RADIATION DAMPING PARTITIONS AND RF-FIELDS

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Abstract

In his classical paper "Radiation Effects in Circular Accelerators" K.W. Robinson has shown that the damping partitions in a circular machine cannot be changed by a radio-frequency field. The proof given there is quite general and valid for any RF-field, provided it is not so strong that it changes the equilibrium orbit appreciably. However, the physical mechanisms which prevent the possibility to alter the damping partitions is not very transparent. As a consequence, so-called "damping cavities" are still proposed from time to time. Here we want to illustrate the underlying physical principles by going in detail through the beam dynamics for the simple case of a cavity operating in a dipole (deflecting) mode and which is located in a region of finite dispersion in a storage ring.

I. INTRODUCTION

In his famous classical paper on radiation damping [1], K.W. Robinson gives a general proof that the radiation damping partition numbers J_E, J_x, J_y of the three modes of oscillation of particles in a circular accelerator cannot be changed with an RF-field. This is shown in a very general way in the Appendix of his paper, with the only restriction that the radio frequency fields should not be so strong that they change the equilibrium orbit appreciably. The proof is so elegant and complete that not much can be added. However, just due to this generality, it is also not very transparent as far as the underlying physical principles are concerned. Probably for this reason, proposals for so-called "damping cavities" are still made from time to time. K.W. Robinson himself told one of us that he once found such a cavity already in the design stage on the drawing board during a visit to an accelerator laboratory.

The effect of such cavities in linear accelerators and proton synchrotrons has also been investigated by H.G. Hereward [2].

To illustrate clearly the mechanisms which prevent the change of damping partition numbers by RF-fields, we consider here a simple RF cavity operating in a dipole mode. Such a cavity produces a longitudinal electric field with a transverse gradient as shown in Fig. 1. We assume that it is placed at a position with finite horizontal dispersion D , and therefore particles of different energy deviation $\delta\mathcal{E}$ will traverse it at different horizontal offsets

$$x = D\frac{\delta\mathcal{E}}{\mathcal{E}_0} = D\,\epsilon$$

Here $\mathcal{E}_0$ is the nominal particle energy, and the relative energy deviation $\epsilon = \delta\mathcal{E}/\mathcal{E}$, has been introduced to make some later expressions more compact. Since the longitudinal cavity field depends on the distance x from the axis, particles of different energy will thus gain different amounts of energy in one traversal. However, there is also a magnetic field associated with this cavity mode. This leads to a transverse deflection which changes the path length of a particle in one revolution. We will show that these two actions of the cavity, i.e. acceleration and deflection, cancel each other as far as their effect on the damping partitions is concerned.

The treatment will not be rigorous or general, but should better illustrate the mechanisms involved. The exact and general proof has already been given by K.W. Robinson.

II. The dipole mode cavity

For acceleration of particles by an RF-field, one normally uses a cavity which oscillates in the so-called monopole mode (m=0). Such a mode has a longitudinal electric field E_z which is - to a good approximation - homogeneous close to the axis.

One can also operate a cavity in a dipole mode (m=1), having a longitudinal electric field with a gradient such that its strength increases with transverse distance from the axis as shown in Fig. 1. In the neighborhood of the axis, the increase is approximately linear and can be written

$$E_z = \frac{x}{a}\hat{E}\cos(\omega_d t) = \frac{\partial\hat{E}}{\partial x}x\cos(\omega_d t).$$

Here ω_d is the frequency of the cavity oscillation which we will choose to be a harmonic of the revolution frequency in the storage ring, and a is an effective cavity radius. We can get the associated magnetic field from Maxwell's equation

$$\dot{\mathbf{B}} = -\mathrm{curl}\,\mathbf{E} \;\rightarrow\; B = B_y = \frac{1}{\omega_d}\frac{\partial\hat{E}}{\partial x}\sin(\omega_d t). \qquad (1)$$

First we calculate the energy gain due to the electric field, and the deflection due to the magnetic field, of a particle with charge e which is going through such a cavity of length ℓ. Both will depend on the distance x from the axis and on the time of traversal t. We phase the cavity such that the synchronous particle with nominal energy $\mathcal{E}_0$ passes through the dipole mode cavity at the synchronous time t_s, when the electric field gradient is at its maximum and the magnetic field goes through zero. Since it is on-axis and synchronous, it will experience neither acceleration nor deflection. Another particle, with a different energy, is now assumed to traverse the cavity at the distance $x = D\epsilon$ from the axis, at a time which deviates by $\tau = t - t_s$ from the synchronous one. The relative energy gain for this particle is

$$\Delta\epsilon = \frac{e\ell\hat{E}}{a\mathcal{E}_0}\cos(\omega_d\tau).$$

We assume now that the bunch length is much smaller than the wave length of the cavity oscillation, i.e. $\omega_d\tau \ll 1$ and approximate the cosine by unity to obtain

$$\Delta\epsilon \approx \frac{e\ell\hat{E}}{a\mathcal{E}_0}x = \frac{e}{\mathcal{E}_0}\frac{\partial\hat{V}}{\partial x}x = Kx, \qquad (2)$$

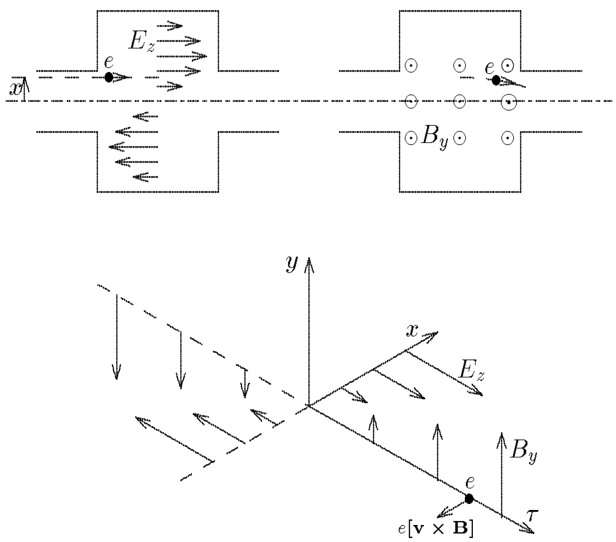

Figure. 1. Dipole mode cavity. Top: The electric field a the synchronous time and the magnetic field a quarter oscillation later. Bottom: Situation of the fields around the synchronous time.

where we used $\hat{V} = \ell \hat{E} = a\partial \hat{V}/\partial x$ and

$$K = \frac{e}{\mathcal{E}_0} \frac{\partial \hat{V}}{\partial x}.$$

Next we calculate the deflection angle $\Delta x'$ due to the magnetic field of the cavity.

$$\Delta x' = -\frac{e\ell B_y c}{\mathcal{E}_0}.$$

With the relation between the electric and magnetic field (1) this can be expressed as

$$\Delta x' = -\frac{ec}{\mathcal{E}_0 \omega_d} \frac{\partial \hat{V}}{\partial x} \sin(\omega_d \tau) \approx -\frac{e}{\mathcal{E}_0} \frac{\partial \hat{V}}{\partial x} c\tau = -Kc\tau. \quad (3)$$

III. Longitudinal beam dynamics in an electron storage ring

First we recapitulate some basic longitudinal beam dynamics considerations for a circular accelerator without a dipole mode cavity. For this we consider a model storage ring consisting of two 180^0 arcs with some focusing structure, and an accelerating cavity operating in the monopole mode. We locate this cavity in a dispersion free region in order to avoid any synchro-betatron coupling effects. The accelerating cavity has a peak voltage $\hat{V}_{RF}$, and oscillates with frequency $\omega_{RF} = h\omega_0$, where h is the harmonic number and ω_0 the revolution frequency. The focusing structure in the arcs creates a momentum compaction factor α_c which determines the relative change of the revolution time T_0

$$\frac{\Delta T}{T_0} = \frac{\Delta \tau}{T_0} = \alpha_c \epsilon.$$

The energy gain per revolution of a particle going through the RF-cavity at the time $t = t_s + \tau$ is given approximately by

$$\Delta\epsilon \approx \frac{e\hat{V}_{RF}}{\mathcal{E}_0} \left[\sin(h\omega_0 t_s) + \cos(h\omega_0 t_s) h\omega_0 \tau\right].$$

The deviation τ from the synchronous time t_s is assumed to be small such that $h\omega_0 \tau \ll 1$. It is convenient to introduce the "synchronous phase angle" $\phi_s = h\omega_0 t_s$, which for electron rings must lie between 90^0 and 180^0 ($\cos\phi_s < 0$) for stability.

We further include the effect of synchrotron radiation which produces an energy loss proportional to the squares of the particle energy and of the magnetic field in the dipoles of the ring $U \propto \mathcal{E}^2 B^2$. Calling U_s the energy loss of a particle with nominal energy, we get the loss for a particle with a small relative energy deviation ϵ

$$U = U_s + \frac{2U_s}{\mathcal{E}_0}\epsilon.$$

Putting these equations together, this yields for the relative energy change per revolution

$$\Delta\epsilon = \frac{e\hat{V}_{RF}\sin\phi_s}{\mathcal{E}_0} + \frac{e\hat{V}_{RF}\cos\phi_s}{\mathcal{E}_0} h\omega_0\tau - \frac{U_s}{\mathcal{E}_0} - \frac{2U_s}{\mathcal{E}_0}\epsilon.$$

For the synchronous particle $\tau = \epsilon = 0$, but for equilibrium we should also have $\Delta\epsilon = 0$, which gives the condition

$$U_s = e\hat{V}_{RF}\sin\phi_s.$$

We assume that the changes in ϵ and τ are small over one revolution, such that they can be described by a differential form. We use $\dot{\epsilon} \approx \Delta\epsilon\omega_0/2\pi$ and $\dot{\tau} = \Delta\tau\omega_0/2\pi$ to get the equations

$$\begin{aligned} \dot{\epsilon} &= -\frac{\omega_0}{2\pi}\frac{2U_s}{\mathcal{E}_0}\epsilon + \omega_0^2 \frac{he\hat{V}_{RF}\cos\phi_s}{2\pi\mathcal{E}_0}\tau \\ \dot{\tau} &= \alpha_c \epsilon \end{aligned}$$

which have as solution a damped oscillation

$$\epsilon(t) = \mathrm{e}^{-\alpha t}\cos(\omega_s t + \phi).$$

with the synchrotron frequency ω_s and the damping rate α

$$\omega_s = \omega_0 \sqrt{-\frac{h\alpha_c e\hat{V}_{RF}\cos\phi_s}{2\pi E}} \quad , \quad \alpha = \frac{\omega_0}{2\pi}\frac{U_s}{\mathcal{E}_0}. \quad (4)$$

In this derivation, approximations have been made assuming $\omega_s \ll \omega_0$ and $\alpha_c \ll \omega_s$.

IV. Effects of the dipole mode cavity

We now include the effects of a dipole mode cavity on beam dynamics. This cavity is supposed to be located in a straight section, which may be opposite to the accelerating cavity, and where the dispersion D is nonzero. As we have seen before, such a cavity will give both an energy increase (2) and a deflection (3)

$$\Delta\epsilon = Kx \quad , \quad \Delta x' = -Kc\tau \quad \text{with} \quad K = \frac{e}{\mathcal{E}_0}\frac{\partial \hat{V}}{\partial x}.$$

The deflection has also a longitudinal effect which is important. The angle $\Delta x'$, which is created by the cavity, leads to a horizontal orbit distortion, and therefore to a change of the path length L of the trajectory of a particle travelling around the ring[3]. Since this well known effect is an essential part of the mechanism under investigation here, we shall give its detailed derivation in the following.

We study the trajectory of a particle which passes at a radial distance x from the equilibrium orbit through a bending magnet with radius of curvature ρ . Due to the deflection by the magnetic field, its path length will increase by an amount ΔL over that of a particle on the equilibrium orbit

$$\Delta L = \int_0^s \frac{x}{\rho} ds.$$

We will make use of the differential equations for the betatron trajectory $x(s)$ and for the dispersion $D = D_x(s)$

$$x'' + k\, x = 0 \;\;,\;\; D'' + k\, D = \frac{1}{\rho}$$

where the $x' = dx/ds,\; x'' = d^2x/ds^2$ and $k = k(s)$ is the horizontal focusing function. With these expressions we can rewrite the integral for the path length

$$\Delta L = \int_0^s \frac{x}{\rho} ds = \int_0^s (D'' + k\, D) x ds$$

$$= -\int_0^s (D'' + k\, D)\frac{x''}{k} ds = -\int_0^s \frac{D''x''}{k} ds - \int_0^s Dx'' ds.$$

Introducing for short the quantity $g = D'x - Dx'$, and thus $g' = D''x - Dx''$, we obtain

$$\begin{aligned}\Delta L &= -\int_0^s \frac{D''x''}{k} - \int_0^s D''x ds + \int_0^s g' ds \\ &= \;\; -\int_0^s \frac{D''}{k}(x'' + k\, x) ds + g|_0^s = g|_0^s.\end{aligned}$$

since $x'' + k\, x = 0$. Therefore the increase of path length is given by

$$\Delta L = [D'x - Dx']_s - [D'x - Dx']_0.$$

An important application of this expression is lengthening of the closed orbit due to a deflection by an angle θ at the origin. The transverse deflection there is $x'_0 = x'_C + \theta$, where x'_C is the deflection after one turn around a ring with circumference $C = 2\pi R$. Since the closed orbit is stationary, we should have $D_0 = D_C$ and $x_0 = x_C$. The path lengthening due to a stationary deflection at a location with dispersion D becomes

$$\begin{aligned}\Delta L = (D'x - Dx')_C - (D'x - Dx')_0 = \\ = D_0(x'_0 - x'_C) = D\theta.\end{aligned}$$

The lengthening of the orbit vanishes for $D = 0$, except for higher order effects in θ which we have neglected.

We now apply this to the deflection $\theta = \Delta x'$ by the dipole mode cavity. We assume again that the synchrotron motion is slow and changes the orbit distortion only adiabatically, and thus we neglect synchro-betatron coupling. In this case we get for the orbit lengthening in good approximation

$$\Delta L = D\Delta x' = -DKc\tau \;\text{ and }\; \Delta\tau = \frac{\Delta L}{c} = -KD\tau$$

Since the orbit is longer there is an increase $\Delta\tau = \Delta L/c$.

Combining the energy and the path length changes caused by the dipole mode cavity, we get for the differential equations

$$\begin{aligned}\dot{\epsilon} &= \frac{\omega_0}{2\pi}\left(-\frac{2U_s}{\mathcal{E}_0} + KD\right)\epsilon + \omega_0^2 \frac{he\hat{V}_{RF}\cos\phi_s}{2\pi\mathcal{E}_0}\tau \\ \dot{\tau} &= \alpha_c\epsilon - \frac{\omega_0}{2\pi}KD\tau\end{aligned}$$

Seeking solutions of the form e^{rt} we calculate the determinant

$$\begin{vmatrix} \frac{\omega_0}{2\pi}\left(-\frac{2U_s}{\mathcal{E}_0} + KD\right) - r & \omega_0^2 \frac{he\hat{V}_{RF}\cos\phi_s}{2\pi\mathcal{E}_0} \\ \alpha_c & -\frac{\omega_0}{2\pi}KD - r \end{vmatrix} = 0$$

and get the characteristic equation

$$\begin{aligned}r^2 - r\frac{\omega_0}{2\pi}\left(\frac{-2U_s}{\mathcal{E}_0} + KD - KD\right) \\ +\omega_s^2 + \left(\frac{\omega_0}{2\pi}\right)^2\left(\frac{2U_s}{\mathcal{E}_0} + KD\right)KD = 0.\end{aligned}$$

We see that in the central term, which is responsible for the damping, the two effects of the dipole mode cavity cancel. We find the solution $r = -\alpha_d \pm i\omega_{sd}$ with a slightly changed synchrotron frequency

$$\omega_{sd} = \sqrt{\omega_s^2 - \alpha^2 + \frac{\omega_0}{2\pi}\left(-\frac{U_s}{\mathcal{E}_0} + KD\right)KD}$$

However, the damping rate

$$\alpha_d = \frac{\omega_0}{2\pi}\frac{U_s}{\mathcal{E}_0}$$

is exactly the same as for the case without dipole mode cavity given by (4). Combining the two roots we can write the solution in the form

$$\epsilon = \hat{\epsilon}\mathrm{e}^{-\alpha t}\cos(\omega_{sd} + \phi) \approx \hat{\epsilon}\mathrm{e}^{-\alpha t}\cos(\omega_s + \phi).$$

where $\hat{\epsilon}$ is the initial relative energy spread.

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