Lattice Function Measurement with TBT BPM Data

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At Fermilab a procedure using data from TBT BPM system to measure the lattice function of a synchrotron has been developed. The betatron oscillation recorded by the BPM system is fitted to obtain beam parameters x, x', y, y', and Δp/p. These TBT beam parameters (x, x') or (y, y') are fitted to ellipses to obtain the lattice function β, α, and the emittance associated with the betatron amplitude. Fitting the BPM data gives information useful for diagnosing BPM system calibration, noise level and polarity. Other benefits include TBT tune calculation and x-y coupling analysis.

I. Introduction

Single BPM Turn-by-Turn (TBT) beam position data of betatron oscillation is typically used to do FFT tune analysis. Multiple BPM TBT data can be used to go one step further, to the measurement of lattice function. Such procedure has been developed at Fermilab [1] and will be demonstrated here with BPM data taken from Fermilab Main Ring sector D3 & D4 at 150 GeV beam energy. A sketch of BPM layout is shown in Figure 1.

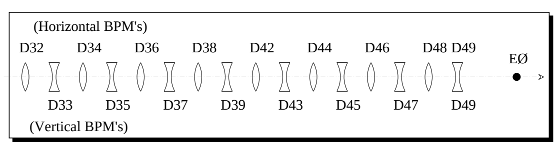

Figure 1: The Layout of BPM's in the D3 & D4 section of Fermilab Main Ring. The horizontal BPM's are indicated above and the vertical BPM's below. The EØ location is the chosen reference location and has no BPM instrumentation.

The procedure fits the BPM data to get the beam parameters x, x' in the horizontal plane, and y, y' in the vertical plane. The fitted TBT beam parameters are in effect coordinates of phase space points which follow elliptical path, as determined by the lattice function. Figure 2 is an example in the horizontal plane. By analyzing these phase space points the lattice function can be calculated. As part of the analysis the deviation from the fit provides useful information either for diagnosing BPM system or for confirming the beam line modeling.

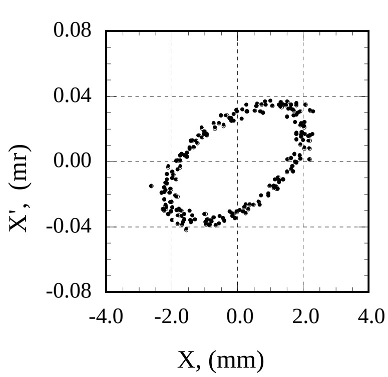

Figure 2: The phase space plot of fitted beam parameters (x, x') at EØ. Each point represents one turn of the proton beam. There are a total of 200 consecutive turns on the plot.

With the normalized phase space it is possible to calculate the TBT tune of the machine with as few as 20 turns of data. Having beam parameters in both planes simultaneously also allows new approach in understanding the local and global x-y coupling effect.

II. The mathematics for analysis

The algorithms used in analyzing the TBT BPM data are quite elementary but are included here to help clarify things that are being discussed here.

A. *Fitting for phase space parameters*

The beam line transfer matrix between the reference zero-th location and the i-th BPM locations where the data is taken is shown in the equation below:

$$\begin{pmatrix} x_i \\ x_i' \\ \delta \end{pmatrix} = \begin{bmatrix} T_{11}^i & T_{12}^i & T_{13}^i \\ T_{21}^i & T_{22}^i & T_{23}^i \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_o \\ x_o' \\ \delta \end{pmatrix}$$

δ (= Δp/p) is the percentage deviation of the average beam momentum from the reference momentum as determined by the bending field. The inclusion of δ in the fit is necessary to account for the momentum error. The x_o, x_o', and δ on the right hand side are the beam parameters being fitted at the reference location. The transfer matrix elements are assumed known from beam line model. The fitting is done by minimizing the summed quantity:

$$S = \sum_i (p_i - x_i)^2 = \sum_i (p_i - x_o \cdot T_{11}^i - x_o' \cdot T_{12}^i - \delta \cdot T_{13}^i)^2 \cdot$$

The summation index "i" runs through all the BPMs used in the calculation. The "p_i" is the i-th BPM reading. This same process is performed for every turn of beam data.

The horizontal BPM data is fitted for beam parameter x, x', and Δp/p at the reference location. The vertical plane data is fitted for y and y'. The result is then propagated to other BPM location using equation above for calculation of deviations between BPM data and the fitted result.

B. *Ellipse and lattice function*

To fit for α, β, and ε with a given number of data points the procedure starts with normalized phase space coordinate transformation:

$$\begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \begin{pmatrix} x_n/\sqrt{\beta} \\ \sqrt{\beta} \cdot x_n' + \alpha \cdot x_n/\sqrt{\beta} \end{pmatrix}.$$

The trajectory will be circular by definition. Re-scale this coordinate space with a factor $1/\sqrt{\beta}$ to get:

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \frac{1}{\sqrt{\beta}} \cdot \begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \begin{pmatrix} x_n/\beta \\ x_n' + (\alpha/\beta) \cdot x_n \end{pmatrix} = \begin{pmatrix} a \cdot x_n \\ x_n' + b \cdot x_n \end{pmatrix},$$

where $a = 1/\beta$ and $b = \alpha/\beta$. A circular trajectory in this (U,V) phase space is still expected. Fitting the (U,V) points to the best circular path by minimizing the equation:

$$S = \sum_n [R^2 - (a \cdot x_n)^2 - (x_n' + b \cdot x_n)^2]^2 .$$

The summation on "n" is over the number of turns used to fit the phase space ellipse. The value for α, β, and ε can be solved accordingly. The ε is simply the area enclosed by the ellipse which is $\varepsilon = \beta \cdot R^2$, in π-*mm-mr*.

C. Tune calculation

With fitted lattice function β and α the normalized phase space angle can be calculated cumulatively as:

$$\theta_n = \tan^{-1}[X_n, Y_n] + \Delta\Phi_n,$$

where $\Delta\Phi_n$ is an adjustment in increment of 2π so that θ_n is successively increasing. The TBT tune is simply:

$$\nu_n = \theta_n - \theta_{n-1}.$$

Because of data precision the TBT tune is not expected to be noise free. A more precise estimate of the tune can be obtained by performing a linear fit of θ_n against the turn number. The slope then gives the tune of the machine.

III. Data analysis

For the analysis of the data to be shown here the reference location is chosen to be at the EØ straight at the down stream of M:HPD49, as indicated in Figure 1. Only horizontal data will be shown.

A. Beam parameters from BPM data

The fitting procedure is as outlined in section II.A. Figure 3 is a single turn BPM position data and the position according to the fitted result. The fitted EØ location beam parameters x and x' are already shown in Figure 2 with a total of 200 consecutive turns. The deviation between data and the fit can be a measure of the noise level in the BPM system or a possible error in the known transfer matrix .

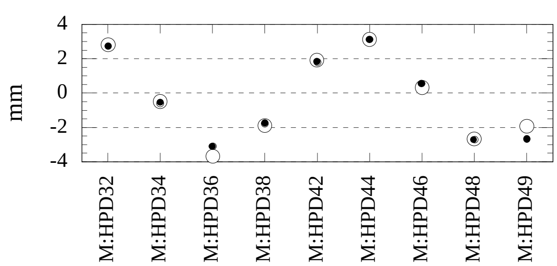

Figure 3: The horizontal BPM position is plotted for each BPM in the beam line. The open circles are the actual BPM data and the solid dots are the calculated beam positions based on the fitted x and x' at EØ.

The deviation statistics is collected and examined in two ways. The "TBT RMS error" is the fit statistics across all the BPMs for every turn. The "BPM error" is the statistics on individual PBM over a number of turns.

1. TBT RMS error

The each turn Root-Mean-Square deviation of data from the fit is plotted in Figure 4. This error is expected to be independent of the turn number and should be consistent with the expected RMS due to BPM electronics noise and the digitization resolution.

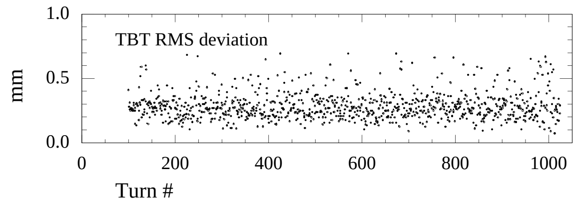

Figure 4: Turn-by-turn RMS deviation of BPM data from the fitted result.

2. BPM deviation error

The individual BPM deviation from the fit is shown in Figure 5. The average of position deviation is expected to be nearly zero and would indicate problems if otherwise. The RMS on the BPM deviation error is shown as the vertical error bars. Generally the RMS in the BPM deviation is a good indicator of individual BPM problems, which could be due to noise, poor calibrations, or even wrong signal polarity.

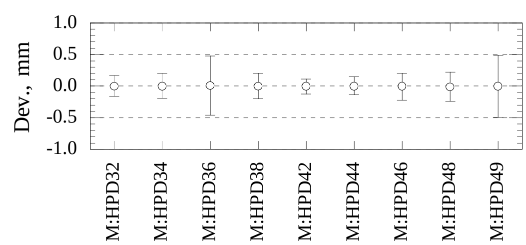

Figure 5: The individual BPM deviation statistics collected for about 200 turns. The open circle represent the average of deviations and error bar is the RMS of the deviation.

B. Fitting the phase space ellipse

It takes five parameters to describe an ellipse, two for the centroid of ellipse in the x-x' space, one for the area of the ellipse, and the last two which has the information on the lattice function α and β. The equations shown in section II.B assumes that ellipse is centered at the origin. By using the average of all data points as origin that assumption is approximately correct and perfect when the number of turns used is beyond 50.

The result of fitting ellipse at EØ location is shown in Figure 6 with the resulted value for ε, α, β, and a calculation of the tune list on the right. The lattice function at other BPM location can be constructed by propagating the fitted beam parameters at the EØ reference location using the transfer matrix. The result is plotted in Figure 7. In (a) is the beta function and (b) the alpha function.

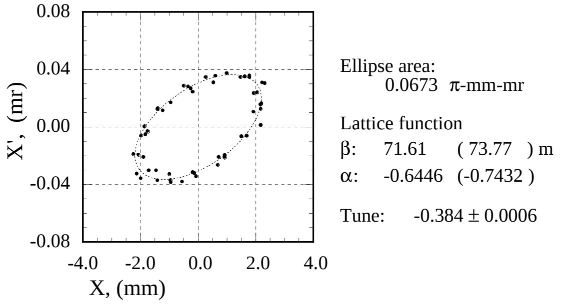

Figure 6: X and X' phase space plot for 50 consecutive turns. The fitted ellipse is shown in dotted lines and the fitted values are shown to the right. The design lattice function is shown enclosed within the parenthesis.

An analytical error analysis is not done because of the algorithm used to fit the ellipse. Instead, the variation of all possible sample of lattice function is used as the estimated error. For example, with "n" number of turns used to fit the lattice function from a data set of "N" total consecutive turns there is a set of (N – n + 1) possible samples. The statistics on this set of possible samples gives the mean and the RMS deviation plotted in Figure 7.

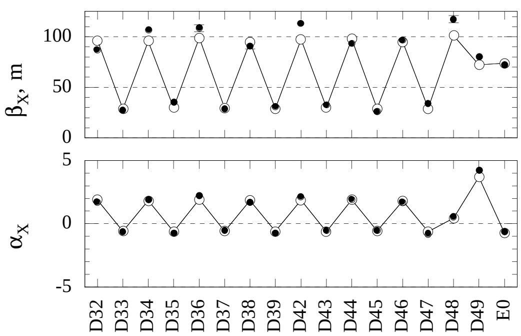

Figure 7: The lattice function at the BPM locations as derived from the data is shown in solid dots. The sigma of the fitted beta distribution is shown as error bars. The open circles with connecting lines are from the SYNCH calculation.

C. *Tune calculation*

The TBT tune of the machine can be calculated as mentioned in section III.C. However, because of the nature

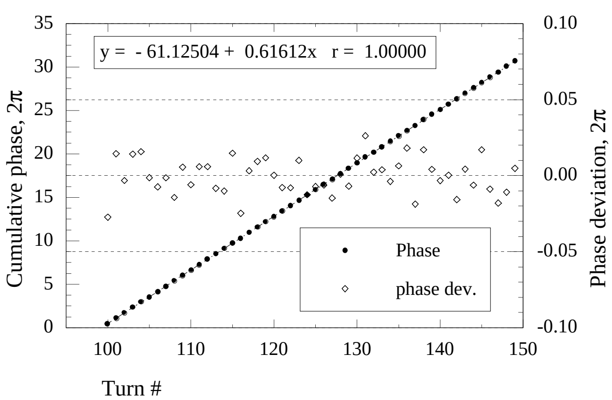

Figure 8: TBT tune calculation. The normalized phase space angles are plotted in solid dots with the vertical scale shown on the left hand side. The phase angle deviations are plotted in open diamonds with the vertical scale shown on the right.

of BPM data noise, a better tune calculation is done by fitting the normalized phase space angle θ_n to the turn number. Figure 8 shows the θ_n and the linear fit result with linear correlation function of 1. The slope gives the horizontal machine tune value of .384 . The deviation of θ_n from the linear fit is shown in open diamonds with its values range from -.03 to +.03, in unit of 2π.

Typically this method will need about 30 turns of data to get a resolution of about 1 part in a thousand. The conventional FFT method would require 1024 turns for a comparable accuracy.

D. *TBT Dp/p*

The beam momentum error $\Delta P/P$ is obtained as part of the fitting procedure to horizontal BPM data and is plotted in Figure 9. This information can be used to correct beam position and angle to account for the $\Delta P/P$ error.

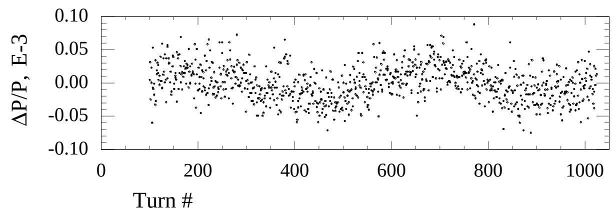

Figure 9: The TBT $\Delta P/P$ from fitting horizontal BPM data.

IV. Conclusion

Knowing the lattice function is important for matching lattice functions between different machines and for the understanding of the machine as well. The technique introduced here provides both the β and α value needed for a complete matching. The measurement can be done within relatively short time. Some thing no other method of lattice function measurement can do.

The result of this procedure appears reproducible and the procedure possesses some ability of self-diagnosing. It is also inherently susceptible to systematic effect caused by the integrity of data and the use of the bad transfer matrix as well. There is systematic effect associated with fitting the ellipse which has been studied with simulated data. With future improvement to the analysis software the TBT BPM data could be the ideal instrument to machine lattice measurement.

REFERENCES

[1] MJ Yang, Lattice Function Measurement with TBT BPM Data, FERMILAB-TM-1922, April 1995.