*CRYSTALLINE BEAMS**

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A beam of confined charged particles, that are cooled to the extreme of the space-charge dominated regime, where the relative motion of particles within the beam is small compared to their Coulomb potential energies, will crystallize in a unique form of condensed matter [1]. Such a system of particles can be simulated using the method of Molecular Dynamics, which explicitly includes the interaction between all pairs of particles and uses repeating cells to simulate the effects of a long beam. Within the molecular dynamics simulations typically a few thousand particles are followed in time, allowed to equilibrate, and then the velocities are gradually scaled down while still allowing the system to maintain equilibrium. To reach a cold equilibrium value requires 10-100 thousand iterations, corresponding to real times on the order of a few thousand betatron periods.

I. THE BASIC CRYSTALLINE STRUCTURES

The configuration of ions in such a cold beam will depend primarily on the number of particles per unit length relative to the strength of the average confining force (whose magnitude determines the betatron frequency). For few particles, the Coulomb repulsion is weak and at low (transverse as well as longitudinal) temperatures the beam particles will arrange themselves in a string on the beam axis. As the density of particles is increased, at a critical density, the repulsion between the ions will force them into a two-dimensional zig-zag pattern, and then at a yet higher density into three-dimensional arrays on the surface of a cylinder. The radius of the cylinder grows and eventually a new string forms on the axis, then gradually, more and more concentric equally-spaced cylindrical shells of particles appear. Such a system is illustrated in figure 1. The number of shells is proportional to the square root of the number of particles, each shell has the same surface density, and the particles form a triangular pattern on the mantle of the cylinder characteristic of two-dimensional Coulomb solids [2]. Thus while the system is ordered, it does not have the normal type of crystallographic order with a unit cell repeating by translation along the crystal axes. These configurations appear to be close to the classical 'ground states' of these systems; though the true quantum-mechanical ground states (having the same configurations) with no phonon excitations, are at considerably lower temperatures. This type of order was predicted to occur in all cold confined ionic systems such as in laser-cooled ion traps, where the confinement is three-dimensional [3] and where spherical or spheroidal shells of

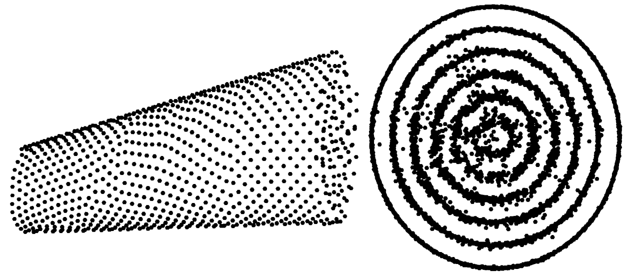

Figure 1. The result of a simulations showing a segment of beam with 5 shells. On the left is a perspective drawing of the beam with only the particles that appear on the surface shown, on the right is a projection of the beam segment onto a perpendicular plane. Typical spacings between particles and shells are on the order of tens of microns.

particles have been observed. At some point, in the interior of very large systems, the shell structure should transform into a body centered cubic lattice. Shell structure has been seen in a special "ring ion trap" [4] for stationary ions with rf quadrupole focusing which has the "beam" geometry, and the detailed patterns and the transitions between them were quantitatively confirmed.

Unequal Confining Field

In the case where the focusing forces are not equal in the two (vertical and horizontal) directions, the above shell structure is modified and the shells become elliptical in cross section instead of circular, but the pattern is otherwise very similar. As the eccentricity increases it becomes possible for the innermost structure to become planar, larger than the simple zig-zag for the cylindrically symmetric case.

II. NORMAL MODES OF THE CYLINDRICAL LATTICE

A lattice confined by a force field that is constant in time is, of course, a idealized simplification. Before considering a more realistic focusing lattice it is appropriate to discuss the simple degrees of freedom associated with a cylindrically confined beam. While many complex modes are possible, the details of these modes will depend on particular configurations. The hydrodynamic multipole modes that are characteristic of a charged fluid are more universal and of these two that are illustrated in figure 2 appear to be dominant for all but the thinnest (one- or two-dimensional) beams.

Monopole Mode

A cylindrically syummetric volumne oscillation of the beam is seen at the plasma frequency (which is sqrt)2) times the betatron frequency for a single ion in the same

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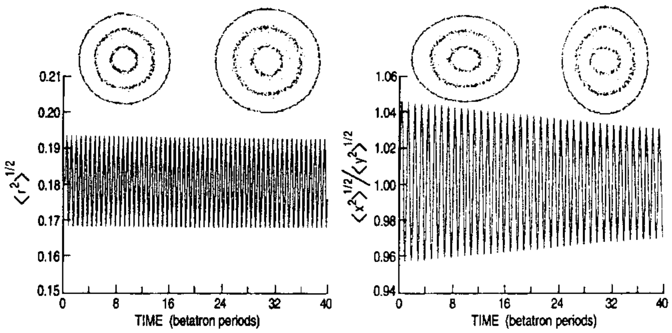

Figure 2. Simulations of the two normal modes discussed in the text followed in time; the left shows the monopole mode, the right the quadrupole.

focusing field). For a sufficiently cold beam this mode appears to be a true eigenmode — with no mixing into thermal excitation, as is illustrated in figure 2.

Quadrupole mode

The other mode that is readily excited is a volume-conserving quadrupole mode, where the beam oscillates between elliptical cross sections, elongated alternately in the horizontal and vertical directions. The frequency of this mode is equal to the betatron frequency and its damping, also shown in figure 2, is indicative of interactions with more complicated excitations.

III. MORE REALISTIC APPROXIMATIONS TO AN ACCELERATOR LATTICE

Several aspects of an accelerator need to be considered in order to approach the reality of a storage ring or similar device in which a moving cold beam may be confined. So far no effect has been found that suggests an insurmountable obstacle to achieving the condensed state of the beam discussed above.

Focusing Lattice

The fact that the accelerator lattice consists of finite focusing elements means that the confining force is not constant in time. However, the question is the time scale in which the force changes and whether the periodicity of these changes is at a harmonic of the eigenmodes of the system. While the periods within an accelerator lattice are typically shorter than the normal mode frequencies, harmonics of the lattice frequency may excite the normal modes and thus place some restrictions on particular lattices, but in general they do not significantly alter the conclusions for a static confining force mentioned above. The ordered system does undergo a slight micro-motion: oscillations with the periods imposed by the time-dependence of the focusing lattice and analogous to the "micromotion" in rf ion traps. The ordered system is a plastic one and these oscillations do not seem to couple into the random thermal degrees of peaks. The question of what constitutes a random "temperature" and what collective motion is somewhat fuzzy.

Cooling and Longitudinal-Transverse Coupling

In the Molecular Dynamics simulations cooling is a simple matter, applied uniformly to the beam under the most favorable circumstances. In a real accelerator, cooling is much less straightforward. In storage rings, where the coldest beams are obtained, unidirectional cooling is applied in a relatively short section on the beam. Simulations suggest that this may not be a serious problem — what matters is that the random kinetic energy of the beam is removed. For a hot beam the beam radius is a function of the transverse temperature, but once the temperature of the beam is low enough such that the beam radius is within an order of magnitude of the space-charge limit the coupling between the longitudinal and transverse degrees of freedom is sufficient to make the longitudinal cooling effective, as long as the number of particles is sufficient to produce multiple shells in the cold limit. When the number of particles is low (corresponding to a string when they are cold) the coupling becomes much weaker and it is perhaps problematic whether present longitudinal cooling techniques will be able to produce an ordered beam in the string-like regime.

Bending

The bending of the beam in dipole elements is an inevitable consequence of a storage ring. The co-moving coordinate frame thus becomes non-inertial and cooling the beam to a constant **linear** velocity causes an apparent shear, with particles on the inside of the bend moving forward and ones outside the mean ray moving backward. This shear presents a problem, and suggests that there is a limit to normal cooling techniques. Whether the condensed state of a particular beam cooled to a constant velocity can withstand the forces of bending in a particular ring depends largely on the strength of the focusing as compared to the bending shear — and is thus determined by the betatron tune ν_β for the ring. Simulations summarized in figure 3 indicate that with a small betatron tune in the vicinity of ν_β = 2-3 only a single or two-shell beam will withstand the shearing distortion, but with larger storage rings with betatron tunes of 30 or more many shells can survive. The details of the accelerator lattice can slightly modify these conditions [5]. When the shear is overcome, and if strong longitudinal cooling is applied, the beam separates into a set of strings that slip past each other, and are ordered in a triangular pattern with respect to each other as shown in figure 4. Such ordering is characteristic of sheared colloids, but it is not clear whether the cooling in a beam could ever be sufficiently strong to produce such a system. Ideally, one would like to have a beam cooled to a constant angular velocity, where only the fluctuations in discrete bending elements would cause a much smaller problem of shear oscillations. The stability of such systems has been explored in simulations for relatively modest beams [5].

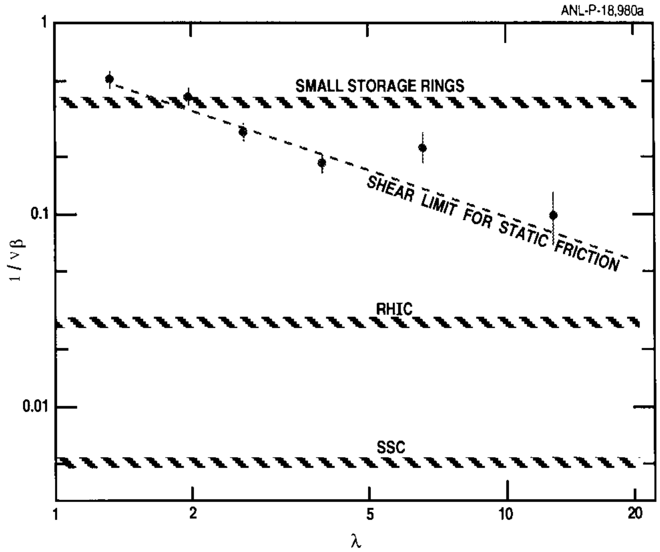

Figure 3. Limits (in $1/\nu_\beta$) at which an ordered beam can withstand bending shear as a function of the linear particle density λ. A beam with a single shell would be in the vicinity of $\lambda = 1$-2. The points with error bars represent the results of simulation.

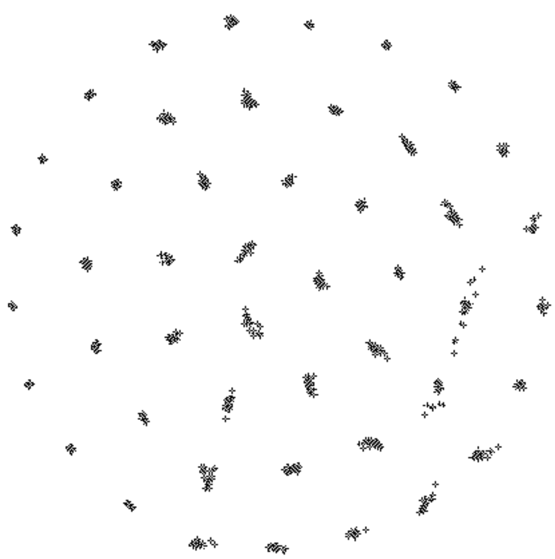

Figure 4. The projection onto a perpendicular plane of particles in a simulated beam beyond the shear limit with continuous cooling.

Ordering in Bunched Beams

In a storage ring it is also possible to cool a bunched beam. This is a case of a three-dimensional confinement similar to ion traps, but with the longitudinal confining force much less than the transverse ones. The fact that the bunching is applied in perhaps only one place in the ring matters even less than the periodicity of the transverse focusing lattice, because the relevant time scale is much longer: the period for synchrotron oscillations. A cold beam bunch is a very elongated spheroid in the cold hydrodynamic limit. Simulations show that for discrete particles the beam will be multi-shelled in the center of a bunch, as before, gradually thinning out, and eventually ending in one-dimensional strings at the ends as shown in figure 5.

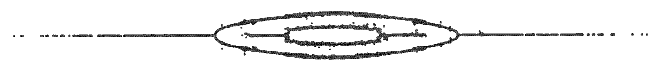

Figure 5. Simulation of a beam bunch, roughly 20 cm long and 0.2 mm in radius.

IV. EXPERIMENTAL PROGRESS AND FUTURE PROSPECTS

Considerable progress has been made in recent years in the cooling of beams in small storage rings. With laser cooling, work at the TSR in Heidelberg [6] has reached longitudinal temperatures in the fractional Kelvin regime, work at ASTRID in Arhus in the milliKelvin regime [7] which is in the crystalline regime, however the transverse temperatures are not known. The suppression of Schottky noise has been observed for continuous beams at ASTRID [8] and the space-charge limit in the length of cooled bunched beam has also been seen [9], along with related work with electron cooling at Indiana [10]. The transverse temperature and thus the three-dimensional ordering are not firmly established, some simulations indicate that the longitudinal-transverse coupling is strong for the space-charge limited beams that have apparently been achieved, and then the transverse temperature could be similar to the longitudinal one. However better diagnostics on the transverse properties of the beam are needed and with these more quantitative measurements will become possible.

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