wave function $\check{\psi} := C^*(\mathbf{E} - i\mathbf{H})$ and its adjoint $\check{\psi}^\dagger := C(\mathbf{E}^T + i\mathbf{H}^T)$. Then, $\mathcal{H}^- := -(c/\hbar)(\mathbf{s}, \mathbf{p})$.

Only waves corresponding to positive energy $\mathcal{E}$ of the photon have a physical counterpart in classical electromagnetic fields[7] and thus need to be selected from the solutions of (9) and (10). Among these, the ones with positive sense of rotation of the electric vector with respect to the direction of propagation (i.e. left-circularly polarized waves in the usual notation, corresponding to positive helicity of the photon) are delivered by (9), while those with negative sense of rotation (i.e. right-circularly polarized waves, corresponding to negative helicity) are described by (10).

Here, it will be more comfortable, to make use of a one-to-one correspondence of solutions from (9) and (10), restricting to solutions of (9)

$$i\hbar\partial_t\psi = \mathcal{H}\psi, \tag{11}$$

where $\mathcal{H} := (c/\hbar)\chi(\mathbf{s}, \mathbf{p})$, and χ distinguishes between states of positive and negative helicity.

IV. Normalization and Statistical Interpretation

The quantum Schrödinger picture can be related to the classical Maxwell picture through a statistical interpretation. Multiplication of the Schrödinger equation (9) with $\psi^\dagger$ and of the adjoint equation with ψ after substraction delivers,

$$\partial_\mu \mathrm{w}_\mu + \partial_t \mathrm{w} = \mathrm{q}, \tag{12}$$

where $\mathrm{w}(\mathbf{x}, t) := \psi^\dagger\psi = (\psi^*, \psi)$, and $\mathbf{w}(\mathbf{x}, t) = (\mathrm{w}_\mu) := -ic[\psi^*, \psi]$, and[8] $\mathrm{q} := -8\pi\mathrm{Re}(\psi^*, \zeta)$, where Re( ) stands for the real part.

If the wave packet of the particle constituting the electric charge does *not* grow too fast within time intervals considered, w is *quasi-source free*. In that case, normalization

$$\int_V \psi^\dagger\psi\, d^3\mathbf{x} = 1 \tag{13}$$

is possible with the help of

$$|C|^2 = 1/\int_V \{\mathbf{E}^2 + \mathbf{H}^2\} d^3\mathbf{x}, \tag{14}$$

where $\mathbf{V} \subseteq \mathbf{R}_3(\mathbf{x})$ is an appropriately chosen normalization volume in 3-dimensional coordinate space[9]. Then, w is interpretable as the *position probability density* of the photon and w_μ as the corresponding *position probability current* and the expectation value for energy, e.g., is

$$<\mathcal{H}> = \int_V \psi^\dagger\mathcal{H}\psi d^3\mathbf{x}. \tag{15}$$

Multiplying (9) with $\psi^*_\varrho\varepsilon_{\kappa\varrho\mu}$ and subtraction of the complex conjugate delivers

$$\partial_\nu \mathrm{w}_{\mu\nu} + \partial_t \mathrm{w}_\mu = \mathrm{q}_\mu, \tag{16}$$

where $\mathrm{w}_{\mu\nu} := (\psi^*, \psi)\delta_{\mu\nu} - (\psi^*_\mu\psi_\nu + \psi^*_\nu\psi_\mu)$ and[10] $\mathrm{q}_\mu := C\{\psi^*_\mu(\nabla, \psi) + \psi_\mu(\nabla, \psi^*)\}$.

[4] For example, in: A. Messiah, Quantum Mechanics, Vol.I & II, North Hollandish Publ.Comp. (1970).

[5] Magnetic charges and currents are admitted for reasons of symmetry, but their appearance is not essential in the following argumentations. Brackets [,] are for the vector product, (,) for the scalar product. In what follows, Latin indices are running from 0 through 3, Greek indices are running from 1 through 3. ^{T} stands for transposition and * for complex conjugate. $\mathcal{H}$ will be used for the Hamiltonian to distinguish it from the magnetic vector **H**. Also, $\mathcal{E}$ will be used for the energy of a photon to distinguish it from the energy density of the electromagnetic field ε. $\varepsilon_{\lambda\mu\nu}$ are the Levi-Civita symbols.

[6] With this definition of ψ through **E** and **H** we have not given privilege to any of these two field vectors **E** or **H** since the norm of the wave function, as well as expectation values are numerically invariant, and the field equations are form-invariant under the transformation $\psi \rightarrow \psi' = e^{i\alpha}\psi$, so that $\mathbf{E} \rightarrow \mathbf{E}' = \mathbf{E}\cos\alpha - \mathbf{H}\sin\alpha$, and $\mathbf{H} \rightarrow \mathbf{H}' = \mathbf{E}\sin\alpha + \mathbf{H}\cos\alpha$, (with corresponding transformation rules for the source terms). Moreover, these invariances also exist under *unitary transformations*, $\psi \rightarrow \psi' = \mathcal{A}\,\psi$, with $\mathcal{A} \in U(3)$, from where it looks 'natural' that the corresponding irreducible representations 'appear' as 'elementary entities' which take part in electromagnetic interaction.

[7] In addition, one has to consider wave functions which are independent from the coordinates in space and time, in the normalization volume. They correspond to the limit of vanishing photon energy, $\mathcal{E} \rightarrow 0$.

[8] Corresponding to $q = -8\pi|C|^2\{(\mathbf{E}, \mathbf{j}^{(e)}) + (\mathbf{H}, \mathbf{j}^{(m)})\}$.

[9] Singularities from point-like sources can be avoided by taking into account the finite extension of the wave packet of the corresponding physical particle.

[10] Corresponding to $\mathrm{q}_\mu = 8\pi c|C|^2\{E_\mu\varrho^{(e)} + H_\mu\varrho^{(m)}\}$

Since, in a source-free region, each component of $\mathbf{w} = (c/\hbar)\psi^\dagger \mathbf{s}\psi$ obeys the continuity equation (16), $\mathbf{w}$ may also be interpreted as a density, in this case, of $\mathbf{c} := (c/\hbar)\mathbf{s}$.

For an interpretation of the conserved quantity $\mathbf{c}$, we note that with (11) the only possible eigenvalue of $\mathbf{c} = (c/\hbar)\mathbf{s}$ is $+c$. Thus, c_μ may be understood as the velocity operator of the photon with $c_\mu c_\mu = c^2$. $\mathbf{w}$ can be seen as the velocity density and $\mathrm{w}_{\mu\nu}$ as the corresponding velocity current.

We thus arrive at the classical continuity equation $\partial_\mu S^\#_\mu + \partial_t \varepsilon^\# = 0$ for the density $\varepsilon^\# := \mathrm{N} < \mathcal{H} > \mathrm{w}$, and the current $S^\#_\mu := \mathrm{N} < \mathcal{H} > w_\mu$, of *expected* energy, and at the classical continuity equation $\partial_\nu \sigma^\#_{\mu\nu} + \partial_t P^\#_\mu = 0$, for the density $P^\#_\mu := \mathrm{N} < \mathcal{H} > \mathrm{w}_\mu/c$, and the current $\sigma^\#_{\mu\nu} := \mathrm{N} < \mathrm{H} > \mathrm{w}_{\mu\nu}/c$, of *expected* momentum, where N is the ratio of classical field energy in the normalization volume to the expectation value of photon energy.

V. Reproduction of the Equation of Motion

We now have the means to reinterpret radiation generation and radiation reaction within a field of arbitrary shape in terms of *many photon Thomson scattering*.

If, in the MRS, $O(V')$ is the surface separating a (sufficiently small spherical) scattering volume $V' \subset V$ from $V \backslash V'$, then the rate of *expected* momentum transfer onto the charged particle is

$$K_{MRS\mu} = - \oint_{O(V')} \sigma^\#_{\mu\nu} d^2 o_\nu . \qquad (17)$$

Through arguments analogous to those applied earlier [2], [5], [6], contributions from the external field and from the Coulomb field deliver the Lorentz (Coulomb) force

$$\mathbf{K}^{LOR}_{MRS} = e\mathbf{E}^{EXT}_{MRS}, \qquad (18)$$

and transformation of (18) from the MRS to an arbitrary IS leads to the covariant form of the Lorentz force

$$K^{LOR}_j = m\eta_0 F^{EXT}_{jk} u^k, \qquad (19)$$

while contributions from the radiation field and from the Coulomb field reproduce the radiation reaction force

$$\mathbf{K}^{RAD}_{MRS}(t) = \tau_0 d\mathbf{K}^{LOR}_{MRS}(t)/dt. \qquad (20)$$

Transformation of (20) analogously leads to

$$K^{RAD}_j = m\tau_0 G^{EXT}_{jk} u^k, \qquad (21)$$

and thereby back to the classical equation of motion (2) with (5).

VI. A Possible Mechanism for the Formation of Jets and the Generation of Gamma Ray Bursts by Rotating, Magnetized Neutron Stars

Near the surface of a rotating, magnetized neutron star, the magnetic field may be extremely strong [7], typically of the order of $10^{12}G$, and also the electric field may be very strong, though considerably less, typically of the order of $10^{10}G$. Under such conditions particles tend to follow magnetic field lines and, as I have suggested earlier [6], [5], an upper limit of particle energy is created *locally* by radiation reaction.

In the polar region, $\theta = 0$, of an *orthogonal rotator*, this limit is $\max(\gamma^{\mathrm{ortho}}) \cong 2.4 \cdot 10^3$ for the electron, and $\max(\gamma^{\mathrm{ortho}}) \cong 3.6 \cdot 10^5$ for the proton[11].

For a *parallel rotator*,

$$\max(\gamma^{para}) \cong 2\sqrt{r_L/c\tau_0} \cdot \cdot (\frac{r_N}{r_T}) \operatorname{ctg}\theta_0 \quad \frac{\sqrt[4]{\cos^2\theta_0(3\cos^2\theta_0+1)}}{(1+\cos^2\theta_0)}. \qquad (22)$$

In the polar region, $\theta = 0$, of a parallel rotator, the upper limit of the Lorentz factor is $\max(\gamma^{\mathrm{para}}) \cong 2.9 \cdot 10^3 \operatorname{ctg} \theta_0$ for the electron, and $\max(\gamma^{\mathrm{para}}) \cong 4.4 \cdot 10^5 \operatorname{ctg} \theta_0$ for the proton.

Unlike the orthogonal rotator, the parallel rotator develops a very narrow *nozzle* around the axis, $\theta = 0$, through which very energetic particles can be ejected from the surface. This mechanism may play a rôle in both, the generation of gamma ray bursts[12] as well as the formation of jets.

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[11] As in [8], numerical values given here are for the 'standard set of parameters', e.g., for the radius of the neutron star: $r_N = 10^6 cm$, for the light radius: $r_L = 4.8 \cdot 10^8 cm$, and for the 'typical radius': $r_T = 2.4 \cdot 10^{13} cm$ (electrons) and $r_T = 5.6 \cdot 10^{11} cm$ (protons), corresponding to $\nu = 10\ sec^{-1}$ and $\mu = 10^{30} Gcm^3$.

[12] One may think of thin beams of very energetic particles from spinning neutron stars distributed in the volume of the galactic halo, which randomly hit the location of the observer. But, of course, one has to keep in mind that this suggestion is based on results from particle dynamics in vacuum fields ('stage one'). As mentioned before, modifications of the fields are expected through plasma effects.