THE DYNAMIC BETA EFFECT IN CESR*

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Abstract

The change in beta due to the beam–beam interaction — the 'dynamic beta effect' — has been observed in the Cornell Storage Ring CESR by comparing the observed luminosity with the observed vertical beam heights. Under current colliding beam conditions the resulting changes in horizontal beta around the ring have exceeded $\Delta\beta_x/\beta_x = 0.5$ and the horizontal tune shift parameter ξ_x has exceeded 0.05.

I. ANALYSIS

In a colliding beam storage ring the Twiss parameters are affected by the quadrupolar focusing of the beam–beam interaction. Like any quadrupole error this 'dynamic beta' effect is enhanced by running near a half–integer or integer resonance. Following Chao[1], the dynamic beta effect can be analyzed by writing the 1–turn transfer matrix from IP to IP as

$$\begin{pmatrix} \cos\mu & \beta\sin\mu \\ -\frac{1}{\beta}\sin\mu & \cos\mu \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\mu_0 & \beta_0\sin\mu_0 \\ -\frac{1}{\beta_0}\sin\mu_0 & \cos\mu_0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} , \tag{1}$$

where β_0 and μ_0 are the 'unperturbed' beta and tune without the beam–beam interaction. In Eq. (1) the beam–beam interaction strength of $1/f$ is given by

$$\frac{1}{f_{x+}} = \frac{2N_- r_e}{\gamma\sigma_{x-}(\sigma_{x-} + \sigma_{y-})} , \tag{2}$$

with analogous formulas for f_{x-}, f_{y+}, and f_{y-} where x and y refer to the horizontal and vertical planes and $+$ and $-$ refer to the positrons and electrons respectively. In Eq. (2) N is the number of particles in a beam, r_e is the classical electron radius, γ is the usual relativistic factor, and σ is the beam size. The beam–beam parameter ξ is defined by

$$\xi \equiv \frac{\beta_0}{4\pi f} , \tag{3}$$

ξ is just the focusing strength of one beam on the other normalized by β_0. It is sometimes convenient to define another beam–beam parameter κ by

$$\kappa \equiv \frac{\beta}{4\pi f} = \frac{\beta}{\beta_0}\xi . \tag{4}$$

Combining Eqs. (1), (2), (3), and (4) gives

$$\cos\mu = \cos\mu_0 - 2\pi\xi\sin\mu_0 \tag{5}$$

*Work supported by the National Science Foundation

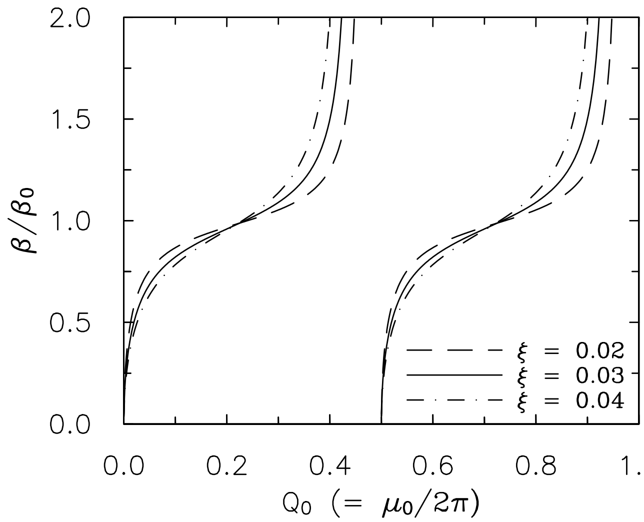

Figure 1. β relative to β_0 as a function of tune for three different values of ξ. The top scale shows the tune in kHz.

$$= \cos\mu_0 - 2\pi\kappa\frac{\beta_0}{\beta}\sin\mu_0 , \tag{6}$$

$$\sin\mu = \frac{\beta_0}{\beta}\sin\mu_0 . \tag{7}$$

Eliminating μ from Eqs. (5) and (7) gives

$$\frac{\beta}{\beta_0} = \left(1 - (2\pi\xi)^2 + 2(2\pi\xi)\cot\mu_0\right)^{-1/2} . \tag{8}$$

Alternatively, in terms of κ, one finds

$$\frac{\beta}{\beta_0} = \sqrt{1 + \frac{(2\pi\kappa)^2}{\sin^2\mu_0}} - (2\pi\kappa)\cot\mu_0 . \tag{9}$$

Figure 1 shows β/β_0 as calculated from Eq. (8) as a function of $Q_0 \equiv \mu_0/2\pi$ for three different values of ξ. As can be seen from the figure, for tunes just above an integer or half–integer resonance, the dynamic beta effect causes a reduction in β. This, of course, is what is desired for increased luminosity. As an example, the Cornell Electron/positron Storage Ring CESR is currently operating with a design horizontal tune of 9.52. Under the assumption that ξ is in the vicinity of 0.03 (see below) this implies that there is a large reduction in beta of $\beta_x/\beta_{x0} \sim 0.5$. Additionally, with the present CESR vertical tune of 9.60, the reduction in vertical beta is $\beta_y/\beta_{y0} \sim 0.8$.

Along with the change in β at the IP there will also be a beta–wave throughout the ring. If the beam–beam interaction is small enough, one can use first order perturbation theory (cf. Sands[2] Eq. 2.105) to obtain

$$\frac{\Delta\beta(s)}{\beta_0(s)} = \frac{\Delta\beta(\mathrm{IP})}{\beta_0(\mathrm{IP})} \cdot \frac{\cos(2\phi_0(s) - \mu_0)}{\cos\mu_0} , \tag{10}$$

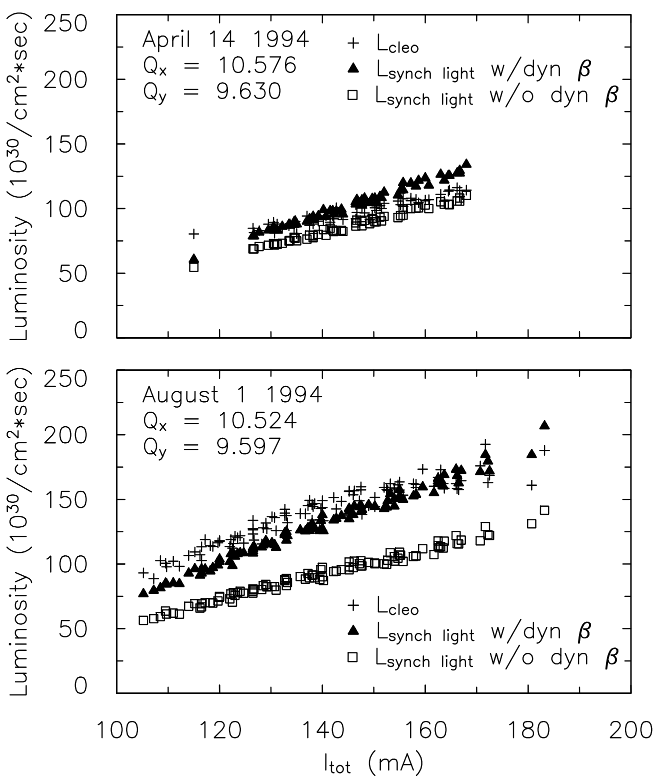

Figure 2. Luminosity as a function of total current for two days of HEP running.

where $\phi_0(s)$ is the phase advance from the IP to point s. With the present CESR horizontal tune of 9.52 this translates into a horizontal beta–wave with amplitude $|\Delta\beta_x|_{max}/\beta_{0x} \approx 0.5$ One consequence of this beta–wave is that it changes the horizontal emittance function $\mathcal{H}_x(s)$ (cf. Sands[2] Eq. 5.71) and this will affect the horizontal emittance.

II. SYNCH LIGHT LUMINOSITY

As a fast tuning aid in CESR the luminosity is monitored via a calculation that uses the observed electron and positron beam heights. The observed beam heights are obtained via the synchrotron light generated at two specific locations in the arcs. Since it modifies the betas the neglect of the dynamic beta effect can throw off the 'synch light' luminosity calculation. Conversely by comparing the synch light luminosity with the actual luminosity recorded by the CLEO detector the presence of the dynamic beta effect can be verified.

The synch light luminosity is calculated from the equation

$$\mathcal{L} = \frac{f_{rev}}{4\pi\,\sigma_x\,\sigma_y} \sum_{i=1}^{n_b} N_{i+} N_{i-}\,, \tag{11}$$

where f_{rev} is the revolution frequency, n_b is the number of bunches, N_{i-} and N_{i+} are the number of positrons and electrons respectively in the i^{th} bunch, and the beam sigmas are calculated from the equations

$$\sigma_x = \sqrt{\epsilon_x \beta_x}\,, \quad \text{and} \tag{12}$$

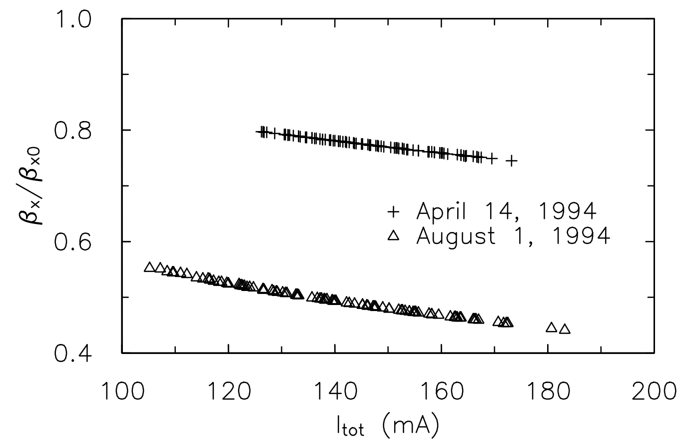

Figure 3. β_x/β_{x0} as a function of total current.

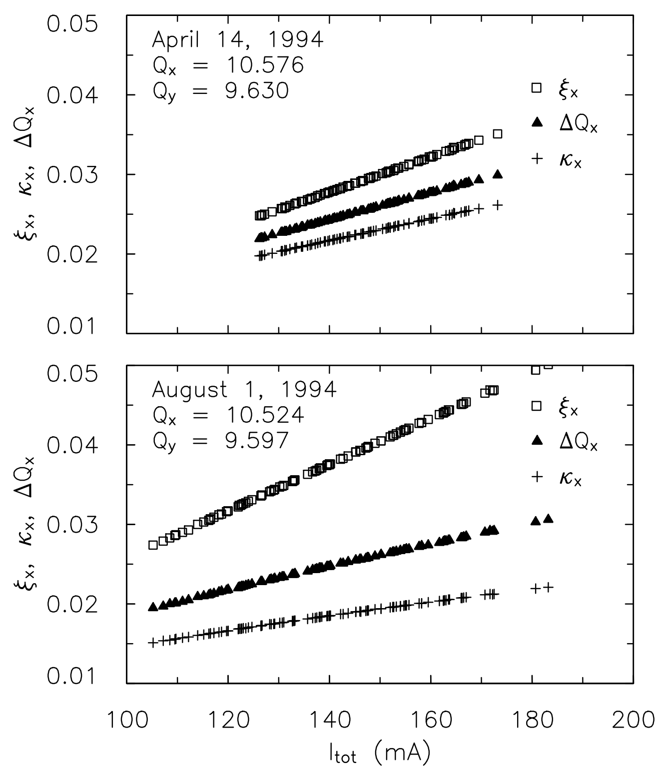

Figure 4. ξ_x, κ_x, and ΔQ_x as a function of total current.

$$\sigma_y = \sigma_y(ls) \cdot \sqrt{\frac{\beta_y(\mathrm{IP})}{\beta_y(ls)}}\,, \tag{13}$$

where (ls) stands for the light source point.

Figures 2 through 4 show data from two days of normal HEP running: April 14, 1994 and August 1, 1994. [for a complete report see Sagan[4].] The April 14 run was at an old operating point with tunes of $Q_{x0} = 10.574$ and $Q_{y0} = 9.632$ while the August 1 run had tunes of $Q_{x0} = 10.523$ and $Q_{y0} = 9.597$. Figure 2 shows the luminosity as a function of total electron and positron current. The three sets of data shown correspond to: (A) Data from the CLEO detector, (B) The luminosity as calculated from the synchrotron light monitors neglecting the dynamic beta effect, and (C) The luminosity as calculated from the syn-

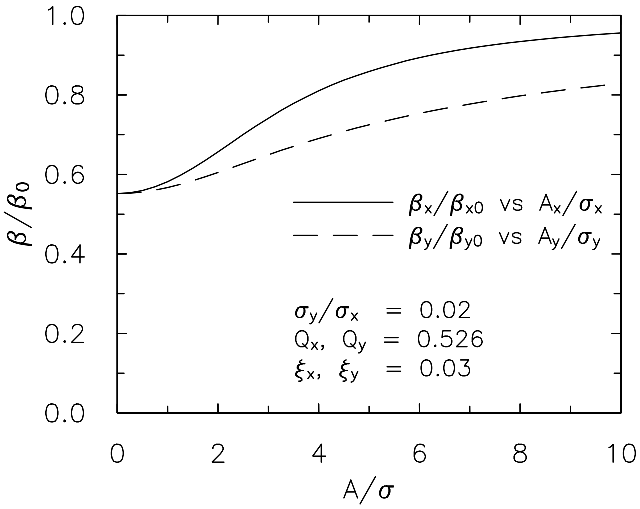

Figure 5. Relative β/β_0 as a function of oscillation amplitude for the horizontal plane (solid line) and the vertical plane (dashed line).

chrotron light monitors including the dynamic beta effect. For the April 14 run the three curves are too close together to decide whether including the dynamic beta effect gives a better fit to the CLEO data. However, For the August 1 run, since the horizontal tune is closer to a half–integer, it is clear that one must take the dynamic beta effect into account. For the August 1 run the change in ϵ_x with beam current was significant exceeding 50% at the highest currents. The fact that there is still a discrepancy between the synch light calculation and CLEO can be explained by the neglect of other effects such as the hourglass effect[3].

Figure 3 shows β_x normalized by the unperturbed β_{x0} as a function of total current. For the August 1 run the reduction in β_x is quite dramatic, being over a factor of 2 at the larger currents.

The difference between ξ_x, κ_x, and $\Delta Q_x \equiv (\mu_x - \mu_{x0})/2\pi$ is shown in figure 4 which shows ξ_x, κ_x, and ΔQ_x as a function of total current. For the April 14 run the tunes are far enough away from the half–integer resonance so that the dynamic beta effect is small and $\kappa_x \approx \xi_x \approx \Delta Q_x$. On the other hand, for the August 1 run, there is a large difference between the three. For the August 1 run ξ_x varies linearly with current up to the largest currents where it exceeds 0.05. ΔQ_x and κ_x however, are significantly lower than ξ_x and they show some slight signs of 'saturation' at the highest currents.

III. AMPLITUDE DEPENDENCE

In terms of single particle dynamics the beam–beam force is nonlinear beyond 1σ either horizontally or vertically. The fact that the beam–beam force starts to fall off beyond 1σ results in a monotonic decrease of the effective quadrupolar focusing strength with increasing particle oscillation amplitude. This results in the dynamic beta effect being amplitude–dependent with large amplitude particles being relatively unaffected by the dynamic beta effect. This implies that the deleterious effects of reduced single particle lifetime that are associated with a lower $\beta(\text{IP})$ are not present with dynamic beta. In other words, the dynamic beta effect is materially different from using a lattice with a lower $\beta(\text{IP})$.

The amplitude dependence of the dynamic beta effect was explored with a simple 1–dimensional particle tracking program which used linear arcs and the full nonlinear beam–beam kick[4]. Particles were seeded at different amplitudes and tracked for 300 turns. For a single particle the resulting motion in phase space was fitted to an ellipse and a value for β extracted. Figure 5 shows the dependence of β/β_0 on oscillation amplitude A for both the horizontal and vertical planes under the conditions $Q = 0.526$, $\sigma_y/\sigma_x = 0.02$, and, in the linear region, $\xi = 0.3$. As can be seen, β is insensitive to changes in amplitude for the particles with oscillation amplitudes below about 2σ. This implies that the amplitude dependent effects on the luminosity are small. In the tails of the beam, where $A_x \gtrsim 10\sigma_x$ or $A_y \gtrsim 50\sigma_y$, the dynamic beta effect is seen to be small.

References

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