

Asymmetric Hopf bifurcation for proton beams with electron cooling*

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Abstract

We observed maintained longitudinal limiting cycle oscillations, which grew rapidly once a critical threshold in the relative velocity between the proton beam and the cooling electrons was exceeded. The threshold for the bifurcation of a fixed point into a limit cycle, also known as a Hopf bifurcation, was found to be asymmetric with respect to the relative velocity. This asymmetry of Hopf bifurcation was found to be related to the electron beam alignment with respect to the stored proton beam.

I. Introduction

Recently, we have reported an experimental observation of the Hopf bifurcation in the synchrotron phase space when the relative velocity between the proton beam and the cooling electrons is greater than a threshold value[1], [2]. We have found that the threshold of bifurcation is related to the “temperature” of the cooling electrons, or equivalently the rms velocity spread of cooling electrons seen by the proton beams. In these observations, we were puzzled by the asymmetry of the bifurcation with respect to the relative velocities (See Fig. 1 of Ref. [1] and Figs. 4 and 5 of Ref. [2]).

This paper reports results of experimental studies on beam motion when the energy of the synchronous proton is varied, while holding the electron energy constant and varying the electron beam direction relative to the proton beam. In particular, we investigate the effect of electron beam alignment on the asymmetry of Hopf bifurcation and the shape of the cooling drag force on Hopf bifurcation amplitude. Our experimental results are compared with results from numerical simulations, where the onset of the limit cycle instability is related to the temperature of the electron beam.

II. Experimental Methods and Results

The IUCF Cooler Ring is a hexagonal shaped storage ring with a circumference of 86.8 m. The experiment was done with a 45 MeV proton beam injected and then stored in a 10 s cycle time. After 5 s from the start of the cycle, the 6-D phase space coordinates were digitized at 10-revolution intervals for 16384 points. The nominal rf cavity frequency was 1.03168 MHz with the harmonic number $h = 1$. At this energy, the phase slip factor

η of the Cooler Ring was about -0.86 . The beam was typically a single bunch of about 4×10^9 protons with a typical length of about 100 ns FWHM at a rf peak voltage of about 41 V. For experimental results reported in this paper, the rf voltages were set at 85 V and 128 V respectively. Since measurements of longitudinal motion were being made, the phase lock feedback loop for the rf, which is normally on, was switched off. Damping of synchrotron oscillations while operating under these conditions occurred entirely due to the electron cooling.

The difference equations describing the longitudinal motion are

$$\delta_{n+1} = \delta_n - \frac{2\pi v_s^2}{h\eta} (\sin \phi_n - \sin \phi_s) - f(\delta_n) \quad (1)$$

$$\phi_{n+1} = \phi_n + 2\pi h\eta\delta_{n+1}, \quad (2)$$

where η is the phase slip factor, ϕ_s is the phase of the synchronous particle, h is the harmonic number, $f(\delta)$ is the damping force, provided in our case by electron cooling, v_s is the small amplitude synchrotron tune at a zero synchronous phase, and the subscripts n refer to the revolution number.

A. The damping force

The damping force $f(\delta)$ produced by the electron cooling is the result of a statistical exchange of energy in Coulombic collisions between the protons and relatively cold electrons as they travel together at the same velocity in the accelerator. In practice, electron cooling in synchrotrons is normally done in relatively short straight sections due to cost and space limitations. At IUCF the electron beam is mixed with the proton beam for distance of only 2.2 m or about 2.5% of the circumference of the ring. The electron beam radius is about 1.27 cm and the cathode temperature is about 1300°K or $kT_{\text{cath}} = 0.11$ eV, where k is the Boltzmann constant. The maximum electron beam current is 4 A.

Assuming an electron beam with an isotropic phase space Maxwellian velocity distribution, the damping force in the non-magnetized binary collision theory is given by

$$f(\delta) = \frac{4\pi\alpha\Delta_{e\parallel}}{\omega_0} g(\zeta), \quad (3)$$

with a kinematic factor given by

$$g(\zeta) = \frac{3\sqrt{\pi}}{4\zeta^2} \left[\text{erf}(\zeta) - \frac{2\zeta}{\sqrt{\pi}} e^{-\zeta^2} \right], \quad (4)$$

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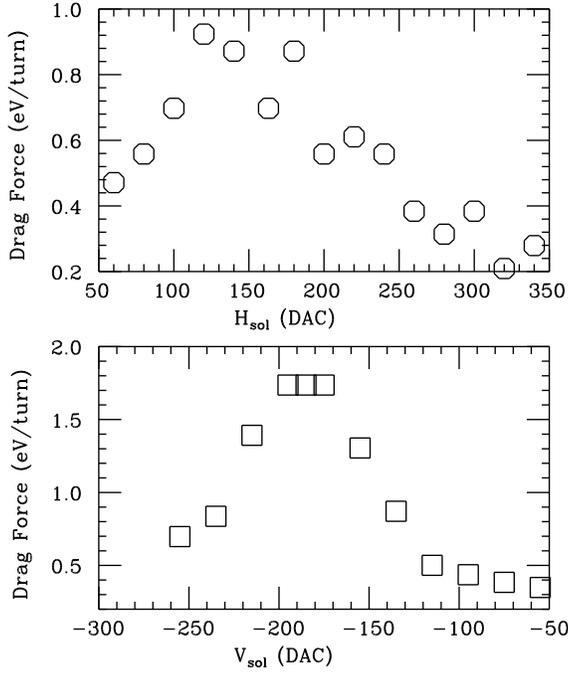


Figure. 1. The drag force measured with respect to electron beam alignment by using the horizontal and vertical dipoles H_{sol} and V_{sol} .

where $\Delta_{e\parallel} = \sigma_e/\beta c$, $\zeta = (\delta - \delta_e)/\Delta_{e\parallel}$, α is the $1/e$ damping rate for small relative velocities with units s^{-1} . From our previous measurements [1], [2], we have $\Delta_{e\parallel} \approx 3 \times 10^{-4}$ for the IUCF electron cooling system at 45 MeV of proton kinetic energy.

The effective temperature of the electron beam is related to the rms electron velocity spread by

$$kT_{\parallel\text{eff}} = \frac{1}{2}m_e\sigma_e^2 = \frac{1}{2}m_e\beta^2c^2\Delta_e^2. \quad (5)$$

Because of the adiabatic acceleration, the longitudinal effective electron temperature is much smaller than that of the cathode temperature. Since there is no adiabatic damping in the transverse phase space, the effective transverse temperature remains 0.11 eV. The equivalent momentum deviation is $\Delta_{e\perp} = 2.1 \times 10^{-3}$. For comparison, the bucket height of the synchrotron phase space at a rf voltage of 128 V is about 1.0×10^{-3} . The cooling drag force for relative velocities between protons and cooling electrons lying between the longitudinal and transverse rms values is of particularly interest.

In machines where the electron beam is magnetically confined by a solenoidal field, as it is in the IUCF Cooler Ring, the damping force can be enhanced by an effect called magnetized cooling. Magnetized cooling can be substantial for small relative velocities, where electrons are trapped in magnetic field lines, the effective longitudinal and transverse cooling rates can be greatly enhanced.

B. Optimization of electron cooling system

Since the threshold of Hopf bifurcation is related to the rms velocity spread of cooling electrons along the proton trajectory in the cooling electron cloud, the asymmetry of the Hopf bifurcation

can easily be explained by different velocity spreads seen by the proton beam having different momentum closed orbit. Since electrons having identical speed as that of protons have a much smaller momentum rigidity, the electron beam alignment can easily be adjusted by superimposing a horizontal or a vertical dipole field to the solenoidal field. A method to obtain optimal alignment is described as follows.

When the rf frequency is shifted, the beam, which is originally at the center of the rf bucket (i.e., $\delta = 0$ and $\phi = 0$), will be dragged away from the origin and begin to undergo a synchrotron oscillation. If the damping force were linear over the entire range of v_{rel} , the proton beam would damp to a new fixed point attractor ϕ_{FP} , i.e. the synchronous phase angle, where

$$\phi_{\text{FP}} \approx \frac{2\alpha}{\omega_0 v_s} \frac{h\eta\delta_e}{v_s}, \quad (6)$$

where δ_e is the fractional momentum spread of the proton traveling at the velocity of cooling electrons with respect to the synchronous particle of the rf cavity. This would correspond to the situation where the proton beam was continually losing energy due to the damping force, but with it continually being made back by the rf cavity, or vice versa. Because $\alpha \ll \omega_0 v_s$, the resulting ϕ_{FP} , which is equivalent to the synchronous phase angle ϕ_s of the beam, is very small.

In order to increase the sensitivity of measuring the synchronous phase angle ϕ_{FP} shown in Eq. 6, we choose the rf voltage with $V_{\text{rf}} = 10$ V, where The corresponding synchrotron tune is 1.25×10^{-4} at $h = 1$. The rf synchronous phase angle is measured by stepping away from the reference frequency by 200 Hz. Figure 1 shows the drag force, which is $eV_{\text{rf}} \sin \phi_{\text{FP}}$, vs the horizontal and vertical alignment, where the lower plot shows the drag force vs the vertical alignment with $H_{\text{sol}} = 160$ dac. We obtained a maximum drag force at about 1.8 eV/turn. From Eq. 6, we obtain $\alpha = 45 s^{-1}$, which agrees well with the value of $40 s^{-1}$ obtained from an earlier measurement by using the harmonic modulation to the HVPS (see Sec. III A of Ref. [2]).

C. Hopf Bifurcation amplitude with a Nonlinear Damping Force

To investigate Hopf bifurcation due to the nonlinear damping force, the electron velocity was displaced from the proton velocity to produce a nonzero relative velocity. This was done by changing the rf cavity frequency, where a step of 1 Hz resulted in changing the fractional proton velocity by about 1×10^{-6} . If the electron velocity is equal to the proton velocity when the rf cavity frequency is f_0 , then the fractional momentum deviation of the electron beam from the proton beam, δ_e , at the new rf frequency f is given by

$$\delta_e = \frac{(f - f_0)}{\eta f_0}. \quad (7)$$

The maximum synchrotron phase amplitude $\hat{\phi}$, and the maximum fractional momentum deviation $\hat{\delta}$ are measured at 5 s after the start of an injection cycle to allow the initial transient oscillations to damp out. As shown in our earlier reports [1], [2], the Hopf bifurcation amplitudes can also be measured with a BPM sum signal on an oscilloscope.

Choosing the optimal drag force with $H_{\text{sol}} = 160$ dac and $V_{\text{sol}} = -185$ dac, the measured bifurcation amplitude is shown

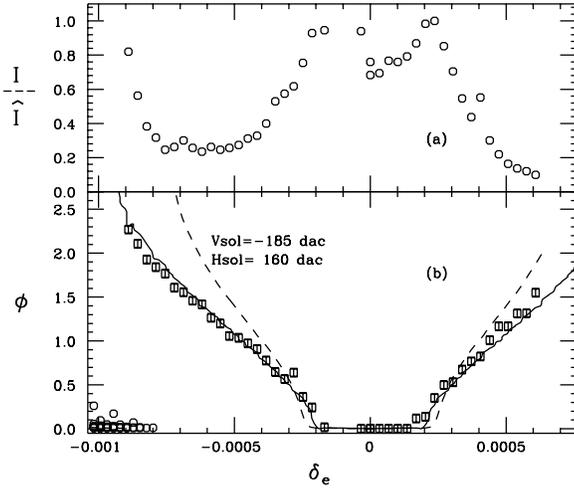


Figure 2. The stored beam current and the stable synchrotron amplitudes vs δ_e . Note the close correlation between the stored beam current and the Hopf bifurcation threshold. The dashed line corresponds to numerical simulations with non-magnetized cooling force while the solid line is obtained from a parameterization with a magnetized cooling force.

in Fig. 2b and the corresponding beam intensity is shown in Fig. 2a. In particular, Fig. 2b shows a nearly symmetric Hopf bifurcation amplitude vs δ_e .

More significantly, a close correlation between the storage beam current and the Hopf bifurcation threshold indicates the importance of this phenomena in realistic beam operation. It is worth pointing out that the dashed line in Fig. 2b shows the theoretical prediction of Hopf bifurcation amplitude based on the kinematic factor of Eq. 4 with $\Delta_{e\parallel} = 3 \times 10^{-4}$. Two obvious disagreements are (1) the experimental threshold is lower than that predicted by theory and (2) the Hopf bifurcation amplitudes observed are much lower than those predicted by theory.

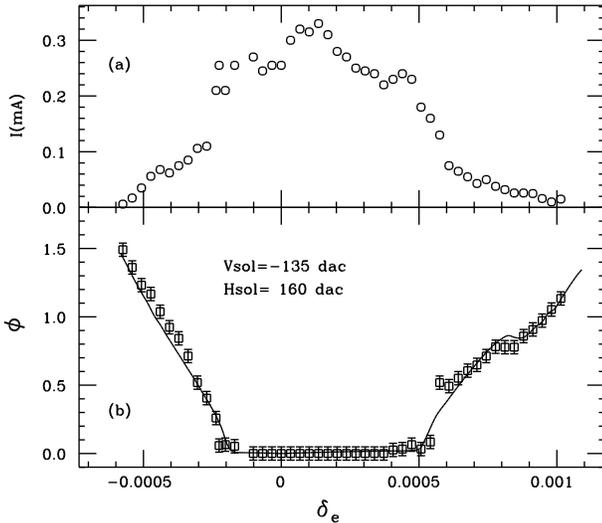


Figure 3. Similar to that of Fig. 2 except with a non-aligned electron beam. Note that the asymmetric Hopf bifurcation is evident.

Using the values of $H_{sol} = 160$ dac and $V_{sol} = -135$ dac that do not optimize the drag force, the measured Hopf bifurcation curve is shown in Fig. 3b. It is evident that the Hopf bifurcation amplitude is asymmetric with respect to relative momentum deviation. Similarly, We have also observed a close correlation between the loss of beam intensity with respect to the Hopf bifurcation threshold shown in Fig. 3a.

A strong asymmetry in the Hopf bifurcation threshold may indicate that the off momentum closed orbit of the proton beam would experience different electron “temperature”, or equivalently different velocity spread of the cooling electron cloud. In order to increase the momentum aperture of proton beam, alignment between the proton and the electron beams is important.

III. Conclusion

In conclusion, we have studied the effect of electron beam alignment on the Hopf bifurcation. When the electron and proton beams were aligned, the Hopf bifurcation amplitudes became symmetric with respect to the relative velocity between the cooling electrons and the synchronous proton. The non-magnetized drag force model with the kinematic factor of Eq. (4) fails to fit the data of Hopf bifurcation amplitudes. The data may be employed to determine the slope of drag force in the region $(\Delta_{e\parallel}, \Delta_{e\perp})$, which can provide essential characteristics of the magnetized cooling.

References

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