*MEASUREMENTS OF THE OCTUPOLE-INDUCED AMPLITUDE-DEPENDENT FREQUENCY SHIFT IN SPEAR**

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Abstract

Four octupoles are used in SPEAR to provide the frequency spread for Landau damping of coupled-bunch motions at high current. With the planned implementation of a new low-emittance lattice, the effectiveness of the octupoles needs to be quantified. The recent development of a multi-dimensional turn-by-turn phase-space monitor and the availability of an accurate frequency analysis technique have made measurement of the octupole-induced amplitude-dependent frequency shift in the new SPEAR lattice possible. This paper presents the data collection and analysis procedures, and compares experimental results to model-based simulations.

I. INTRODUCTION

The SPEAR Synchrotron Light Source has four octupoles which were designed and used for the high-energy physics collider configuration. These magnets have been reactivated recently to provide an amplitude-dependent frequency shift for the Landau damping of coupled-bunch motion in SPEAR. The result has been a dramatic improvement in transverse beam stability at high current, and raises the possibility of a 20-percent increase in useful delivered current.

The success of the octupoles in stabilizing coupled-bunch motions has prompted interest in determining their effectiveness in a new NOQ3 lattice [1] that is planned for future operations. The NOQ3 lattice has the defocusing quadrupole family removed from the insertion doublet, reducing that region from a D/2-O-F-O-D-O structure to D/2-O-F-O. Although the optical functions in the arcs stay fixed, the tunes and the IR optics will differ significantly. We wish to measure the octupole-induced amplitude-dependent frequency shift of this new lattice.

Using a synchro-betatron phase-space monitor [2], the transverse dynamics of an excited electron bunch was tracked turn-by-turn. The data were stored and post-processed using a technique called numerical analysis of fundamental frequency (NAFF) [3] to extract characteristic oscillation frequencies. Relating these frequencies to the average oscillation amplitudes and octupole strength gave a representation of the amplitude-dependent frequency shift in the NOQ3 lattice. The analysis of these measurements and model-based simulations are presented herein.

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II. MEASUREMENT HARDWARE

The 6-D phase-space monitor in SPEAR is capable of recording turn-by-turn amplitude of the synchrotron and betatron oscillations of an excited electron bunch for up to 15000 turns. At present, only the transverse unit of the monitor is used.

Four 8-bit, 2-channel LeCroy 6840 waveform digitizers acquire the data. Each channel has a bandwidth of 100 MHz, a maximum sample clock rate of 40 Megasamples/second, a memory of 128 Kilosamples, and an effective resolution of ± 0.125 mm. The layout allows the digitizers to be triggered serially by a VAX software command. In turn, one of the digitizers signals to gate through one pulse of the 1.28-MHz SPEAR revolution clock to trigger a horizontal kicker. The kicker has a pulse width of approximately 2 μs FWHM, and will excite a single-bunch twice on consecutive turns.

The transverse BPM signals at two different locations are stretched by passive filters, and processed by RF hybrid junctions to produce two sets of signals: The horizontal difference (Δx), the vertical difference (Δy), the SUM (proportional to the stored beam current), and the TRIGGER. The latter is used to clock the LeCroy 6840 waveform digitizers which sample the other three. For each of the two BPMs, the ratios Δ(x/SUM) and (Δy/SUM) give the single-turn, current-independent horizontal and vertical displacements. Figure 1 shows a typical transverse tracking result for a single-bunch at one BPM.

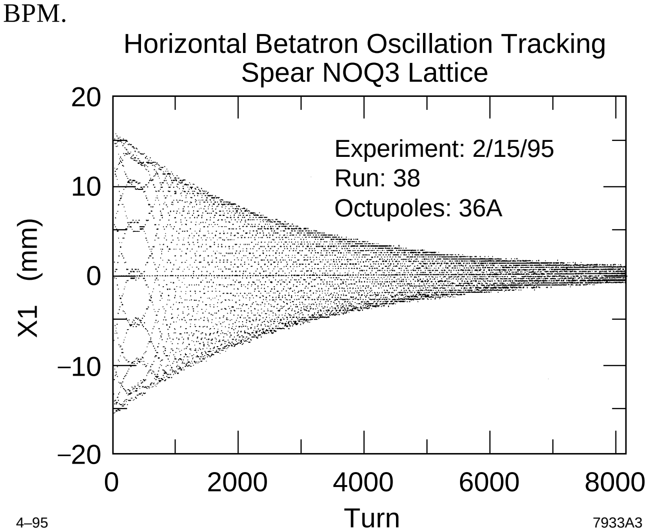

FIGURE 1. Turn-by-turn tracking of an excited single-bunch beam.

III. ANALYSIS

Given a set of tracking data as shown in Fig. 1, one may perform an FFT to determine the oscillating frequency of the bunch centroid. This method of frequency analysis has an accuracy of $2\pi/n$ where n is the number of data points used in the procedure. To resolve frequencies to an accuracy of 1×10^{-3}, which is marginal for analysis of the amplitude-dependent frequency shift in SPEAR, requires more than 6200 turns. However, the typical bunch-centroid damping time in SPEAR is relatively short, approximately 2500 turns for the data set shown in Fig. 1. In this case, the FFT method of frequency analysis is not adequate. We therefore employ the more accurate NAFF technique for the purpose of frequency extraction (see section IIIB below).

A. Data Preparation

The action-angle variables (J,Φ) were used as the basis for data analysis. We first transformed the horizontal beam position data from a pair of BPMs into the Courant-Snyder normalized coordinates (x,p_x). From there, a second coordinate transformation takes the data into the $J-\Phi$ space. These transformations are relatively straight forward. Given the horizontal displacements x_1 and x_2 at BPM_1 and BPM_2 and assuming that there are only dipoles and quadrupoles between the BPMs, x_1 and x_2 are related by [4]

$$x_2 = \sqrt{\frac{\beta_{x2}}{\beta_{x1}}}\left(\cos\mu_{12}+\alpha_{x1}\sin\mu_{12}\right)x_1+\sqrt{\beta_{x1}\beta_{x2}}\left(\sin\mu_{12}\right)x_1' \quad (1)$$

where x_1' is the angle the beam made with respect to the design orbit at BPM_1, β_{xi} is the value of the horizontal betatron amplitude function at the i^{th} BPM, $\alpha_{x1}=-\beta_{x1}'/2$, and μ_{12} is the betatron phase-advance. Equation (1) can be solved for x_1',

$$x_1' = \left(\frac{1}{\beta_{x1}}\right)\left[\left(\frac{\sqrt{\beta_{x1}/\beta_{x2}}}{\sin\mu_{12}}\right)x_2-\left(\cot\mu_{12}+\alpha_{x1}\right)x_1\right]. \quad (2)$$

The normalized momentum p_x is defined as: $p_x \equiv \alpha_x x+\beta_x x'$. Substituting Eq. (2) for x_1', we find

$$p_{x1} = \left(\frac{\sqrt{\beta_{x1}/\beta_{x2}}}{\sin\mu_{12}}\right)x_2-\left(\cot\mu_{12}\right)x_1. \quad (3)$$

Figure 2 displays the result of transforming the data from Fig. 1 and a companion set measured simultaneously at a second BPM into the normalized phase-space coordinates (x,p_x). The values for β_{xi}, α_{x1}, and μ_{12} were taken from a model-based simulation. Notice a gradual reversal of the spiraling direction, which is a manifestation of the amplitude-dependent frequency shift.

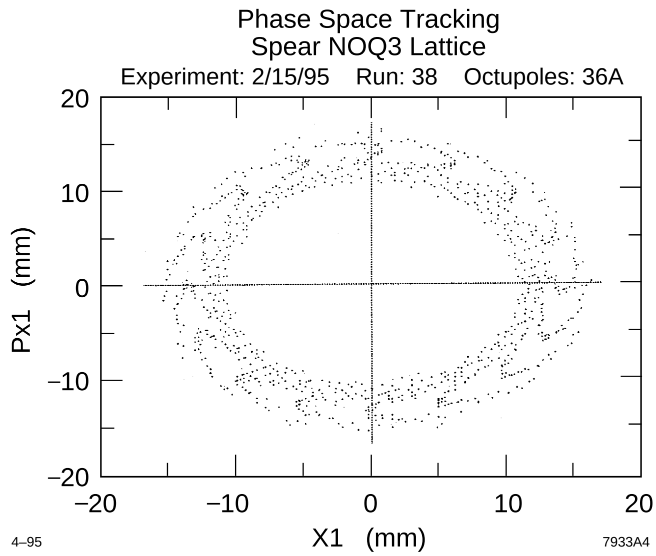

FIGURE 2. Phase-space tracking of a single electron bunch in SPEAR.

The coordinate transformation to (J,Φ) follows from the Courant-Snyder invariant,

$$J=\frac{1}{2}\left(\frac{\left(\alpha_x+1\right)}{\beta_x}x^2+2\alpha_x xx'+\beta_x x'^2\right), \quad (4)$$

which reduces to

$$J=\frac{1}{2\beta_x}\left(x^2+p_x^2\right). \quad (5)$$

The corresponding angle Φ is

$$\Phi=\tan^{-1}\left(\frac{p_x}{x}\right). \quad (6)$$

B. NAFF Method

The Fourier series expansion of a function $f(t)$ that is piecewise regular over an interval of $[-T,T]$ is

$$\tilde{f}=\sum_{-\infty}^{\infty}c_n e^{in\left(\frac{\pi}{T}\right)t}, \quad n\in\{\ldots,-1,0,1,\ldots\} \quad (7)$$

where

$$c_n=\frac{1}{2T}\int_{-T}^{T}f(t)e^{-in\left(\frac{\pi}{T}\right)t}dt. \quad (8)$$

This expansion projects $f(t)$ onto the orthogonal basis-vectors $\{\exp(in\pi t/T)\}$. If the function $f(t)$ is periodic, say

$$f(t)=ae^{\imath\nu t} \quad (9)$$

where a is a complex amplitude, the projection gives

$$c_n = a\frac{\sin[(\nu - n\pi/T)T]}{(\nu - n\pi/T)T}. \quad (10)$$

We approximate the fundamental frequency of $f(t)$ by an $n\pi/T$ that corresponds to the maximum value of c_n. If ν is not an integer multiple of π/T, this approximation is only accurate to π/T.

We can find ν much more precisely by solving for an ω that maximizes the projection integral

$$I = \frac{1}{2}\int_{-T}^{T} f(t)e^{-i\omega t}dt. \quad (11)$$

For the above example, this integral is simply

$$I = a\frac{\sin[(\nu - \omega)T]}{(\nu - \omega)T}, \quad (12)$$

which has a maximum value at $\omega = \nu$. For cases where $f(t)$ may have more than one frequency component, the projection method still works; however, the precision depends on the separation of the frequency components since the continuum of vectors $\{\exp(i\omega t/T),\ \omega \in \Re\}$ is not an orthogonal set and leakage may occur between the frequency components. As long as the separation between any two frequency components of $f(t)$ is larger than a few π/T, the distortion between the frequencies will be minimal and the NAFF method is more accurate than an FFT.

Usually the function $f(t)$ is not known a priori; only its sampled values over the interval $[-T, T]$ are available. In this case, assuming that there is no aliasing and the sampling time is small so that one can compute integrals involving $f(t)$ from the data, the projection integral in Eq. (11) can be evaluated numerically using for example an elementary algorithm of nth stage, extended trapezoidal rule.

C. Analysis of Amplitude-Dependent Frequency Shift

For this experiment, SPEAR was operated at 2.3 GeV in the NOQ3 configuration. A single-bunch 3.44-mA beam was excited by a kicker powered to 4.0 KV. The octupole currents were 36 Amps during the measurement designated as RUN 59, and 33 Amps during the measurement designated as RUN 60. The data were transformed into $x - p_x$ and then $J - \Phi$ before being subdivided into bins of 512 consecutive points for frequency analysis. We analyzed each bin using the NAFF algorithm and correlated the resulting frequencies to the average amplitude.

Figure 3 summarizes the results of the NAFF analysis. Model simulations using TRACY [5] are shown as solid lines. The top curve corresponds to a *simulated* octupole current of 70.4 Amps, and the bottom, 58.5 Amps. Assuming discrepancies come entirely from horizontal beta-beating in SPEAR, the analysis suggests an average beating of 36-percent at the octupole sites.

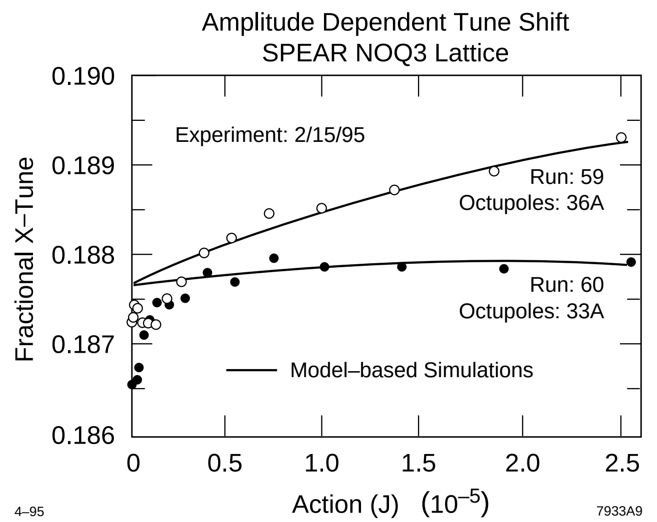

FIGURE 3. A comparison of the measured and simulated octupole-induced amplitude-dependent frequency shift in the SPEAR NOQ3 lattice.

IV. CONCLUSION

The recent development of a turn-by-turn phase-space monitor in SPEAR and implementation of the NAFF algorithm has made measurement of the octupole-induced amplitude-dependent frequency shift in the new SPEAR NOQ3 lattice possible. Discrepancies between the measured and simulated results show a possible beta beating of as much as 36 percent.

V. ACKNOWLEDGMENTS

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VI. REFERENCES

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