EFFECT OF PARAMETRIC RESONANCES ON THE BUNCHED BEAM DILUTION MECHANISM[*]

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Abstract

Experimental measurements of bunch dilution resulting from a modulating secondary rf cavity will be discussed. We found that parametric resonances played indeed an important role in the bunch dilution mechanism. The rms bunch length vs time did not satisfy the Einstein relation. Thus the bunch dilution may not be explained by a simple diffusion mechanism.

I. INTRODUCTION

The double rf system has been used to alleviate the space charge effect by reducing the peak current [1], [2]. It has also been used to overcome multibunch instabilities by modifying the time structure of the bunch, which changes the effective impedance experienced by the beam, and more importantly to increase the synchrotron tune spread of the beam for achieving an enhanced Landau damping [3], [4]. In past few years, there have been some theoretical studies on the double rf system for small amplitude synchrotron oscillations [6], which are valid only for synchrotron phase amplitude $\hat{\phi} \leq 50^\circ$. More recently, a method has been advanced to solve the double rf system, without small amplitude approximations, in the presence of external coherent harmonic modulations [7]. In particular, an analytic solution has been obtained for the case where the harmonic ratio is two [8]. Furthermore, a secondary high frequency rf system has also been used for controlled longitudinal beam emittance dilution [5].

Longitudinal phase space dilution is important in the operation of many synchrotrons. In particular, the longitudinal phase space dilution can minimize the negative mass instability across the transition energy. A common procedure is to modulate a secondary high frequency rf system. The phase modulation with a proper modulation frequency leads to a controlled phase space blowup before the transition energy crossing. This paper discusses experimental measurements of the evolution of bunch profile while a modulating secondary rf system is acting on the beam.

II. EXPERIMENTAL TECHNIQUE AND CALIBRATION

The experimental procedure started with 90 MeV H_2^+ strip-injected into the Indiana University Cyclotron Facility (IUCF) Cooler Ring, resulting in a proton kinetic energy of 45 MeV. The revolution frequency for the synchronous particle, f_0, was 1.03168 MHz. The frequency of the primary rf cavity was 1.03168 MHz with $h_1 = 1$, and the harmonic number of the secondary rf cavity was $h = 9$. The ratio of the harmonic numbers was chosen to be equal to 9 so that the secondary rf cavity would work as a perturbation to the primarily rf caivity. The primary rf voltage was set at about 300 V, which resulted in a synchrotron frequency of about 705 Hz while operating with the primary rf cavity alone. The total beam current was about 100 μA, or equivalently 6×10^8 protons per bunch. The accelerator was operated with a cycle time of 10 s. The injected beam was electron-cooled for about 3 seconds. The cooling rate has been previously measured to be about 3 ± 1 s^{-1} [9], [10], which is equivalent to a cooling time of about 300 ms.

The Hamiltonian for the double rf system is given by

$$H_0 = \frac{1}{2}\nu_s\delta^2 + \nu_s\left[(1-\cos\phi) - \frac{r}{h}(1-\cos[h\phi+\Delta\phi(t)])\right], \tag{1}$$

where $\nu_s = \left(\frac{h_1 e V_1|\eta|}{2\pi\beta^2 E}\right)^{1/2}$ is the small amplitude synchrotron tune of the primary rf system alone, $r = \frac{V_2}{V_1}$ is the ratio of the rf voltages, and $h = \frac{h_2}{h_1}$ is the ratio of the harmonic numbers. The synchrotron tune is the number of synchrotron oscillations per revolution. The conjugate phase space coordinates (ϕ, δ) are respectively the phase of the particle relative to that of the synchronous particle and the normalized off-momentum variable $\delta = -\frac{h|\eta|}{\nu_s}\frac{\Delta p}{p_0}$, where $\eta = -0.86$ is the phase slip factor. The phase of the secondary rf cavity is modulated by $\Delta\phi = A\sin\nu_m\theta + \Delta\phi_0$, where A is the modulation amplitude and $\Delta\phi_0$ is a constant.

The rf voltages of the two cavities were calibrated individually by measuring their synchrotron frequencies, when each cavity was operated alone. Since there is a lower limit of higher harmonic cavity, we choose $V_1 = 300$ V, which gives $\nu_s = 705$ Hz for the primary rf system. At this rf voltage, the rms bunch length is about 20 ns at a beam current of about 100 μA.

To measure the evolution of the bunch distribution function, we use a BPM sum signal passing through a low loss cable, called *elephant trunk.* The elephant trunk cable is a 7/8" heliax high bandwidth, low attenuation cable, made by Andrews Corp. The amp is a Burr-Brown 3553 FET buffer amp. We use a 1 Mohm resistor to ground on the front end of the circuit. Taking into account the capacitance of the pickup, it gives us a 1 kHz high pass filter. Setting the acceptance of the amp close to dc can effectively eliminate the overshoot that is commonly seen on 50 ohm amps. The amp has a bandwidth of 150 MHz.

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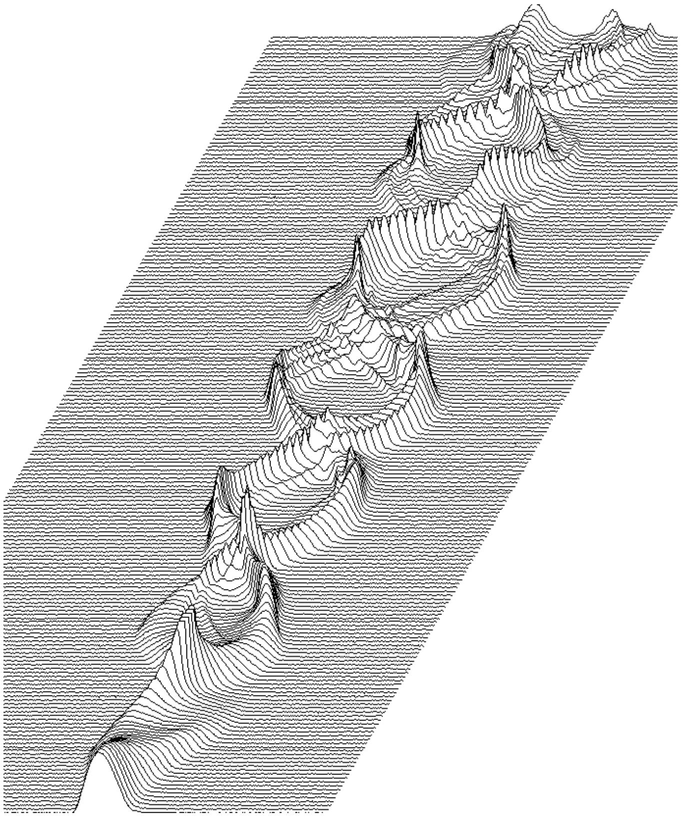

Figure 1. The bunch evolution profile as a function of time obtained from fast digtal oscilloscope. The time resolution is 1 ns and the interval between each bunch profile is about 25 μs. The modulating frequency is 1.3 kHz. We note that the effect of parametric resonances is evident from the bunch profile splitting.

The signal is digitized by a fast sampling scope at a time step of 1 ns with adjustable interval of revolution periods. We have written a program in the PC to control the data taking and data transfer from the scope to PC via GPIB control card. A data processing and replay system has been developed in the X Window environment to benefit from its portability. The digitized data can be replayed in a movie real-time style or projected in 3-D. Furthermore, the digitized data can be integrated to estimate beam loss in each run. The rms beam size can also be calculated by using the relation,

$$\sigma_t^2(t) = \oint \rho(t')(t-t_0)^2 dt. \tag{2}$$

Figure 1 shows an example of the evolution of bunch profile as a function of time when the secondary rf cavity is modulated at a frequency of 1.3 kHz. We note in particular that the bunch profile was observed to split into many beamlets which rotated about the center of the synchrotron phase space. Depending on the modulating frequency, the characteristics of bunch evolution would vary. In order to analyze these data systematically, we calculate the rms beam size of the beam.

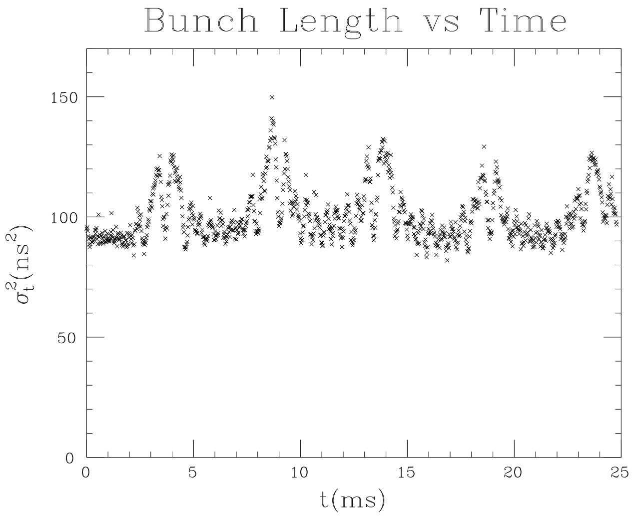

Figure 2. The rms bunch length as a function of time when the modulation frequency is 200 Hz. Note that the bunch length shows a characteristic 200 Hz modulational peaks. The bunch shape is distorted without dilution. Since the modulation frequency is far from parametric resonances, the beam response is small.

A. Background subtraction and the evolution of the rms distribution

The initial rms bunch length is about 20 ns while the final rms bunch length can be as large as 80 ns. The digitized trace of each bunch profile is 512 ns. Thus any noise in the digitized signal will greatly distort the evolution of rms value. Thus a reliable background subtraction is necessary. This is ensured by the 512 ns sampling time of each bunch profile which is long enough to guarantee at least 100 ns pretrigger of pure background whose level is then easily estimated and subtracted for each profile. Also, a computational method is used to cut the background noise preceding and trailing the beam profile signal in order to eliminate the error introduced by these random noise in evaluation of the rms value; the error is observed to be overwhelmingly large and buries the true rms value of the beam bunch due to the quadratical contribution of these noise that spread widely towards the two ends of the sampling window.

Figure 2 shows an example of the evolution of the $\sigma_t(t)^2$ vs time t. Note that the rms size of the bunch remains constant, while the modulation frequency of 200 Hz is visible in the bunch evolution spectrum. On the other hand, when the modulation frequency was near harmonics of synchrotron frequencies, we observed a sizable emittance blowup shown in Fig. 3. Note in particular that the evolution of the bunch rms size shows two characteristic slopes. From the bunch profile evolution shown in Fig. 1, it seemed that some major parametric resonances played key roles in transporting particles into traps of small islands created by the modulating secondary rf system.

The particle transporting mechanism depends sensitively on the modulation frequency. Figure 4 shows the slope of the initial growth as a function of the modulation frequency.

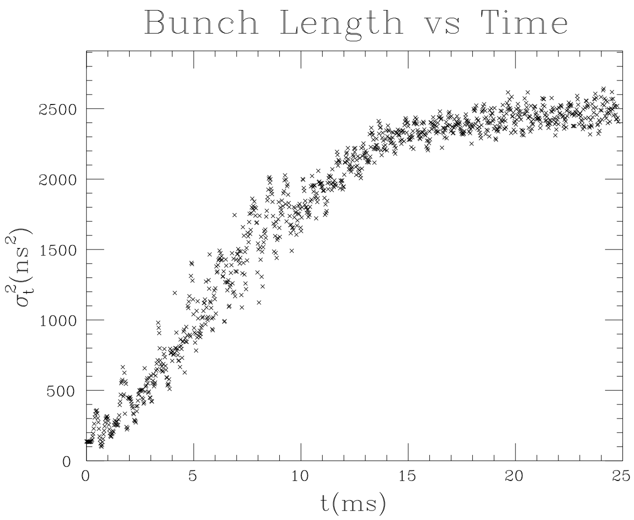

Figure 3. The rms bunch length shown as a function of time when the modulation frequency is 2.5 kHz, where the 4th order parametric resonances are important. Note in particular that there growth rate of the rms bunch length shows two distinct diffusion like behavior. The fast growth region corresponds to the parametric resonance dominant regime and the slow growth region resembles the statistically randomized motion.

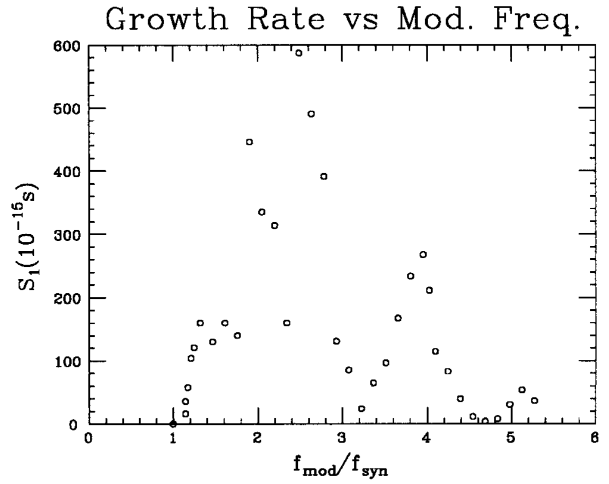

Figure 4. The measured initial growth rate plotted as a function of the modulation frequency. Note the sensitivity of the modulation frequency.

III. CONCLUSION

We have made systematic experimental observation of the bunch profile evolution when a secondary rf field is modulated. The measured bunch profile shows characteristics of dominant parametric resonances playing the role of particle transporting mechanism. Once the bunch is distributed uniformly, the beam bunch distribution function becomes less sensitive to these strong dominant resonances. We also found that the initial diffusion rate depended sensitively on the modulation frequency. In many cases, the bunch beam dilution mechanism does not exhibit the characteristics of diffusion. The abnormal diffusion played an important role in the bunch beam dilution mechanism.

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