NONLINEAR SPACE CHARGE EFFECT OF GAUSSIAN TYPE BUNCHED BEAM IN LINAC *

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The nonlinear space charge effect of bunched beam with Gaussian type longitudinal distribution is discussed in this paper. Some useful formulae are derived for calculating the potential induced by a cylinder model of space charge in the waveguide of a linac with longitudinal density distribution of Gaussian type combining with transverse density distributions of Kapchinskij-Vladimirskij, waterbag, parabolic and Gaussian types, respectively.

I. INTRODUCTION

Theoretical and experimental studies on the intense beam have already found that the bunched beams always show some distributions [1]. Therefore, the nonlinear space charge effect due to the different distributions of the bunched beam is a very important subject in the intense beam research.

In Ref.[2], the general calculation formulae for the nonuniform density space charge effect in the waveguide of electron linac have been given. And in Ref.[3], we discussed the space charge effect of the disk model and the cylinder model with different transverse distributions: Kapchinskij-Vladimirskij (K-V), waterbag (WB), parabolic (PA), and Gaussian (GA), but uniform distribution in longitudinal direction. Furthermore, in Ref.[4] we developed the space charge effect of the two models with longitudinal distributions of waterbag or parabolic types combining with the above four different transverse distributions. It should be pointed out that the Gaussian distribution is of much more general importance, since, according to the central limit theorem of statistical mechanics, any processes of a random, statistically independent nature acting on a particles' positions that obey a Gaussian distribution [5]. Therefore, in this paper, the nonlinear space charge effect of longitudinal Gaussian type bunched beam combining with the above four different transverse distributions has been discussed according to the general formulae derived in the Ref.[4].

II. SPACE CHARGE EFFECT OF BUNCHED BEAM WITH LONGITUDINAL GAUSSIAN DISTRIBUTION

We use the cylinder model of space charge with b and $\pm L/2$ as its boundaries in r and z axes, respectively, in the waveguide of a linac.

The function for longitudinal Gaussian distribution can be described as follows:

$$\rho(z)=e^{-z^2/2\beta^2}, \tag{1}$$

where β is the truncated distance of the longitudinal distribution.

A. Transverse uniform (K-V) distribution

The volume charge density distribution function of the space charge bunch is

$$\rho(r,z)=\rho_{kv,ga}e^{-z^2/2\beta^2}, \tag{2}$$

with $\rho_{kv,ga}=q/\sqrt{2}\,\pi^{3/2}b^2\beta$ and the total charge q .

Now expanding the eq.(1):

$$e^{-z^2/2\beta^2}=1+\sum_{n=1}^{\infty}\frac{(-1)^n z^{2n}}{2^n n!\,\beta^{2n}} \tag{3}$$

and substituting eq.(2) into the general formulae derived in Ref.[4], one can get the potentials:

$$\varphi_{1,2}=\frac{2ab\rho_{kv,ga}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_1(k_l b)J_0(k_l r)}{(k_l a)^3 J_1^2(k_l a)}P_{ga}e^{-k_l|z|}\ , \tag{4}$$

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$$\varphi_3=\frac{2ab\rho_{kv,ga}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_1(k_lb)J_0(k_lr)}{(k_la)^3J_1^2(k_la)}Q_{ga}\ , \tag{5}$$

where

$$P_{ga}=sh\frac{k_lL}{2}+\sum_{n=1}^{n=\infty}A(n)\left[\sum_{k=1}^{n}B(k)sh\frac{k_lL}{2}-\sum_{k=1}^{n}C(k)ch\frac{k_lL}{2}\right]\ , \tag{6}$$

$$Q_{ga}=(1-e^{-k_lL/2}chk_lz)+\sum_{n=1}^{\infty}A(n)\Big\{\left(e^{-k_l|z|}-e^{-k_lL/2}\right)chk_lz+ +\sum_{k=1}^{n}\left[D(k)-B(k)e^{-k_lL/2}chk_lz\right]-\sum_{k=1}^{n}C(k)e^{-k_lL/2}chk_lz\Big\}\ , \tag{7}$$

and, the subscript "1,2" stands for the potentials in the region of (|z|>L/2), "3" stands for the potential in the region of (|z|<L/2), and with

$$A(n)=\frac{(-1)^n(2n)!}{n!(2\beta^2k_l^2)^n} \tag{8}$$

$$B(k)=\frac{(k_lL/2)^{2k}}{(2k)!} \tag{9}$$

$$C(k)=\frac{(k_lL/2)^{2k-1}}{(2k-1)!} \tag{10}$$

$$D(k)=\frac{(k_l|z|)^{2k}}{(2k)!} \tag{11}$$

It should be pointed out that the potentials are derived as usual in the frame of reference moving with the space charge bunch with the same velocity. As for the relativistic space charge effect, the formulae should be transformed into the relativistic case according to Ref.[6].

B. Transverse WB distribution

The volume charge density distribution function of the space charge bunch is

$$\rho(r,z)=\rho_{wb,ga}\left(1-\frac{r^2}{b^2}\right)e^{-z^2/2\beta^2}, \tag{12}$$

with $\rho_{wb,ga}=2q\ /\ \sqrt{2}\pi^{3/2}b^2\beta$.

The potential formulae for the space charge bunch with waterbag distribution in both longitudinal and transverse directions can be derived as follows:

$$\varphi_{1,2}=\frac{4a^2\rho_{wb,ga}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_2(k_lb)J_0(k_lr)}{(k_la)^4J_1^2(k_la)}P_{ga}e^{-k_l|z|}\ , \tag{13}$$

$$\varphi_3=\frac{4a^2\rho_{wb,ga}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_2(k_lb)J_0(k_lr)}{(k_la)^4J_1^2(k_la)}Q_{ga}\ . \tag{14}$$

C. Transverse PA distribution

The volume charge density distribution function of the space charge bunch is

$$\rho(r,z)=\rho_{pa,ga}\left(1-\frac{r^2}{b^2}\right)^2e^{-z^2/2\beta^2}\ , \tag{15}$$

with $\rho_{pa,ga}=3q/\sqrt{2}\pi^{3/2}b^2\beta$.

The potentials can be obtained as follows:

$$\varphi_{1,2}=\frac{16a^3\rho_{pa,ga}}{\varepsilon_0b}\sum_{l=1}^{\infty}\frac{J_3(k_lb)J_0(k_lr)}{(k_la)^5J_1^2(k_la)}P_{ga}e^{-k_l|z|}\ , \tag{16}$$

$$\varphi_3=\frac{16a^3\rho_{pa,ga}}{\varepsilon_0b}\sum_{l=1}^{\infty}\frac{J_3(k_lb)J_0(k_lr)}{(k_la)^5J_1^2(k_la)}Q_{ga}\ , \tag{17}$$

D. Transverse GA distribution

The volume charge density distribution function of the space charge bunch is

$$\rho(r,z)=\rho_{ga,ga}e^{-r^2/2\alpha^2}e^{-z^2/2\beta^2}\ , \tag{18}$$

with $\rho_{ga,ga}=q/(2\pi)^{3/2}\alpha^2\beta,\ \alpha^2=\langle x^2\rangle$.

The potentials can be derived as follows:

$$\varphi_{1,2}=\frac{2\rho_{ga,ga}\alpha^2}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_0(k_lr)e^{-k_l^2\alpha^2/2}}{(k_la)^2J_1^2(k_la)}P_{ga}e^{-k_l|z|}\ , \tag{19}$$

$$\varphi_3 = \frac{2\rho_{ga,ga}\alpha^2}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_0(k_l r)}{(k_l a)^2 J_1^2(k_l a)} e^{-k_l^2\alpha^2/2} Q_{ga} \ , \qquad \textbf{(20)}$$

It is obviously that when L/2 approaches to infinity, we have: $P_{ga} = sh\frac{k_l L}{2}, Q_{ga} = (1 - e^{-k_l L/2} chk_l z)$. Therefore, the above potential formulae eqs.(4), (5), (13), (14), (16), (17), (19), and eq.(20) degenerated into the potentials induced by the cylindrical space charge with longitudinal uniform distribution while uniform(K-V), waterbag, parabolic and Gaussian distributions in transverse direction, respectively, in Ref.[3]. Furthermore, as L/2 approaches to zero, the above equations agree to the potential of a point charge in Ref.[7].

III. REFERENCES

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