

# Colliding Crystalline Beams

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Crystalline Beams\* are an ordered state of an ensemble of ions, circulating in a storage ring, with very small velocity fluctuations [1,2]. They can be obtained from ordinary *warm* ion beams with the application of powerful cooling techniques (stochastic, electron, laser, ...). Depending on the focussing properties and dimensions of the storage ring, and on the ion beam density, several ground states are possible [3-4]. All of them can be visualized as a bundle of  $n_s$  symmetrically distributed, parallel *strings* [5]. The longitudinal ion separation  $\lambda$  is the same for all *strings*. The minimum temperature that can be achieved depends on the background noise of the cooling technique used. It is required for stability that the vibration amplitude of the ions is only a fraction of the separation  $\lambda$ .

## I. COLLIDING CRYSTALLINE BEAMS

There are several advantages in using colliding Crystalline Beams. First, it is possible to obtain more-compact beams. Second, most importantly, the particles in each beam occupy well-defined and rigid positions. The uncertainty on the transverse location of each *string* is considerably smaller than the longitudinal spacing  $\lambda$ . By carefully aligning the two beams to pair countermoving *strings* in the collision, it is possible to increase the luminosity by a considerable factor. Indeed the luminosity in this case is proportional to the number of *strings* in each beam

$$L = n_s L_s \quad (1)$$

where

$$L_s = N_s^2 f_{enc} / 4\pi \sigma_{eff}^2 \quad (2)$$

is the contribution to the luminosity from each pair of countermoving *strings*;  $f_{enc}$  is the frequency of encounter,  $\sigma_{eff}$  is an effective cross section which is the common transverse dimension of the two *strings* during collision, and  $N_s$  is the number of circulating ions per *string* engaged in the collision at one time, assuming that the two countermoving beams are identical.

## II. COLLISION BETWEEN STRINGS

The easiest example to investigate is the case of two identical Crystalline Beams sharing the same storage ring. The two beams are moving in the opposite direction along the common closed orbit. We require that the ions in the two beams have the same magnetic rigidity, mass number  $A$ , and charge state  $Q$ . But they must have opposite sign of the electric charge, if the

bending along the closed orbit is due to a common axial magnetic field. Thus, one beam is made of partially-stripped, positively-charged ions, and the other beam is made of the same ions with an excess of electrons (negative ions). We shall not debate here how this can be realized in practice, but only assume that it is possible.

It is also sufficient to investigate the case of collision between paired *strings* and neglect the presence of the others in the background which are taken to be at a sufficiently large distance. We shall assume that the two countermoving *strings* are initially separated vertically by some distance  $2b$ , and want to investigate the behavior and the stability of the motion of the ions when the separation distance  $b$  is reduced and the *strings* are brought closer for collision.

## III. EQUATIONS OF MOTION

If the two *strings* were moving in the same direction, the configuration expected is that of a *vertical zigzag* [4] of which the properties are known. In the present case, the two strings move parallel to each other, but in opposite direction, each with velocity  $\beta c$ . As the ions of one *string* pass along the reference ion in the other *string*, the motion of this is modulated periodically in all directions. We shall assume that the modulation has an amplitude small compared to the separation  $\lambda$ , and linearize the equations of motion which apply, for symmetry, to all particles. They are [5]:

$$b'' + K_v(s) b + K_{sc} f_3(\alpha, \tau) b / g_0 = 0 \quad (3)$$

$$a'' + K_h(s) a = h(s) \delta \quad (4)$$

$$\sigma' = h(s) a - \delta / \gamma^2 \quad (5)$$

$$\delta' = -(\gamma^2 \lambda K_{sc} / 2 g_0) [df_1(\alpha, \tau) / d\tau] \quad (6)$$

The force acting on the test particle is the one generated by all the ions of the countermoving *string*. Those ions on the same *string* do not contribute to the total force. In Eq.s (3-6) the independent variable is the path length  $s$  along the common closed orbit. A prime denotes differentiation with respect to  $s$ .  $b$  is the vertical displacement from the median plane of symmetry,  $a$  is the radial displacement,  $\sigma$  the path length difference, and  $\delta$  the momentum deviation from the reference closed orbit conditions.  $K_{v,h}$  are the focussing functions which we approximate with constants  $(v_{v,h} / R)^2$ , where  $v_{v,h}$  are the betatron tune numbers and  $R$  is the average closed orbit radius.  $h(s)$  is the curvature function, which we also approximate with  $1/R$ .  $\beta$  and  $\gamma$  are respectively the velocity and energy relativistic factors.  $g_0 = 1.3$  is a constant. The space-charge constant [5]

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$$K_{sc} = (2/R^2) (\lambda_c/\lambda)^3 \quad (7)$$

where the critical spacing [5]

$$\lambda_c = (g_0 Q^2 r_0 R^2 / A \beta^2 \gamma^5)^{1/3} \quad (8)$$

and  $r_0 = 1.535 \times 10^{-18}$  m is the proton classical radius. Finally, the form factors

$$f_n(\alpha, \tau) = \sum_i [\alpha^2 + (i - \tau)^2]^{-n/2} \quad (9)$$

where the aspect ratios  $\alpha = 2b/\lambda\gamma$  and  $\tau = 2s/\lambda$ . The form factors are periodic with respect to  $\tau$  with a periodicity of one.

#### IV. CASE OF LARGE SEPARATION

In the limit of large separation among the two *strings*, that is  $\alpha > 1$ ,  $f_3$  is about constant, whereas  $df_1/d\tau$  oscillates around zero. In the smooth and linear approximation, ignoring the curvature of the storage ring, we can decouple the vertical motion from the longitudinal one. Because the two beams have opposite charge sign and  $f_3 > 0$  for all values of  $\alpha$  and  $\tau$ , it is seen that the third term at the l.h. side of Eq. (3) adds to the focusing of the storage ring. We can estimate the betatron tune-shift as

$$\delta v_v = K_{sc} \langle f_3 \rangle R^2 / 2 v_0 g_0 \quad (10)$$

where  $\langle f_3 \rangle \sim 2/\alpha^2$ . If the tune-shift does not exceed half of the distance  $\Delta v_v$  to the nearest half-integral stopband the motion is stable [5].

At the same time, particles oscillate longitudinally ( $\sigma$ ) around their stationary position with an amplitude

$$\sigma_0/\lambda = \lambda^2 K_{sc} \Delta / 8 g_0 \quad (11)$$

where  $\Delta < 0.01$  for  $\alpha > 1$  and is a slow function of  $\alpha$ . Thus, for large separation between the countermoving *strings*, the motion is stable, and the configuration of each *string* is not disrupted.

#### V. CASE OF CLOSE COLLISION

As the two *strings* are brought together in collision, we reach a range of aspect ratios  $\alpha \ll 1$ . In this range, the form factor  $f_3$  essentially vanishes over the full period of  $\tau$ , except that periodically, at  $\tau = \text{integer}$ , it transforms to a kick. In the smooth approximation, we can spread the kick evenly over one period. Solving Eq. (3) in the same approximation again leads to the tune-shift (10). Since  $\langle f_3 \rangle \sim 2/\alpha^2$ , Eq. (10) sets a limit to the aspect ratio  $\alpha$  below which the relation cannot be satisfied. At the limit of stability of individual *strings*,  $\lambda \sim \lambda_c$ , there is a threshold value of the aspect ratio

$$\alpha^2 > 1/v_v \Delta v_v = \alpha_{trans}^2 \quad (12)$$

below which the configuration of the two *strings* is destroyed. As a consequence, a large betatron tune  $v_v$  and a large periodicity of the storage ring is desired (so that  $\Delta v_v \sim v_v/2$ ) to allow the lowest possible threshold value  $\alpha_{trans}$ .

As the separation  $2b$  between the two *strings* reduces, the longitudinal force acting on the test particle has still an oscillatory behavior around zero, but with a waveform grossly distorted from a pure sine-wave. The amplitude of the oscillation increases as  $\Delta \sim 1/2 \alpha^2$  for  $\alpha \ll 1$ . Inserting this at the r.h. side of Eq. (11) gives the amplitude of the longitudinal oscillations. To avoid disruption of both strings, we require  $\sigma_0 < \lambda/2$ . That is, at the limit of individual *string* stability, when  $\lambda \sim \lambda_c$ , the aspect ratio cannot be reduced below the threshold value

$$\alpha^2 > (\lambda/R)^2 / 4 g_0 = \alpha_{long}^2 \quad (13)$$

#### VI. CONCLUSIONS

In conclusion, two *strings*, moving in the opposite direction in the same storage ring, separated vertically by the aspect ratio  $\alpha$ , have a stable configuration with the ions performing small-amplitude oscillations around their otherwise stationary position, provided that the vertical separation is sufficiently large. Two criteria are to be satisfied, namely Eq.s (12 and 13). In a typical storage ring, it is found that the limitation is caused mainly by the transverse forces, and that the smallest separation that can be achieved between the two *strings* is set by the betatron tune

$$\alpha = 2b/\lambda\gamma \sim 1/v_v \quad (14)$$

We have seen that according to Eq. (2) the luminosity depends on the effective cross section  $\sigma_{eff}$  of the two *strings* colliding with each other, given by the background noise of the cooling method used. For the effective collision of the two *strings*, it is also required that the separation  $b$ , set by Eq.s (12 or 13), is sufficiently smaller than  $\sigma_{eff}$ .

#### VII. REFERENCES

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