

# OBSERVATIONS OF THE EFFECTS OF THE BEAM-BEAM INTERACTION ON THE ORBITS OF STORED BEAMS IN CESR<sup>§</sup>

E. Young, Wilson Synchrotron Laboratory, Cornell University, Ithaca, NY 14853 USA

Luminosity in the Cornell Electron Storage Ring (CESR) depends critically on the degree of overlap of the counter-rotating beams at the interaction point (IP). However, due to the pretzel<sup>†</sup>, this overlap is sensitive to changes in quadrupole and sextupole strengths. The beam-beam interaction (BBI) perturbs the closed orbits and depends on the transverse distance between the opposing beams as they pass each other. If a real time measurement of this small orbit change were possible, then this effect could be used to determine whether the bunches remain in collision while tuning CESR for high luminosity. Our purpose was to determine whether this would be possible with the present beam position detector system.

## I. THE BEAM-BEAM INTERACTION FOR FLAT BEAMS

The vertical kick felt by a test particle due to the transverse electric field of a two-dimensional Gaussian charge distribution is given by the equation below. Complete derivations are available in the references [1], [2], [3], and [4].

$$\Delta y' = A \operatorname{Re} \left\{ w \left( \frac{\mu + i\rho v}{d} \right) - e \left( \frac{\mu^2}{2} - \frac{v^2}{2} \right) w \left( \frac{\rho\mu + iv}{d} \right) \right\} \quad (1)$$

where  $A$ ,  $d$ ,  $\mu$ ,  $v$ , and  $\rho$  are

$$A = -\frac{\sqrt{2\pi}Nr_e}{\gamma\sqrt{(\sigma_x^2 - \sigma_y^2)}}, \quad d = \sqrt{2(1-\rho^2)} \quad (2)$$

$$\mu = \frac{x}{\sigma_x}, \quad v = \frac{y}{\sigma_y}, \quad \rho = \frac{\sigma_y}{\sigma_x},$$

$N$  is the number of particles in the distribution,  $\sigma_x$  and  $\sigma_y$  are the transverse sizes of the distribution, and  $x$  and  $y$  are the horizontal and vertical distances of the test particle from the center of the Gaussian charge distribution.  $w(z)$  is the complex error function defined by the following equation.

$$w(z) = \exp \left\{ -z^2 \left( 1 + \frac{2i}{\pi} \int_0^z \exp(u^2) du \right) \right\} \quad (3)$$

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<sup>†</sup> The pretzel in CESR is a closed orbit with kinks in the horizontal plane in order to insure that counter-rotating beams of electrons and positrons remain separated at parasitic crossing points. Thus, the beams are off axis horizontally through most of the magnets.

The complex error function must be evaluated numerically, and we have used a computer program [5] to do so based on the routines developed in reference [1]. Figure 1 shows the vertical beam-beam kick as a function vertical displacement for a beam aspect ratio of  $\sigma_y/\sigma_x = 1/10$ .

For fitting measured data to the theory a simpler expression for the kick would be convenient. We can expand the error function and the exponential in equation (1) about the origin and keep as many terms as necessary to obtain good agreement with equation (1). For small vertical displacements and assuming that the horizontal displacement is zero, the linear approximation of equation (1) is

$$\Delta y' = -\frac{2Nr_e}{\gamma(\sigma_x + \sigma_y)\sigma_y} y \quad (4)$$

To 3rd order the approximation is

$$\Delta y' = A \left\{ \left[ \frac{2(1-\rho)}{\sqrt{\pi}d} - \frac{2(1-\rho)^2}{\sqrt{\pi}d^3} \mu^2 \right] v + \frac{2(-1+3\rho^2-2\rho^3)}{3\sqrt{\pi}d^3} v^3 \right\} \quad (5)$$

where  $A$ ,  $d$ ,  $\rho$ ,  $\mu$ , and  $v$  are the same as in equation (2). Equation (4) and the first term of equation (5) are the same for  $\mu = 0$ . These approximations are also plotted in figure 1.

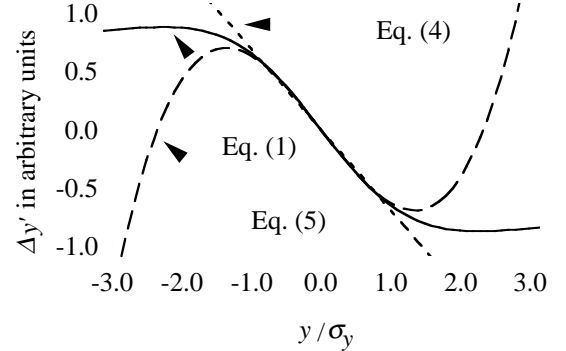


Figure 1: The vertical beam-beam kick from a flat, Gaussian electron beam as a function of vertical distance from the center of the beam.

As is discussed in the references [2], [6], and [7], if the current of one bunch is significantly smaller than that of the other bunch (the weak-strong approximation), the strong beam remains undisturbed by the presence of the weak beam, while the weak beam can be considered a group of non-interacting particles. The disturbed orbit of the weak beam can then be obtained by substituting the kick from equation (1) in the usual formula for a closed orbit distortion (using the  $N$ ,  $\sigma_x$ , and  $\sigma_y$  of the strong beam).

The above equations apply for the weak-strong case, i.e. one beam has a much smaller current than the other so that its orbit is perturbed, but the strong beam is essentially unperturbed. This is a special case; normally when tuning luminosity in CESR, the bunches have nearly equal currents, so the beams feel kicks of approximately equal strengths, and both beams' orbits and transverse sizes are affected. This will complicate any attempt to measure the closed orbit distortion due to the beam-beam interaction as a function of the overlap at the IP during luminosity conditions and fit that measurement to the theory. In fact, the strong-strong case is usually approached via simulations. However, the qualitative features of the orbit distortion due to the beam-beam interaction in the strong-strong case are the same as in the weak-strong case, i.e. as the displacement between the beams increases, the effect of the beam-beam force increases to a maximum then decreases.

## II. MEASURING THE EFFECT OF THE BEAM-BEAM INTERACTION ON CLOSED ORBITS

The measurement of the orbit distortion due to the beam-beam interaction was performed in a lattice which had a vertical tune of  $Q_y = 9.6298$  and a vertical beta-function at the IP of  $\beta_y^* = 0.0172$  meters. Normally, CESR operates with nine bunches each of electrons and positrons, but for these experiments we filled two positron bunches and one electron bunch; the positron bunches will hereafter be referred to as the colliding and reference bunches. We then brought the beams into collision and measured the difference between the orbits of the positron bunches at 10 different detectors around CESR. Table 1 shows the beam parameters during these measurements. The BBI's at parasitic crossing points were negligible, so the only difference between the orbits of the two positron beams should be due to the BBI at the IP.

Table 1: Beam parameters during the experiments.<sup>‡</sup>

Data set	I <sub>1-</sub>	I <sub>1+</sub>	I <sub>5+</sub>	$\sigma_{e-}$	$\sigma_{e+}$
#1	7.18	7.33	7.59	15.4	11.7
#2a	6.84	7.51	7.73	14.3	9.7
#2b	6.30	3.40	5.64	7.4	9.8

Since the size of the orbit distortion is different at each detector, we should not average the measured orbit differences directly. Doing so would weight the data from each detector by the beta-function at that detector. Instead we will use Equation (6) to normalize the measured orbit distortion at each detector to  $\beta_y^*$ , then average them. The three sets of normalized, averaged data are plotted in Figures 2,3, and 4.

<sup>‡</sup> The current, I, is in mA, and the vertical beam size,  $\sigma$ , is in  $\mu\text{m}$ . The subscripts refer to the bunch number and species. The currents are averages over the measurements. The beam sizes are also averages over the measurements, and for the positrons, an average of the two bunches as well.

$$\Delta y(\text{IP}) = \Delta y(\text{arc}) \sqrt{\frac{\beta_y^*}{\beta_y^{\text{arc}}}} \quad (6)$$

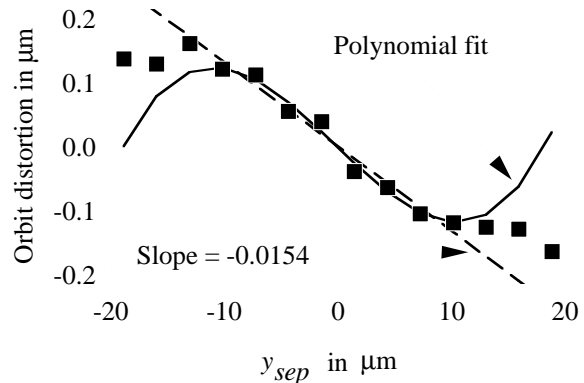


Figure 2: Data set #2b (unequal bunch currents).

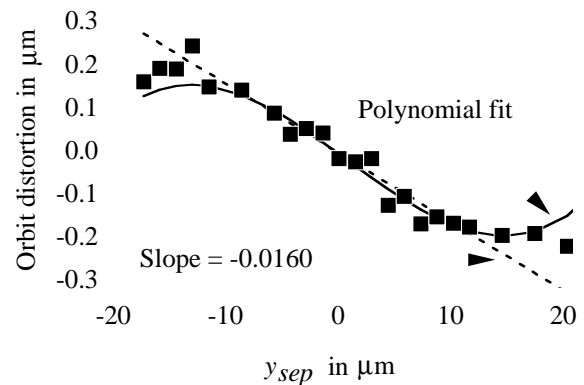


Figure 3: Data set #2a (equal bunch currents).

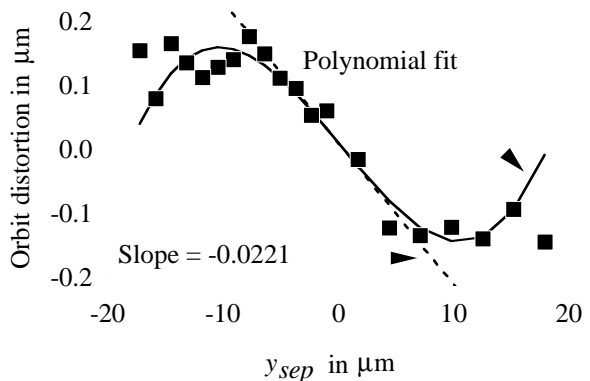


Figure 4: Data set #1 (equal bunch currents).

To vary the vertical separation at the IP we used two electrostatic separators located on the opposite side of the ring from the IP. Ideally, the vertical betatron phase between these separators is  $\pi$  creating a closed vertical bump at the parasitic crossing point opposite the IP and no vertical separation at the IP; however, coupling plus the presence of the pretzel<sup>†</sup> may create a vertical separation at the IP even if the bump is closed. Changing the phase advance uncloses the

bump creating a vertical ripple which changes the vertical separation at the IP. A “knob” in the control room controls a combination of quadrupoles such that we can change the phase advance between these separators without disturbing the rest of the machine. Before attempting to measure the effect of the BBI, we measured the change in the orbit at several points as we varied the phase advance, and calculate the corresponding change in the displacement of the orbit at the IP. To measure the effect of the BBI, we varied the vertical separation at the IP over 35  $\mu\text{m}$  or about  $4\sigma_y$ .

### III. DATA ANALYSIS

To find the phase advance which corresponded to  $y_{sep} = 0$ , we used the fact that  $y_{sep} = 0$  occurs at the inflection point of Equations (1) and (5). Using data set #2b, we fit a cubic polynomial to the data versus  $-\sigma_y < y_{sep} < \sigma_y$  with an arbitrary zero then found the inflection point of that fit. This point was set to  $y_{sep} = 0$ . When data sets #2a and #1 were treated in the same way, their inflection points corresponded to the same setting for the phase advance within the errors of the fit. The coefficients of the fit were scaled accordingly, and are listed in Table 2. The slope of the fitted line and  $m_1$  of the fitted polynomial agree with each other within the limit of the errors of the fits in all cases. As a further check, we compared the ratio  $m_1/m_3$  of the polynomial fit to that predicted by equation (5); they agreed within the limits of error.

Table 2: Coefficients of the fits shown in figures (2)-(4).

Data set	Fit to a line	Fit to a cubic polynomial	
	$m_1$ ( $\times 10^{-3}$ )	$m_1$ ( $\times 10^{-3}$ )	$m_3$ ( $\times 10^{-5}$ )
#1	$-1.54 \pm 0.17$	$-1.70 \pm 0.19$	$3.5 \pm 1.4$
#2a	$-1.60 \pm 0.10$	$-1.88 \pm 0.17$	$5.0 \pm 1.5$
#2b	$-2.21 \pm 0.17$	$-2.18 \pm 0.10$	$7.6 \pm 1.5$

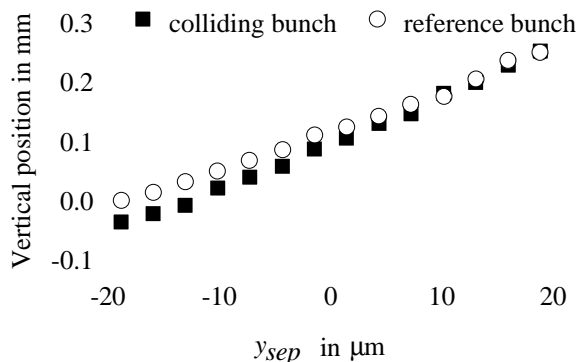


Figure 5: Vertical positions of the colliding and reference positron bunches at one detector from data set #2b.

Our results suggest that in theory this method could be used to determine whether the beams are in collision, however, these measurements were not done under normal

luminosity conditions. Under normal conditions there would not be an extra, non-colliding beam available to use as a reference, and having such an extra bunch during high energy physics runs is undesirable. Figure 5 shows the actual vertical positions of the colliding and reference positron bunches in data set #2b at one detector. Due to limitations of the present beam detector system, the effect of the beam-beam interaction is visible only in the difference between the two bunches. Fitting a cubic polynomial to the colliding bunch’s position and finding the inflection point does not agree with the inflection point of the difference data. In fact, adding the quadratic and cubic terms does not significantly improve the “goodness” of the fit.

### IV. CONCLUSION

The experiment was successful in that the effects of the beam-beam interaction on the closed orbit were observed and measured, and the measurements were consistent with the formulae in section II. However, the need for a reference bunch as demonstrated in figure 5 makes this method impractical for use during normal operating conditions. Two pairs of dedicated detectors on either side of the interaction point (to measure both the position and slope of the electron and positron orbits separately) might be a more reliable method of determining the overlap of the beams at the IP.

### V. REFERENCES

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