DETERMINING ELECTRON BEAM PARAMETERS FROM EDGE RADIATION MEASUREMENT RESULTS ON SIBERIA-1 STORAGE RING

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The paper deals with practical application of the method for electron beam diagnostics by means of edge radiation (ER) [1] on the Siberia-1 storage ring. In this application, each measurement of the ER intensity distribution allows one to determine a linear combination of three second-order moments of particle density distribution in transverse phase space. It is shown that, when combining data on the storage ring magnet lattice with the results of two ER intensity distribution measurements (performed at different distances from bending magnet edges), the totality of beam parameters, including horizontal and vertical RMS transverse sizes, angular divergences, energy spread, emittances and mixed moments can be determined.

I. INTRODUCTION

A possibility to determine angular divergences, transverse sizes and emittances of electron beam in a storage ring from measured intensity distributions of the edge radiation (i.e., the radiation generated at bending magnet edges) was stated in former related works [1] - [3].

Logical foundation of the ER-based method for electron beam diagnostics is very simple. The ER and the radiation emitted within elements of electron beam optics in storage rings was found to be very sensitive to angular divergences and transverse sizes of the electron beam [1]. Therefore, if one can measure the ER intensity distributions and perform precise calculation of the corresponding distributions for particular magnet lattice and measurement system, at different values of the beam parameters, then one can determine the real beam parameters as a result of fitting the measured and computed ER intensity distributions.

In Refs. [1], [2] general attention was paid to methods for effective computation of the ER intensity distributions in view of electron beam parameters, the methods being elaborated especially for the electron beam diagnostics. In Ref. [3] an emphasis was done on experimental equipment for the ER-based diagnostics. Though precise computation and measurement of the ER intensity distributions are very important for the beam diagnostics method concerned, these still are not sufficient for the "full-scale" implementation of the method. The main subject of this paper is treatment of the ER measurement results in order to determine electron beam divergences, transverse sizes and emittances.

In this paper, general attention is paid to the case of "empty" straight section between bending magnet edges (when no hard focusing elements are located within the straight section). This case was chosen for consideration due to two reasons: first, it is more simple for understanding the main features of the ER-based method, and second, the situation takes place in the Siberia-1 450 MeV electron storage ring (Kurchatov Synchrotron Radiation Source, Moscow), where the experimental part of this work was done. The main distinctions from the case concerned one should keep in mind when applying the ER-based method to hard-focusing machines, are discussed at the end of the paper.

II. MEASURING AND FITTING THE ER INTENSITY DISTRIBUTIONS

Scheme of the ER measurement system installed on the Siberia-1 storage ring is shown in Figure 1. The system allows one to measure the ER intensity distributions simultaneously at two different distances from the middle of straight section, $y_1 < y_2 = y_{21} + y_{22}$. One can see from consideration presented in the next chapter that for optimal layout, the reference value separating y_1 and y_2 should be the value of horizontal beta-functions in some point within the straight section: $y_1 < \beta_x < y_2$. In this case the ER intensity distribution at y_1 is more sensitive to beam transverse size, and vice versa, the distribution at y_2 "feels" beam divergence better then the transverse size.

It can be shown analytically and numerically, that in the case of "empty" straight section (see note above) the ER
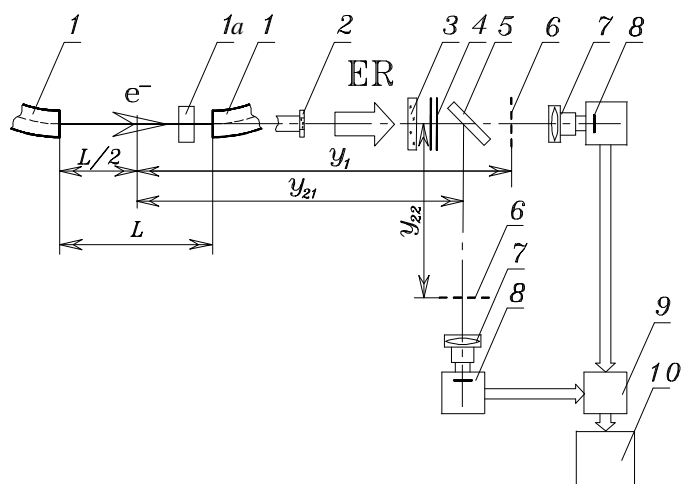

Figure 1: Scheme of the ER measurement system on the Siberia-1.
1- bending magnet; 1a- steering magnet; 2- extraction window; 3- neutral filters; 4- interference filter; 5- semitransparent mirror; 6- object plane; 7- lens; 8- CCD-matrix camera; 9- interface; 10- computer.

intensity distributions depend on RMS electron beam divergences $\sigma_{x'}$, $\sigma_{z'}$ and transverse sizes σ_x, σ_z in combinations:

$$\begin{cases} \sigma_{x'eff}^2 \equiv \sigma_{x'}^2 + \sigma_x^2/y^2 + 2M_{xx'}/y; \\ \sigma_{z'eff}^2 \equiv \sigma_{z'}^2 + \sigma_z^2/y^2 + 2M_{zz'}/y, \end{cases} \quad (1)$$

where $M_{xx'}$, $M_{zz'}$ are second-order central mixed moments of particle density distribution in transverse phase space, y is distance from some "initial point" within straight section or at bending magnet edges, to detector. The values of $\sigma_{x'}$, $\sigma_{z'}$, σ_x, σ_z, $M_{xx'}$, $M_{zz'}$, $\sigma_{x'eff}$, $\sigma_{z'eff}$ refer to the initial point (in most cases, it is more convenient to set the point in the middle of the straight section). It is worth mentioning that the feature is valid for large as well as for small distance y (as compared with the straight section length).

According to the preceding, one value of $\sigma_{x'eff}$ and one value of $\sigma_{z'eff}$ can be potentially determined from the ER intensity distribution measured at one distance y. Actual possibility to determine $\sigma_{x'eff}$ or $\sigma_{z'eff}$ depends on the straight section length L, bandwidth $\Delta\lambda$ of the monochromatic filter used, beam energy, detector dynamic range. Approximately, one can show that for the straight section large enough and the standard commercially available CCD detector, $\sigma_{z'eff}$ can be determined with appropriate precision if it takes place $\sigma_{z'eff} > \sqrt{\Delta\lambda/(2L)}/4$.

Figure 2 illustrates the fitting of measured and computed ER intensity distributions performed to determine $\sigma_{x'eff}$ and $\sigma_{z'eff}$.
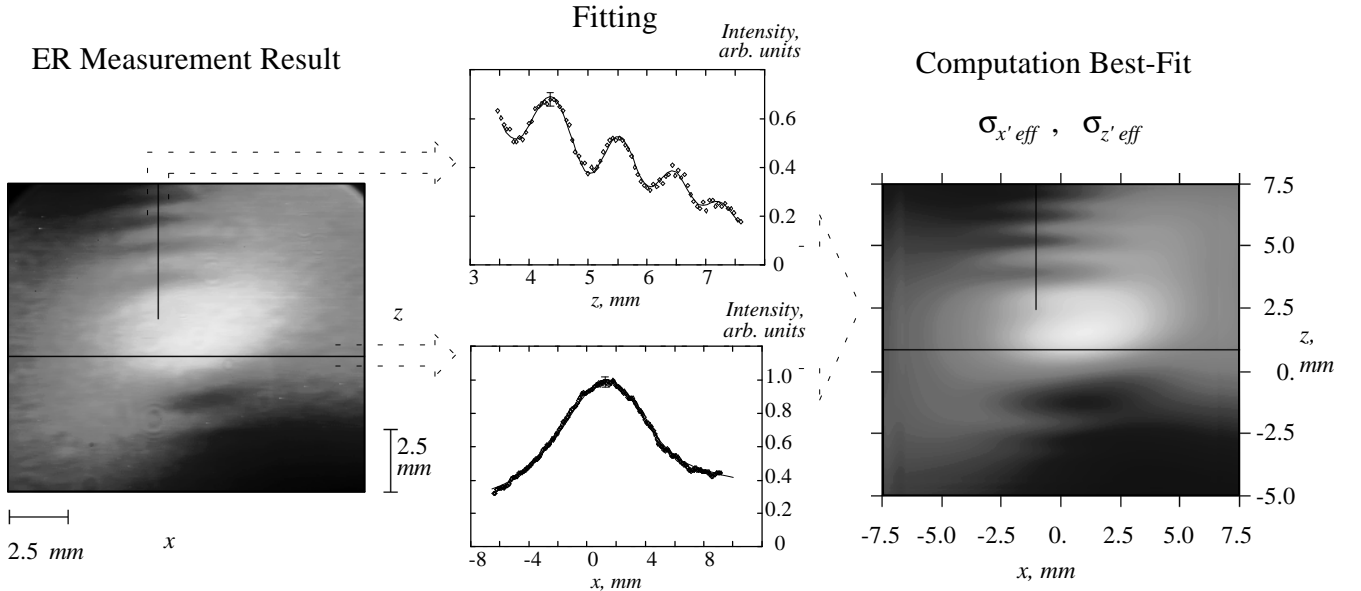
III. DETERMINING ELECTRON BEAM PARAMETERS

Each of the two Eqs. (1) has three unknown beam parameters: $\sigma_{x'}$, σ_x, $M_{xx'}$ ($\sigma_{z'}$, σ_z, $M_{zz'}$). Therefore, formally, one can perform three independent measurements of the ER intensity distributions, at three different distances y, determine three values of $\sigma_{x'eff}$ ($\sigma_{z'eff}$) and solve non-homogeneous system of three algebraic equations with three unknowns: $\sigma_{x'}^2$, σ_x^2, $M_{xx'}$ ($\sigma_{z'}^2$, σ_z^2, $M_{zz'}$).

From the procedure described, vertical beam emittance ε_z can be readily determined: since vertical dispersion is negligible in most cases, then

$$\varepsilon_z = \left(\sigma_z^2\sigma_{z'}^2 - M_{zz'}^2\right)^{1/2}. \quad (2)$$

However, due to horizontal dispersion, the corresponding equation is not valid for horizontal motion (formally, one can find the value $\left(\sigma_x^2\sigma_{x'}^2 - M_{xx'}^2\right)^{1/2}$, yet it is not integral of motion). Besides, some practical problems may take place when realizing the procedure described: it may appear not to be convenient to perform three independent measurements of the ER intensity distributions simultaneously or, if differences between distances y used are not large enough, the errors of the parameters determined may appear to be too large.

In Ref. [3], we used an additional assumption that $M_{xx'} = M_{zz'} = 0$ in the middle of the straight section. This allowed us to use two simultaneous ER measurements instead of three ones. Though rather judicious, the assumption still reduced the precision of data on beam parameters we obtained.

Figure 2: Fitting of measured and computed ER intensity distributions in order to determine $\sigma_{x'eff}$ and $\sigma_{z'eff}$.

◊ - experiment; —— - computation best-fit.

The measurements were done at electron energy E = 400 MeV, wavelength λ = 648 ± 2 nm, y = 199 cm.

Intensity distribution along horizontal line passing through global maximum allows one to determine: $\sigma_{x'eff}$= 0.90 ± 0.07 mrad;

the distribution along vertical line crossing interference oscillations far from the pattern center gives: $\sigma_{z'eff}$= 0.068 ± 0.007 mrad.

More efficient way to determine the electron beam parameters from the ER measurement results is to use more information on the magnet structure. One can use the well-known expressions representing the second-order central moments through Twiss functions and beam emittance (the expressions can be readily obtained, for example, with the Moment method). This approach gives for vertical motion:

$$\begin{cases} \sigma_z^{\,2} = \varepsilon_z \beta_z; \\ \sigma_{z'}^{\,2} = \varepsilon_z \gamma_z; \\ M_{zz'} = -\varepsilon_z \alpha_z; \\ \sigma_{z'_{eff}}^{\,2} = \sigma_{z'}^{\,2} + \sigma_z^{\,2}/y^2 + 2M_{zz'}/y. \end{cases} \quad (3)$$

Here we have a non-homogeneous system of four algebraic equations with four unknowns (σ_z^2, $\sigma_{z'}^2$, $M_{zz'}$ and ε_z); this system should have one solution. In Eqs. (3) the values of the Twiss functions α_z, β_z, γ_z should refer to the same "initial point" as y and $\sigma_{z'eff}$ (practically, it may be more convenient to use representations of α and γ through β and its derivative). According to Eqs. (3), only one measurement of the ER intensity distribution is sufficient to determine vertical parameters of the beam.

For horizontal motion, we have a non-homogeneous system of five algebraic equations with five unknowns (σ_x^2, $\sigma_{x'}^2$, $M_{xx'}$, horizontal emittance ε_x and squared relative RMS energy spread ($\sigma_E/E)^2$):

$$\begin{cases} \sigma_x^{\,2} = \varepsilon_x \beta_x + (\sigma_E/E)^2 \eta^2; \\ \sigma_{x'}^{\,2} = \varepsilon_x \gamma_x + (\sigma_E/E)^2 \eta'^2; \\ M_{xx'} = -\varepsilon_x \alpha_x + (\sigma_E/E)^2 \eta\eta'; \\ \sigma_{x'eff1}^{\,2} = \sigma_{x'}^{\,2} + \sigma_x^{\,2}/y_1^2 + 2M_{xx'}/y_1; \\ \sigma_{x'eff2}^{\,2} = \sigma_{x'}^{\,2} + \sigma_x^{\,2}/y_2^2 + 2M_{xx'}/y_2, \end{cases} \quad (4)$$

where η and η' are values of dispersion function and its derivative at the initial point. Thus, with two measurements of the ER intensity distributions (at two different distances y_1 and y_2) we have not only horizontal divergence, transverse size and mixed moment $M_{xx'}$, but also the horizontal emittance and energy spread, the two last values being integrals of motion. Analysis of the system (4) allows one to see that better precision is attainable when $y_1 < \beta_x < y_2$ (as it was already mentioned in chapter II).

When applied to the Siberia-1 main mode of operation at $E = 450$ MeV, the edge radiation method and the experimental data treatment based on Eqs. (3) and (4) gave us the following electron beam parameters (see Table 1). In the Table 1, the values of $\sigma_{x'}$, σ_x, $M_{xx'}$, $\sigma_{z'}$, σ_z, and $M_{zz'}$ refer to the middle of the straight section we used. The values

Table 1. Siberia-1 transverse beam emittances and related parameters at E = 450 MeV.

ε_x	330 ± 30 nm	ε_z	8.1 ± 1.0 nm
σ_x	2.5 ± 0.3 mm	σ_z	0.11 ± 0.01 mm
$\sigma_{x'}$	0.46 ± 0.05 mrad	$\sigma_{z'}$	0.073 ± 0.009 mrad
$M_{xx'}$	3.2 ± 0.5 nm	$M_{zz'}$	0.0 nm
σ_E/E	$(1.2 \pm 0.2) \cdot 10^{-3}$		

of beam parameters are in gratifying agreement with those published earlier [4].

In presence of strong beam focusing elements within the straight section between bending magnet edges, some corrections should be introduced to the above consideration. In this case, due to different emission conditions for particles at different transverse coordinates, one can not say that the ER intensity distributions depend on beam parameters in combinations given by Eqs. (1). Nevertheless, the ER-based method is supposed to be applicable in this case too, as far as the ER distributions still can be precisely computed in terms of the magnet lattice and beam parameters [1], [2]. The presence of strong quadrupoles could allow one to determine $\sigma_{x'}$, σ_x, $M_{xx'}$, $\sigma_{z'}$, σ_z, $M_{zz'}$ directly from the fitting of measured and computed intensity distributions. Further on, at least two of three first Eqs. (4) should be used to determine horizontal emittance and energy spread in this case.

IV. ACKNOWLEDGEMENTS

V. REFERENCES

[1] O.Chubar, E.Masunov, "Electron Beam Diagnostics by Means of Edge Radiation", Proceedings of the PAC-93, vol. 3, p. 2474.

[2] O.V.Chubar, "Precise Computation of Electron Beam Radiation in Non-uniform Magnetic Fields as a Tool for the Beam Diagnostics", Fifth International Conference on Synchrotron Radiation Instrumentation SRI-94 (Stonybrook, New York, July 18-22, 1994), TuE9, to appear in Proceedings (Rev. Sci. Instum., 1995, Vol. 66).

[3] O.V.Chubar, I.M.Nagornykh, Yu.V.Krylov, "Edge Radiation Based System for Beam Diagnostics on Siberia-1 Electron Storage Ring", Proceedings of the EPAC-94 (London, June 27 - July 1, 1994), vol. 2, p. 1673.

[4] A.N.Artemyev et. al., "Development of the first phase of the specialized synchrotron radiation source Siberia-1", Nucl. Instr. and Meth., 1987, vol. A261, p. 18-21.