Compensation of the "Pacman" Tune Spread by Tailoring the Beam Current†

Miguel A. Furman
Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
Berkeley, CA 94720, USA

Abstract

"Factory"-like e^+-e^- colliders presently under design or construction achieve high luminosity by resorting to large numbers of closely-spaced bunches. The short bunch spacing implies that there are unavoidable parasitic collisions (PCs) in the neighborhood of the interaction point (IP). Since the bunch population of the beam is not uniform due to an intentional ion-clearing gap, the bunches at the head or tail of the train ("pacman bunches") experience different beam-beam tune shifts than those away from the edges ("typical bunches"). We present here a method to minimize the vertical tune spread at the expense of increasing the horizontal tune spread (we assume that there is only one IP). Since the beam-beam dynamics for flat beams typically tolerates a significantly higher horizontal tune spread than a vertical tune spread, this method implies a net advantage. We present our discussion in the context of the PEP-II collider.

I. INTRODUCTION

The PEP-II design [1] calls for head-on collisions with magnetic separation in the horizontal plane. This separation scheme entails unavoidable PCs near the IP whose effects on the beam-beam dynamics have been studied quite extensively [1,2]. The design also calls for an ion-clearing gap equivalent to ~5% of the total beam length. The gaps in the two beams have the same length and are positioned such that head bunch in one beam collides at the IP with the head bunch of the other beam (the two beams have the same bunch spacing and overall length).

The interaction region (IR) is such that a typical bunch experiences four PCs on either side of the IP (for a total of 9 collisions). On the other hand, the pacman bunches (those at the head or tail of the train) do not experience all the collisions, as sketched in Fig. 1.

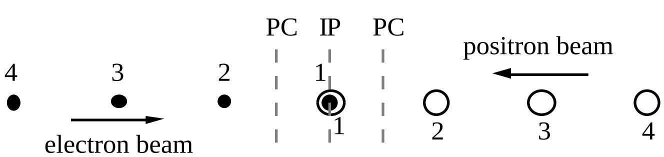

Fig. 1. Sketch of the collision schedule. The dashed lines indicate the location of the IP and first PC. Bunch #1 is at the head of the train in its respective beam.

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As a result, the particles in these bunches have different closed orbits [3] and different beam-beam tune shifts [4] than those in typical bunches. This "pacman tune spread" implies that a working point that may be appropriate for typical bunches might not be good for the pacman bunches and vice versa. In this note we show how to compensate this tune spread for both beams in first order approximation by tailoring the bunch currents. The difference in sign and magnitude between the vertical and horizontal beam-beam parameters at the PCs makes it impossible to compensate vertical and horizontal tune spreads simultaneously. In our particular case, we choose to compensate the vertical tune spreads, which are larger than the horizontal. As a result, the horizontal tune spread is *increased* relative to the nominal (even-bunch-current) case. This increase, however, is not expected to be detrimental, as explained below.

II. COMPENSATION PRINCIPLE

Let us consider only one PC on either side of the IP, as we will in the case of PEP-II (our analysis is extended in a straightforward fashion to the case with more PCs). As a result, there is only one pacman bunch at the head of the train and one at the tail.

Let us focus on the vertical tune shift of the positron beam. Neglecting the dynamical beta function effect [4], the vertical tune experienced by a positron at the center of a bunch is (refer to Fig. 1)

$$\begin{aligned} \nu_{y+} &= \nu_{y+}^{(0)} + \xi_{y+}^{*} + 2\xi_{y+}^{PC} \quad \text{(typical bunch)} \\ \nu_{y+} &= \nu_{y+}^{(0)} + \xi_{y+}^{*} + \xi_{y+}^{PC} \quad \text{(pacman bunch)} \end{aligned} \tag{1}$$

Similar expressions apply to the electron beam, and to the horizontal counterparts of both beams. Here $\nu^{(0)}$ is the lattice (bare) tune and the ξ's are the beam-beam tune shifts at the IP and the PC. The absolute difference between these two equations is the vertical "pacman tune spread" for the positrons, namely

$$\Delta\nu_{y+} = \xi_{y+}^{PC} \tag{2}$$

It is this tune spread (and its counterpart in the opposing beam) that we show here how to eliminate.

Let N_{n-} be the number of particles in electron bunch n and d be the separation between the beams at the PC. Then the well-known expressions for the vertical beam-beam parameters of the positron bunch n are written as

$$\xi_{ny+}^{*} = a_+ N_{n-} \quad \text{and} \quad \xi_{ny+}^{PC} = b_+ N_{n-} \tag{3}$$

where, using standard notation,

$$a_+ = \frac{r_e \beta^*_{y+}}{2\pi\gamma_+ \sigma^*_{y-}\left(\sigma^*_{x-} + \sigma^*_{y-}\right)} \quad \text{and} \quad b_+ = \frac{r_e \beta^{PC}_{y+}}{2\pi\gamma_+ d^2} \tag{4}$$

The schedule of the collisions is shown in Fig. 1. We label the bunches so that #1 is at the head of the train in both beams. Thus we see that bunch #1 experiences only one PC with bunch #2 in the opposing beam, in addition to the main collision at the IP. From Fig. 1 we can read off the beam-beam parameter for each bunch as follows:

$$\begin{aligned} \xi_{1y+} &= a_+ N_{1-} + b_+ N_{2-} \\ \xi_{2y+} &= a_+ N_{2-} + b_+\left(N_{1-} + N_{3-}\right) \\ \xi_{3y+} &= a_+ N_{3-} + b_+\left(N_{2-} + N_{4-}\right) \\ &\vdots \end{aligned} \tag{5}$$

where we assume that the bunch sizes σ_x and σ_y remain at their nominal values under colliding conditions. If we now equate all beam-beam parameters to their nominal value (i.e., in the absence of any beam gap), we obtain

$$\begin{aligned} a_+ N_{1-} + b_+ N_{2-} &= \left(a_+ + 2b_+\right) N_- \\ a_+ N_{2-} + b_+\left(N_{1-} + N_{3-}\right) &= \left(a_+ + 2b_+\right) N_- \\ a_+ N_{3-} + b_+\left(N_{2-} + N_{4-}\right) &= \left(a_+ + 2b_+\right) N_- \\ &\vdots \end{aligned} \tag{6}$$

where N_- is the nominal number of electrons per bunch. Thus we obtain the matrix equation

$$\begin{bmatrix} 1 & \varepsilon_+ & 0 & 0 & \cdots \\ \varepsilon_+ & 1 & \varepsilon_+ & 0 & \cdots \\ 0 & \varepsilon_+ & 1 & \varepsilon_+ & \cdots \\ 0 & 0 & \varepsilon_+ & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y_{1-} \\ y_{2-} \\ y_{3-} \\ y_{4-} \\ \vdots \end{bmatrix} = \left(1 + 2\varepsilon_+\right) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \vdots \end{bmatrix} \tag{7}$$

where we have defined

$$y_{n-} \equiv N_{n-}/N_- \quad \text{and} \quad \varepsilon_+ \equiv b_+/a_+ \tag{8}$$

Eq. (7) has as many entries as there are bunches in the train (1658 in the case of PEP-II). The solution is symmetrical about the middle bunch and is readily found in perturbation theory,

$$\begin{bmatrix} y_{1-} \\ y_{2-} \\ y_{3-} \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 + \varepsilon_+ + \cdots \\ 1 - \varepsilon_+^2 + \cdots \\ 1 + \cdots \\ \vdots \end{bmatrix} \tag{9}$$

where $\cdots$ represents in all entries terms of $O(\varepsilon_+^3)$ or higher.

IV. Application to PEP-II and Discussion

Let us apply our analysis to the case of PEP-II [1], whose basic parameters are listed in Table 1 (LEB=low-energy beam, HEB=high-energy beam). The optics in the IR is symmetrical about the IP and is such that the 1st PC at either side of the IP is much stronger than the others. We are therefore justified in neglecting all other PCs.

From Eq. (2) and Table 1 we see that the nominal vertical pacman tune spread of the positron beam is 0.004, which is ~14% of the main beam-beam parameter at the IP, i.e.,

$$\varepsilon_+ = \xi^{PC}_{y+} \big/ \xi^*_{y+} \approx 0.14 \tag{10}$$

Thus Eq. (9) says that, in order to compensate this tune spread we must increase the number of particles in the first and last bunches of the *electron* train by 14% relative to the nominal value, and decrease the number of particles in the second and next-to-last bunch by 2% relative to the nominal value. These numbers are within the precision reach with which the linac can inject beam [1].

Table 1. Selected PEP-II parameters.

	LEB (e^+)	HEB (e^-)
E [GeV]	3.1	9.0
N	5.63×10^{10}	2.59×10^{10}
β^*_x [m]	0.375	0.50
β^*_y [m]	0.015	0.02
β^{PC}_x [m]	1.433	1.293
β^{PC}_y [m]	26.46	19.85
σ^*_x [μm]	152	152
σ^*_y [μm]	6.1	6.1
ξ^*_x	0.03	0.03
ξ^*_y	0.03	0.03
ξ^{PC}_x	–0.00022	–0.00015
ξ^{PC}_y	0.0041	0.0023
d [mm]	3.5	

A calculation for the electron beam yields a similar solution, obtained from Eq. (9) by replacing $+ \leftrightarrow -$. Because the beam energies in PEP-II are sufficiently high, the beam-beam parameter in one beam does not depend on its own charge; therefore the positron and electron pacman tune spreads can be compensated simultaneously. From Table 1 we obtain $\varepsilon_- = 0.08$, which implies that the number of positrons in the first and last bunches of the train must be increased by 8% relative to the nominal value, while the number of positrons in the second and next-to-last bunches must be decreased by 0.6% relative to the nominal value.

If we were to carry out the same calculation for the horizontal tune spreads we would obtain $\varepsilon_+ = -0.0075$ and $\varepsilon_- = -0.005$. Since both magnitude and sign are different from the solutions presented above for the vertical tune spread, it is obvious that one cannot simultaneously compensate for the vertical and horizontal tune spreads.

As a corollary we conclude that, if we choose to compensate the vertical tune spreads, the horizontal tune spreads become larger than their nominal values. For the LEB we obtain

$$\begin{aligned}\Delta\nu_{x+} &= \left|\xi_{1x+} - \xi_{x+}\right| \\ &= \left|\left(1+\varepsilon_{+}\right)\xi^{*}_{x+} + \left(1-\varepsilon^{2}_{+}\right)\xi^{PC}_{x+} - \xi^{*}_{x+} - 2\xi^{PC}_{x+}\right| \\ &\approx \left|\varepsilon_{+}\xi^{*}_{x+}\right|\end{aligned} \tag{11}$$

which evaluates to ~0.004. The corresponding calculation for the horizontal pacman tune spread of the HEB yields ~0.002. These numbers are a factor ~15–20 larger than their nominal values (see Eq. (2)) and, in fact, are equal to the uncompensated vertical tune spreads. Thus we can say that our method transfers the pacman tune spread from the vertical plane to the horizontal. However, for PEP-II, the horizontal beam dynamics is much less sensitive than the vertical to beam-beam parameter strengths of the same magnitude for an extended region of the tune plane [2]. Therefore, transferring the tune spread from the vertical plane to the horizontal implies a net advantage.

In the more general case, when there are more than two PCs, the method generalizes in a straightforward fashion. If there are altogether n PCs, and if the beam orbits are not symmetrical about the IP, Eq. (6) will couple the currents of n+1 bunches in the opposing beam, there will be n different ε parameters in Eq. (7), and the matrix will have n secondary diagonals. If all the ε parameters are small, as is likely to be the case in any realistic IR design, the solution can be found in perturbation theory.

The increase in the bunch current for the pacman bunches will affect their closed orbit distortion [3]. Here, again, the effects are quite small. In the nominal (non-compensated) case, the closed orbits of the pacman bunches are displaced horizontally from the optical IP by ~5 μm. However, for most values of the horizontal tune, both the LEB and HEB pacman bunches are displaced *to the same side* of the optical IP. As a result, the relative displacement of their centers is ~1-2 μm, which is very small compared to the horizontal beam size $\sigma^{*}_{x} = 152$ μm. If the vertical pacman tune spread is compensated as discussed above, the relative displacement of the pacman bunch centers at the collision point will not increase by more than ~20% from the nominal value of ~1-2 μm quoted above, and therefore will remain small.

The tailored beam current will also have an effect on the induced transient voltages on the RF cavities, and on the stability of the coherent dipole mode of the beams. The ideal case, in which there is no beam gap and all bunches have the same charge is, of course, the simplest. We do not expect that the beam-beam interaction will drive a coherent dipole instability for any reasonable value of the tune. The design of the RF system does take into account the gap. Although these issues remain to be evaluated in detail, we believe that an increase of ~14% in the current of the first and last bunches should not entail serious difficulties, if any.

V. CONCLUSIONS

We have presented a method for the compensation of the vertical pacman tune spread in PEP-II. The method consists in tailoring the beam current in such a way that the pacman bunches have slightly larger charge than the typical bunches. The compensation can be carried out simultaneously in both beams but not in both planes. In fact, a generic feature of the method is a trading off of the vertical tune spread for the horizontal. Thus if the vertical tune spread is compensated, the horizontal tune spread becomes roughly equal to the uncompensated vertical tune spread, which is typically larger than its nominal value. However, the horizontal beam dynamics is much more tolerant than the vertical, so tune spreads of this magnitude should not cause any problems, and the method therefore implies a net advantage. We believe that this beneficial trade-off is a generic feature of flat beams. Thus our technique seems unlikely to be applicable to round beams, such as those encountered in multibunch proton colliders.

VI. ACKNOWLEDGMENTS

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VII. REFERENCES

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