A Semi-analytical Approach to the Design of Low Energy Cylindrically Symmetric Transport Lines

Pedro F. Tavares, Laboratório Nacional de Luz Síncrotron, LNLS, Campinas, Brazil

Abstract

An optimization procedure that determines the best values (i.e., those that minimize beam radius) for the focal distances of solenoidal magnetic lenses in a low-energy cylindrically symmetric transport line is presented. The procedure is based on an analysis of the RMS beam envelope equation, which accounts for space charge as well as emittance effects. Universal beam-spreading curves including emittance effects (a generalization to the well-known space-charge spreading curves) are presented in a form convenient for general use. The results are applied to the design of the gun-to-linac transport channel of the LNLS injector.

I. Introduction

LNLS is building a 1.15 GeV electron storage ring to be comissioned in early 1996 as a synchrotron light source. Injection into the storage ring takes place from a 100 MeV LINAC. The 80 keV 100 ns electron beam pulses are produced in a conventional gridded electrostatic gun and then transported to the accelerating structures by means of three iron-core solenoidal magnetic lenses.

The dynamics of the low-energy cylindrically symmetric electron beam under the action of space-charge and focussing forces along the gun-to-linac transport line can be described by the RMS envelope equation derived by Lee and Cooper[1], which also includes finite emittance effects. Although numerical solutions to this equation can be easily obtained in order to validate a given lay-out (position and excitation of lenses) of the transport line, the question remains as to whether the best possible setting of the lenses (in the sense of providing the minimum beam radius all along the line) has been chosen.

In this paper, I present a simple, semi-analytical approach to find the best possible settings for a series of (thin) magnetic lenses once the transport-line lay-out is chosen. In section (II), the RMS envelope equation is briefly reviewed and solutions are presented for field-free regions in the form of normalised beam spreading curves similar to the well know space-charge beam spread curves. The focussing action of the magnetic lenses is calculated in section (III) in the thin lens approximation and piecewise numerical minimisation of the beam radius is performed between lenses, using the lens focal length as the variable parameter. Finally, in section (IV) the particular case of the LNLS preinjector is presented and the results of a detailed (thick lens) numerical solution are compared with the thin lens approximation.

II. The envelope equation

The RMS radius of a cylindrically symmetric electron beam propagating in a *field-free* region obeys the envelope equation

$$R''(z) = \frac{K}{R(z)} + \frac{\varepsilon_0^2}{R^3(z)}\,, \tag{1}$$

where ε_0 is the RMS radial emittance (a constant of the motion)

$$\varepsilon_0^2 = <r^2><r'>^2 - (<rr'>)^2\,, \tag{2}$$

where the angle brackets denote an average over the electron phase space distribution, and

$$K = \frac{e_0 I_0}{4\pi\epsilon_0 m_0 (\gamma\beta c)^3}\,, \tag{3}$$

where e_0 is the elementary charge, I_0 is the beam current, m_0 is the electron mass, and γ, β are respectively the electron energy in units of the electron rest mass and the electron velocity in units of the velocity of light.

Clearly, the solutions to Eq.(1) may present two kinds of behaviour: an initially diverging beam ($R_0' > 0$) continues to diverge with ever increasing slope, whereas an initially converging beam reaches a waist at a certain point and diverges thereafter. In any case, a waist may be defined with respect to which the solution is symmetric.

Multiplying both terms of Eq.(1) by $2R'(z)$ and integrating from a waist at $z = 0$ (where $R'(z) = 0$, $R = R_0$), I obtain

$$R'^2(z) = 2K\int_{R_0}^{R}\frac{dR^*}{R^*} + 2\varepsilon_0^2\int_{R_0}^{R}\frac{dR^*}{R^{*3}} \tag{4}$$

and yet another integration yields an implict solution for R(z):

$$z(R) = \int_{R_0}^{R}\frac{dR^*}{\sqrt{2K\ln\left(\frac{R^*}{R_0}\right) + \frac{\varepsilon_0^2}{R_0^2}\left[1-\left(\frac{R_0}{R^*}\right)^2\right]}}\,. \tag{5}$$

It is convenient to introduce the dimensionless variables

$$\rho = \frac{R}{R_0}\,, \tag{6}$$

$$\nu = \frac{\epsilon_0^2}{2KR_0^2}\,, \tag{7}$$

$$\xi = \sqrt{2K}\,\frac{z}{R_0}\,, \tag{8}$$

so that Eq.(5) becomes

$$\xi(\rho) = \int_1^{\rho}\frac{d\rho^*}{\sqrt{\ln\rho^* + \nu\left[1-\left(\frac{1}{\rho^*}\right)^2\right]}}\,. \tag{9}$$

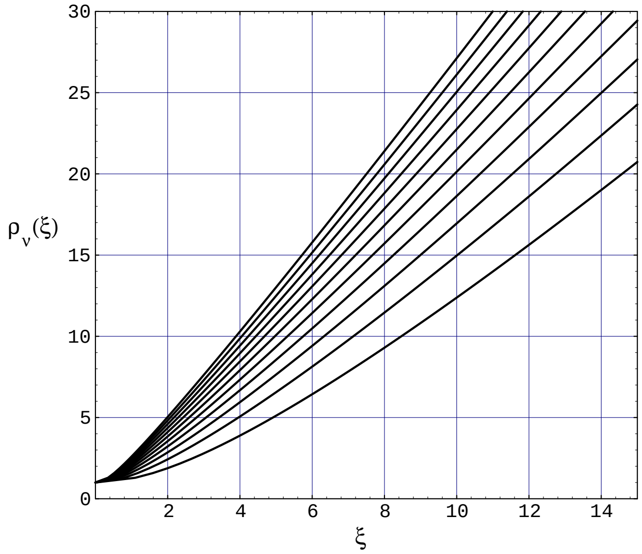

Figure. 1. Normalised spreading curves for a beam under the action of space-charge and emittance. The curves correspond to (from bottom to top) $\nu = 0$, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0.

Finally, it is convenient to change the integration variable to w defined by

$$w^2 = \ln \rho^* \,, \tag{10}$$

so that

$$\xi_\nu(\rho) = \int_0^{\sqrt{\ln \rho}} \frac{2e^{w^2} w \, dw}{\sqrt{w^2 + \nu \left(1 - e^{-2w^2}\right)}} \,. \tag{11}$$

Fig.(1) shows beam spread curves calculated from the equation above for various values of ν. These curves reduce to those plotted by Hutter[2] in the limit of a laminar beam ($\nu = 0$). Once these curves are calculated numerically at a sequence of points, a spline interpolation provides a convenient and fast way of calculating the inverse function $\rho_\nu(\xi)$.

III. Optimisation of lens settings

The generalisation of Eq.(1) to include static axial magnetic fields reads

$$R''(z) = \frac{K}{R} + \frac{\varepsilon_0^2}{R^3} - \Omega^2(z) R \,, \tag{12}$$

where

$$\Omega(z) = \frac{e_0 B(z)}{2m_0 c\gamma\beta} \tag{13}$$

and $B(z)$ is the longitudinal magnetic field. If the magnetic field is confined to a small region around $z = z_i$, its effect may be approximated by a thin lens of focal distance

$$\frac{1}{f} = \int \Omega^2(z) \, dz \tag{14}$$

so that, on crossing the lens, the slope R' changes by $\Delta R' = -R/f$, whereas, in between two lenses, the functions $\xi_\nu(\rho)$ and $\rho_\nu(\xi)$ defined above describe the evolution of the envelope. Fig.(2) shows the qualitative evolution of the envelope along a

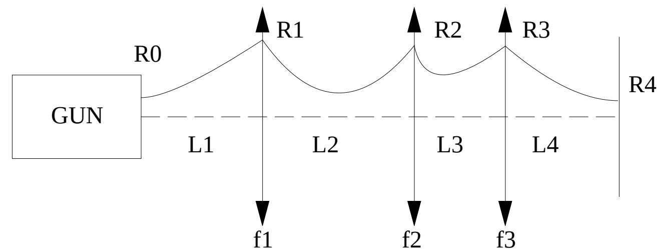

Figure. 2. The beam envelope along a transport line with three thin magnetic lenses.

tranport line containing three thin magnetic lenses. For simplicity, I assume that both the starting point $z = 0$ (at the output of the gun) and the end of the line are waists ($R' = 0$).

The beam radius at the first lens is

$$R_1 = R_0 \rho_\nu \left(\sqrt{2K} L_1 / R_0 \right) \,. \tag{15}$$

I now consider the problem of minimising the beam radius at the second lens (R_2) by adjusting the focal length f_1, or equivalently, by adjusting the position ($z_{wa} = L_1 + z_{ma}$) and radius R_{wa} of a beam waist to be formed after lens 1. The boundary conditions at $z = L_1$ and $z = L_1 + L_2$ are

$$\sqrt{2K} \frac{z_{ma}}{R_{wa}} = \xi_\nu \left(\frac{R_1}{R_{wa}} \right) , \tag{16}$$

$$\sqrt{2K} \frac{|L_2 - z_{ma}|}{R_{wa}} = \xi_\nu \left(\frac{R_2}{R_{wa}} \right) , \tag{17}$$

where the absolute value is introduced to deal with the possibility of the waist being after lens 2. The equations above can be combined to yield

$$R_2(R_{wa}) = R_{wa} \rho_\nu \left(\left| \frac{L_2 \sqrt{2K}}{R_{wa}} - \xi_\nu \left(\frac{R_1}{R_{wa}} \right) \right| \right) \,. \tag{18}$$

Once R_{wa}, z_{ma}, and R_2 are determined by numerical minimisation of the expression above[1], the same reasoning may be applied to the region between the second and third lenses so as to determine the value of f_2 that minimises R_3 given by the equation

$$R_3^\alpha(R_{wb}) = R_{wb} \rho_\nu \left(\left| \frac{L_2 \sqrt{2K}}{R_{wb}} - \xi_\nu \left(\frac{R_1}{R_{wb}} \right) \right| \right) \,. \tag{19}$$

where now the radius at the waist is R_{wb}. A similar reasoning may be used to determine the focal length f_3 that is compatible with the assumption $R_4' = 0$ and that yields the minimum possible R_3 ,by minimising the expression

$$R_3^\beta(R_4) = R_4 \rho_\nu \left(\frac{\sqrt{2K} L_4}{R_4} \right) , \tag{20}$$

[1] Clearly, since ν is actually a function of R_{wa}, a proper treatment would require that the functions $\rho_\nu(\xi)$ and $\xi_\nu(\rho)$ be interpolated as functions of two variables rather than just the 1-dim curves given above. If the emittance effect is a small correction to the space-charge effect, however, one may safely take $\nu =$ constant.

where now the waist position is fixed (at L_4) and the radius at the waist is R_4. The larger of the two values (R_3^α and R_3^β) obtained for the beam radius at the third lens (as determined by independent conditions) must be taken as the design value, since otherwise the two requirements are incompatible and cannot be satisfied simultaneously, so that

$$R_3 = \max\left(R_3^\alpha, R_3^\beta\right). \tag{21}$$

If $R_3 = R_3^\alpha$, then R_4 is obtained by solving the equation

$$R_4 \rho_\nu \left(\frac{\sqrt{2K} L_4}{R_4}\right) = R_3\,, \tag{22}$$

whereas if $R_3 = R_3^\beta$ is larger, then R_4 is just the value obtained from the minimisation in Eq.(20). Similarly, if $R_3 = R_3^\beta$, R_{wb} is obtained from the equation[2]

$$\frac{\sqrt{2K} L_3}{R_{wb}} = \xi_\nu\left(\frac{R_2}{R_{wb}}\right) + \xi_\nu\left(\frac{R_3}{R_{wb}}\right), \tag{23}$$

whereas if $R_3 = R_3^\alpha$, R_{wb} is simply the value obtained in the minimisation of Eq.(19). The positions of the waists are given by

$$z_{wa} = L_1 + \frac{R_{wa}}{\sqrt{2K}} \xi_\nu\left(\frac{R_1}{R_{wa}}\right) \tag{24}$$

$$z_{wb} = L_1 + L_2 + \frac{R_{wb}}{\sqrt{2K}} \xi_\nu\left(\frac{R_1}{R_{wb}}\right). \tag{25}$$

Finally, the slopes just before and after the various lenses are

$$\begin{aligned}
R'_{1_-} &= \sqrt{2K}\sqrt{\ln R_1/R_0 + \nu\left(1 - \left(\frac{R_1}{R_0}\right)^2\right)},\\
R'_{1_+} &= -\sqrt{2K}\sqrt{\ln R_1/R_{wa} + \nu\left(1 - \left(\frac{R_{wa}}{R_1}\right)^2\right)},\\
R'_{2_-} &= \sqrt{2K}\sqrt{\ln R_2/R_{wa} + \nu\left(1 - \left(\frac{R_{wa}}{R_2}\right)^2\right)},\\
R'_{2_+} &= -\sqrt{2K}\sqrt{\ln R_1/R_{wa} + \nu\left(1 - \left(\frac{R_{wa}}{R_2}\right)^2\right)},\\
R'_{3_-} &= \sqrt{2K}\sqrt{\ln R_3/R_{wb} + \nu\left(1 - \left(\frac{R_{wb}}{R_3}\right)^2\right)},\\
R'_{3_+} &= -\sqrt{2K}\sqrt{\ln R_3/R_{wb} + \nu\left(1 - \left(\frac{R_{wb}}{R_3}\right)^2\right)},
\end{aligned}$$

and the focal distances are

$$f_i = \frac{R_i}{R'_{i_+} - R'_{i_-}}\,, \tag{26}$$

with $i = 1, 2, 3$.

[2]Here I assume $z_{mb} = z_{wb} - (L_1 + L_2) < L_3$.

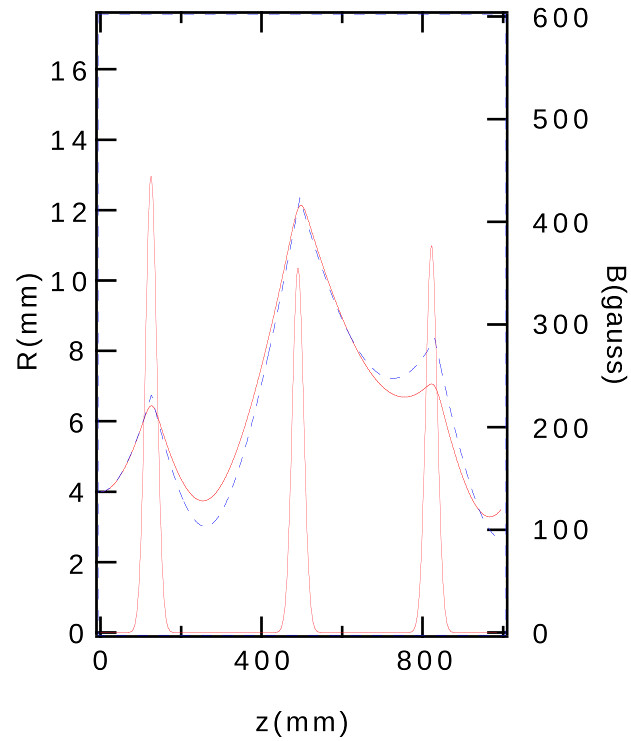

Figure. 3. Beam envelope along the gun-to-LINAC transport line. The dashed line is the result of the thin lens approximation and the solid line was obtained from a full numerical solution of the envelope differential equation with a gaussian model for the longitudinal magnetic field. Also shown is the axial magnetic field.

IV. Results

Fig.(3) shows the results obtained with the method outlined above for an 80 keV 2 Amp beam, with $\varepsilon_0 = 40$ mm mrad. The distances are $L_1 = 12.7$ cm, $L_2 = 33.2$ cm, $L_3 = 32.4$ cm, and $L_4 = 14.21$ cm. The optimised focal distances are $f_1 = 8.5$ cm, $f_2 = 10.5$ cm, and $f_3 = 8.4$ cm.

References

[1] E.P.Lee and R.K.Cooper, *General Envelope Equation for Cylindrically Symmetric Charged-Particle Beams*, Part. Acc., Vol 27 pp. 83 (1976)

[2] R. Hutter, *Beams with Space-charge*, in *Focusing of Charged Particles*, Vol II ed by A. Septier, Academic Press, New York (1967).