HIGH-ENERGY HIGH-LUMINOSITY $\mu^+\mu^-$ COLLIDER DESIGN

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Abstract

We discuss the design of a high luminosity (10^{35} cm^{-2} s^{-1}), high energy ($2 + 2$ TeV) $\mu^+\mu^-$ collider, starting from the proton accelerator needed to generate the muon beams and proceeding through the muon storage ring.

INTRODUCTION

Lepton (e^+e^-) colliders have the valuable property of producing simple, single-particle interactions with little background, and this property is essential in the exploration of new particle states. However, extension of e^+e^- colliders to multi-TeV energies is severely performance-constrained by beamstrahlung, and cost-constrained because two full energy linacs are required[1]. On the other hand μ's (heavy electrons) have negligible beamstrahlung, and can be accelerated and stored in rings.

The liabilities of μ's are that they decay, with a lifetime of 2.2×10^{-6} s, and that they are created through decay into a diffuse phase space; in addition the decay products are likely to create large backgrounds at the final focus points making the detector design a challenge. The first problem is overcome by rapidly increasing the relativistic γ factor; at 2 TeV for example, the lifetime is 0.044 s, sufficient for storage-ring collisions. The second can be dealt with by cooling. The possibility of μ colliders has been introduced by Skrinsky et al.[2], Neuffer[3], and others. More recently, several workshops and collaboration meetings have greatly increased the level of discussion[4],[5]. In this paper we discuss the beam dynamics problems encountered in one particular scenario for a $2 + 2$ TeV collider. Tb.I shows parameters for the candidate design and Fig.1 shows a schematic overview of the machine. This scenario includes a high-intensity μ-source, μ-cooling, and acceleration and storage in a collider. The complete cycle is repeated at 30 Hz.

Table I
Summary of Parameters of 2 + 2 TeV $\mu^+\mu^-$ Collider

Beam energy	TeV	2
Beam γ		19,000
Repetition rate	Hz	30
Muons per bunch	10^{12}	2
Bunches of each sign		1
Normalized rms emittance ϵ_n	mm mrad	50
Average ring mag. field B	T	6
Effective turns before decay		900
β^* at intersection	mm	3
Luminosity ˇ	cm^{-2}s^{-1}	10^{35}

SYSTEM COMPONENTS

Proton Driver

The μ-source driver is a high-intensity rapid-cycling (30 Hz) proton synchrotron. The protons are targeted to produce pions, which are then allowed to decay into the required muons. A recent study[6] suggests that an optimum proton energy may be 10 GeV. In this case, with some conservatism (we allow an extra factor of two for potential loss), we require a total of about 10^{14} protons at 30 Hz. This specification is almost identical to that studied[7] at ANL for a spallation neutron source. The only difference is the number of bunches: 2 of 5×10^{13} instead of 1 of 10^{14}, one of which is for making μ^-, the other for μ^+. Both are brought on to the same target.

In order to minimize the longitudinal emittance of the produced pions it is desirable to target relatively short bunches of protons with rms bunch length less than 3 ns (1 m). An RF sequence must thus be designed to phase rotate the bunch prior to targeting. The total final momentum spread, based on the ANL parameters (95% phase space of 4.5 V s per bunch), is modest (6 %, or 2.5 % rms), but if the compression were to take place in a relatively low-field, fast-cycling synchrotron, then the space charge tune shift just before extraction would be very large (≈ 1.5). A separate superconducting compression ring is thus needed (reducing the tune shift to ≈ 0.15), or some other more exotic solution must be found. Some possible parameters of the main components of the proton driver are given in Tb. II.

Table II
Proton Driver parameters

Linac	Energy	MeV	330
	Gradient	MeV/m	4-5
	Frequency	MHz	1200
Booster 1	Energy	GeV	2.2
	Circ.	m	190
	Frequency	MHz	2.2-3
Booster 2	Energy	GeV	10
	Circ.	m	690
	Frequency	MHz	9
Buncher	Energy	GeV	10
	Circ.	m	70
Final	rms emittance	mm mrad	62
	rms long. phase space	V sec	0.7
	rms bunch length	nsec	3
	rms dp/p	%	2.5

Target and Pion Capture

The target could be Cu (24 cm by 12 mm diameter) or Be (70 cm by 2 cm diameter), although Cu would be preferred because of its higher pion multiplicity. Pions are captured from the target by a high-field hybrid solenoid that surrounds it. A field of 28 T, and radius of 7.5 cm are consistent with what is currently available[8]. The pions can then be matched, using a suitable tapered field[9] into a long (350 m) solenoidal decay channel. A field of 7 T and radius of 15 cm for the decay channel seems reasonable and matches the capture acceptance.

Monte Carlo studies indicate that such a system captures almost 40% of the produced pions. Using the Wang[10] formula for pion production, the program calculates a yield of $0.22\,\mu$'s, of each sign, per initial proton. However, for a Cu target, a higher multiplicity is expected and would consequently give, yet, a higher yield.

Phase Rotation Linac

The pions, captured by a solenoid focus system (and the muons into which pions decay) have a huge energy spread, from 0 - 3 GeV (rms/mean $\approx 100\%$), and would be difficult to transport and to handle in any subsequent system. It is thus proposed to introduce a linac along the decay channel, whose frequencies and phases are chosen to deaccelerate the fast particles and accelerate the slow ones; i.e. to phase rotate the muon bunch. Tb.III gives the parameters of these linacs. After phase rotation the rms bunch

Table III
Parameters of Phase Rotation Linacs

Linac	Length m	Frequency MHz	Gradient MeV/m	Phase degrees
1	50	24	2	36
2	50	24	2	0
3	250	6	2	43
4	60	24	2	81

length is 6 m, and the rms momentum spread is reduced to about 15 %. Unfortunately at such frequencies the linacs cannot phase rotate both signs in the same bunch: hence, the need for two bunches. The phases must be set to rotate the μ^+'s of one bunch and the μ^-'s of the other.

Ionization Cooling

Cooling Theory

For collider intensities, the phase-space volume must be reduced within the μ lifetime. Cooling by synchrotron radiation, conventional stochastic cooling and conventional electron cooling are all too slow. Optical stochastic cooling[11], electron cooling in an plasma discharge[12] and cooling in a crystal lattice[13] are being studied, but are not by any means certain. Ionization cooling of muons[14] seems relatively straightforward.

In ionization cooling, the beam loses both transverse and longitudinal momentum as it passes through a material medium. Subsequently, the longitudinal momentum can be restored by coherent reacceleration, leaving a net loss of transverse momentum. Ionization cooling is not practical for protons and electrons because of nuclear scattering (p's) and bremsstrahlung (e's) effects in the material, but is practical for μ's because of their low nuclear cross section and relatively low bremsstrahlung.

The equation for transverse cooling (with energies in GeV) is:

$$\frac{d\epsilon_n}{ds} = -\frac{dE_\mu}{ds}\frac{\epsilon_n}{E_\mu} + \frac{\beta_\perp(0.014)^2}{2\,E_\mu m_\mu\,L_R} \quad (1)$$

where ϵ_n is the normalized emittance, $\beta_\perp$ is the betatron function at the absorber, dE_μ/ds is the energy loss, and L_R is the material radiation length. The first term in this equation is the coherent cooling term and the second term is the heating due to multiple scattering. This heating term is minimized if $\beta_\perp$ is small (strong-focusing) and L_R is large (a low-Z absorber).

From Eq.1 we find a limit to transverse cooling, when heating due to multiple scattering balances cooling due to energy loss, at $\epsilon_n \approx 0.6\,10^{-2}\,\beta_\perp$ for Li, and $\epsilon_n \approx 0.8\,10^{-2}\,\beta_\perp$ for Be.

The equation for energy cooling is:

$$\frac{d(\Delta E)^2}{ds} = -2\frac{d\left(\frac{dE_\mu}{ds}\right)}{dE_\mu} < (\Delta E_\mu)^2 > + \frac{d(\Delta E_\mu)^2_{\text{strag}}}{ds} \quad (2)$$

Where the first term is the cooling (or heating) due to energy loss and the second term is the heating due to straggling.

Cooling requires that $\frac{d(dE_\mu/ds)}{dE_\mu} > 0$. But at energies below about 200 MeV, the energy loss function for muons, dE_μ/ds, is rapidly decreasing with energy and there is thus rapid heating of the beam. Above 400 MeV the energy loss function increases gently, thus giving some cooling, though not sufficient for our application.

In the long-path-length Gaussian-distribution limit, the heating term (energy straggling) is given by[15]

$$\frac{d(\Delta E_\mu)^2_{\text{strag}}}{ds} = 4\pi (r_e m_e c^2)^2 N_o \frac{Z}{A}\rho\gamma^2\left(1-\frac{\beta^2}{2}\right) \quad (3)$$

where N_o is Avogadro's number and ρ is the density. Since the energy straggling increases as γ^2, and the cooling system size scales as γ, cooling at low energies is desired.

Energy spread can also be reduced by artificially increasing $\frac{d(dE_\mu/ds)}{dE_\mu}$ by placing a transverse variation in absorber density at a location where position is energy dependent, i.e. where there is dispersion. The use of such wedges can reduce energy spread, but it simultaneously increases transverse emittance in the direction of the dispersion. Six dimensional phase space is not reduced. But it does allow the exchange of emittance between the energy and transverse directions, and it can do this either way.

Cooling System

We require a reduction of the normalized transverse emittance by almost three orders of magnitude (from 2×10^{-2} to 3×10^{-5} m-rad), and a reduction of the longitudinal emittance by more than an order of magnitude. This cooling is obtained in a series of cooling cells. Each cell consists of a section of Be ($\approx 0.7\,m$) or Li ($\approx 2\,m$) placed in a region of the lattice with a low $\beta_\perp$, a linac (200 MeV), and a matching bend with dispersion where wedges can be introduced to interchange longitudinal and transverse emittance. The energy would be restricted to a value

between 200 and 400 MeV, so as to avoid the energy dE/dx heating below 200 MeV, but minimize the straggling heating at higher momenta. About 20 such cells would be needed.

For the early cells, when the emittance is still large, a sufficiently low $\beta_\perp$ can be obtained with solenoids. In later cells, when the emittance is lower and a lower $\beta_\perp$ is required, current carrying cooling rods (≈ 2 m long, if Li) which serve both to maintain the low $\beta_\perp$ and reduce the beam energy could be employed. In a Li rod, with surface fields of 10 T (as achieved in Li lenses at Novosibirsk, FNAL and CERN [16]), a $\beta_\perp$ of 1.7 cm can be achieved, and the emittance is reduced to about 10^{-4} m. But this is still a factor of ≈ 3 above the emittance goal of Tb.I. A final stage might consist of short sections of Be at even lower $\beta_\perp$ insertions. Alternatively, the additional transverse emittance reduction can be obtained by cooling more than necessary longitudinally, and then exchanging transverse and longitudinal phase-space with a thick wedge absorber.

In all these cells, lattices are required with adequate momentum acceptance, matching in and out of the low beta insertions, appropriate momentum compaction and control of emittance growth from space charge, wake field and resistive wall effects. In addition it would be desirable to economize on linac sections by forming groups of cells into recirculating loops.

Acceleration

Following cooling and initial bunch compression (of the order of 0.2 m) the beams must be accelerated to full energy (2 TeV). A single linac of this energy would work, but would be expensive, and would not utilize our ability to recirculate μ's in rings. A conventional synchrotron cannot be used because the muons would decay before they were accelerated. A fast cycling synchrotron could be used but, because it would be limited to low magnetic fields, would be very large. The best solution seems to be a recirculating linac (similar to CEBAF). If acceleration is done in 20 recirculations, then only 100 GeV of linear accelerator is required.

In practice, a cascade of at least 3 recirculating linacs (e.g., with maximum energies of 20 GeV, 200 GeV and 0.2 TeV) would be needed. The μ-bunches would be compressed on each of the return arcs, and be bunched finally to the required length of 3 mm at full energy. The two higher energy recirculators must be superconducting for two reasons: the store time is far too long for conventional cavities, and the wall power consumption with conventional cavities would be too high. The total muon beam power is 38 MW. It is hoped to achieve at least 30% efficiency with superconducting cavities, giving a wall power consumption of 127 MW. The gradients assumed are below those assumed for TESLA. They may be over conservative in view of the shorter pulse duration in this application than assumed in TESLA. The muon linac beam dynamics is complicated by transverse HOM because of the large number of muons per bunch, about a factor of 100 higher than electrons in TESLA. The HOM power is estimated to be ≈ 100 W/m. As in the TESLA design, this would required a coupler section to remove this HOM power.

At the higher energies, space charge effects will not be a problem, but as the bunches are compressed wake field and resistive wall effects become serious. Preliminary studies suggest that, with a slight decrease in Q/Z (by widening the irises), and with BNS damping, such effects can be controlled.

μ Storage Ring

After acceleration, the μ^+ and μ^- bunches are injected into the 2-TeV storage ring, with collisions in two low-β^* interaction areas. The beam size at collision is $r = \sqrt{\epsilon_n \beta^*} \approx 2\,\mu m$, similar to hadron collider values. The bunch populations decay exponentially, yielding an integrated luminosity equal to its initial value multiplied by an "effective" number of turns $n_{effective} = 150\,B$, where B is the mean bending field in T. With 9 T superconducting magnets, an average B of 6 T might be obtained, yielding an $n_{effective} \approx 900$. The magnet design is complicated by the fact that the μ's decay within the rings ($\mu \rightarrow e\bar{\nu}\nu$), producing electrons whose mean energy is approximately 1/3 of that of the muons. These electrons travel to the inside of the ring dipoles, but radiate a substantial fraction of their energy, as synchrotron radiation, towards the outside of the ring. A warm tungsten, or other heavy metal, liner of about 2 cm thickness will be required to intercept this radiation.

A relatively conventional lattice has been designed [17], but the rf requirements to maintain the required 3 mm rms bunch length in such a lattice would be large. A low momentum compaction lattice of the type discussed by S.Y. Lee et al[18] might thus be preferred. A preliminary study[19] of resistive wall impedance instabilities indicate that 3 mm bunches of 2×10^{12} muons would have an unacceptable transverse microwave instability. A fully isochronous lattice, with conventional BNS[20] damping, would solve the problem, but is not possible because of the effects of the large angles of trajectories in the insertion regions. The proposed solution is to employ RF quadrupoles to apply the BNS damping[21].

Another problem is the design of chromatic correction for the very low beta ($\beta^* = 3\,mm$) insertions. A triplet design would have maximum beta's of 200-400 km in both directions, and chromaticity ($1/4\pi \int \beta dk$) of 2000-4000. If no correction is employed, as in the lattice in reference [17] then the momentum acceptance ($\approx 10^{-5}$) is much less than that easily obtained by the ionization cooling. It seems clear that a local correction of chromaticity[22] would be required. A preliminary automated[23] study of such a correction system, using a doublet at the final focus, gave momentum acceptances of $\pm 0.1\,\%$ and $\pm 0.6\,\%$ in the two directions, where the β_{max}'s were 1.2 and 0.2 million m respectively. A similar design with the triplet (β_{max}'s both 0.4 million m) would be expected to give about 0.3 % in both directions. More sophisticated designs [24] should do better. But this estimate is only for a single pass device like a linear collider; the performance for a storage ring remains to be seen.

Detector Background

For the physics user there is a problem of background from μ-decays that occur near the intersection point, and from scattering of any muon halo circulating in the ring.

A first Monte Carlo[25],[26] study assumed a final triplet with interspersed strong dipole bending magnets. These magnets, it was hoped, would help deflect background tracks coming from further down the beam. No chromatic correction scheme or machine lattice was included in this study. Background track den-

sities initiated by muon decays are indicated in Tb. IV. In this study it was assumed that the detector pixels in the inner tracker were 20 μm by 20 μm, and in the central tracker: 50 μm by 300 μm. The track densities are high, but they result from very low energy electrons that would be eliminated in any track reconstruction. Given the fine subdivision of the assumed detector,

Table IV

Detector Backgrounds from μ decay. σ is the density of tracks and ρ is the occupancy.

Location	outside		inside	
	σ cm^{-2}	ρ %	σ cm^{-2} %	ρ
inner tracker	170	0.07	480	0.19
central tracker	3.2	0.05	2.3	0.03
outer tracker	1.7	-	0.3	-

the occupancies do not look impossible.

In a second study of this problem [27], it was found that much of the background in the first study had come from synchrotron radiation of electrons in the bending magnets near the intersection point. Removal of these magnets reduced the peak track densities by factors of between 2 and 5, and reduced the total by an even larger factor. Clearly, more studies are needed, but it seems probable that ways will be found to further improve the situation.

These studies have also shown that severe background can be generated by scattering of tails in the muon beam. A collimation system will be required in a straight section far from the detectors (presumably a quarter way around the ring). No such system has yet been designed.

CONCLUSION

- The scenario for a 2 + 2 TeV, high luminosity collider is by no means complete. Much work remains to be done. More theoretical studies are needed on optimization of pion production, muon phase rotation, cooling scenarios, the collider lattice, radiation effects, and detector background. Technical studies are needed on the design of liquid lithium rods, targeting, high field solenoids, low-frequency high-gradient linacs, multibeam magnets for the recirculation, and high field magnets for the collider. But no obvious show stopper has yet been found.
- An experimental demonstration of ionization cooling should be made. A letter of intent[28] for such an experiment has been submitted to the BNL AGS.
- If the problems can be overcome, then a $\mu^+\mu^-$ collider may be the best route to study physics at energies higher than those accessible at the LHC or NLC. A 2 + 2 TeV $\mu^+\mu^-$ machine with a luminosity of 10^{35} $cm^{-2}sec^{-1}$ would have a physics reach greater than either of these machines, yet it would be small enough to fit on the BNL or FNAL sites. Its relative cost, however, remains to be seen.
- Efforts are now needed on the design of a "Demonstration Muon Collider" that would employ an upgraded existing proton source, could have a center of mass energy of 0.5 TeV, and might have a luminosity of the order of

10^{32} $cm^{-2}sec^{-1}$. Such a machine, besides being a stepping stone to a higher energy machine, would have the unique capability of searching for the direct channel production of the supersymmetric Higgs particles A and H.

ACKNOWLEDGMENTS

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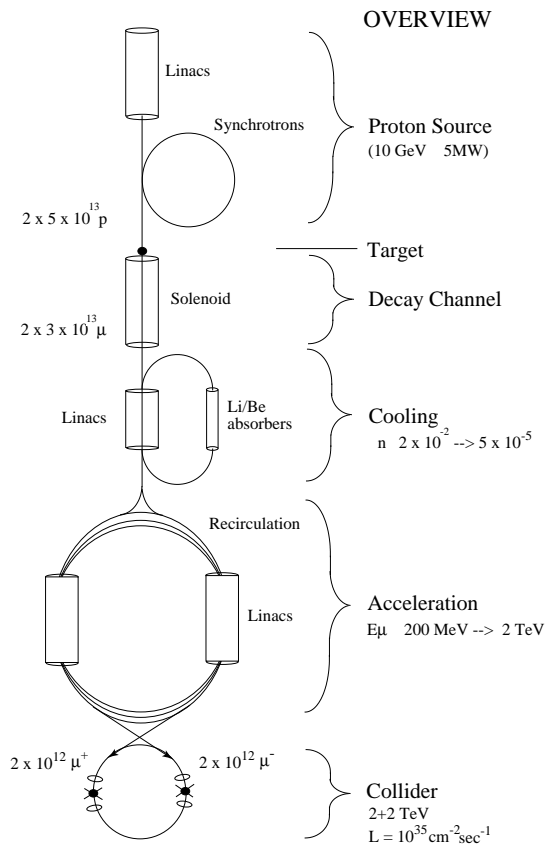[28] R. Fernow et.al., *A possible ionization cooling experiment at the AGS*, BNL-61577 (1994).

Figure. 1. Schematic overview of a $2 + 2\,\text{TeV}\ \mu^+\mu^-$ collider