# RECALIBRATION OF POSITION MONITORS WITH BEAMS

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Abstract

This paper proposes a method to find the geometrical center of a position monitor from its beam signals. The method will be useful in accelerators where the optics is very sensitive to the orbit. Beam test results are also presented.

## I. INTRODUCTION

A beam position monitor system is operated for two kinds of orbit measurements, a relative measurement and an absolute measurement. The former is to measure the orbit displacement from the initial or standard orbit when some optics perturbation is applied. The latter case is to measure orbit position relative to the geometrical monitor center. This function will be essential for maintaining stable operations in a ring where the optics depends strongly on the orbit, particularly at nonlinear optics elements. The dependence will surely appear in future B-factories, where strong sextupole magnets are installed and a small vertical emittance is required.

The output data from a position monitor system usually shows the orbit position relative to the electric monitor center, not the geometrical center. The electric center, however, may drift due to unpredictable imbalance among output signals from the pick-up electrodes, because they must travel through separate paths, cables, connectors, attenuators, switches, and then are measured by detectors.

This paper proposes a method to estimate the imbalance and to find the geometrical monitor center from four output signals of a pick-up unit. It should be noted, however, that the method is not workable in monitor systems where the number of output signals is less than four, as in an AM/PM system.

## II. MODELING OF OUTPUT DATA

Consider a pick-up unit having four electrodes and four processed output data, $V_i$'s, as shown in Fig.1. The data can be given by

$$V_i = g_i \cdot q \cdot F_i(x, y), \quad i = 1, 2, 3, 4$$

where $q$ measures the beam charge, and $x/y$ are horizontal and vertical displacements of the beam relative to the geometrical monitor center. Functions, $F_i(x, y)$, stand for response of four electrodes, and are normalized as $F_i$(0,0)=1. Hence, the origin of arguments of the response function defines the geometrical center. No symmetry condition is required among the response functions. Quantities, $g_i$'s, show overall gains of each electrode. Notice that $g_i$ also includes the impedance imbalance through vacuum connectors, and that the present idea can be applied to calibration of a pick-up unit for modeling its response function.

Further analysis is based on two assumptions. One is that the response functions never change, and can be known well by calibration or calculation. Since the response function depends only

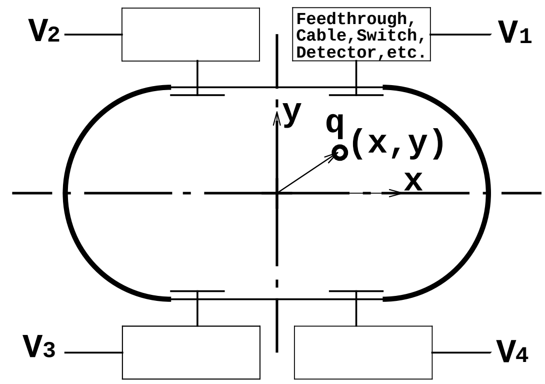


Figure. 1. A model monitor system.

on the geometrical structure of the pick-up unit, this assumption is reasonable. Second assumption is that all of the effects which displace the electric center can be included into $g_i$'s. The second assumption is acceptable at least in a case when the signals are processed with a narrow frequency-band monitor system.

The objective of the paper is to show a possible way to estimate the gains and to recover the geometrical center.

## III. GAIN ESTIMATION

Consider a case when beam positions are measured m times with a pick-up unit, and at each measurement the orbit at the monitor is changed intentionally, for example, with steering magnets. The beam charge may be varied at each measurement although the variation is not important for the gain estimation. Then the data from the i-th electrode at the j-th measurement can be given by,

$$V_{ij} = g_i \cdot q_j \cdot F_i(x_j, y_j).$$

From the fact that only the relative imbalance among the gains contributes to shifting the electric monitor center, $g_1$ can be always set 1 with a proper scaling factor for the beam charge. This means that there exist only 3 unknown gains, $g_2$,$g_3$ and $g_4$. At each measurement 3 unknown parameters, $q_j$, $x_j$ and $y_j$, are generated, but 4 quantities, $V_{1j}$, $V_{2j}$, $V_{3j}$ and $V_{4j}$, can be measured. After the m-th measurement the number of the unknown parameters is 3+3m, whereas that of the known parameters is 4m. When m is larger than 4, the latter number exceeds the former and hence the unknown parameters, including the gains, can be estimated with a nonlinear chi-square method [1].

The present nonlinear model is rewritten as,

$$\begin{aligned} V_{ij} &= g_i \cdot q_j \cdot F_i(x_j, y_j) \equiv V(i, j; \mathbf{a}), \\ & \quad i = 1, ..., 4, \quad j = 1, ..., m, \\ \mathbf{a} &= (g_2, g_3, g_4, q_1, x_1, y_1, ..., q_m, x_m, y_m), \end{aligned}$$

where **a** is the array of fitting parameters. The unknown parameters can be estimated by minimizing the chi-square,

$$\chi^2(\mathbf{a}) = \sum_{i=1}^{4}\sum_{j=1}^{m} \frac{[V_{ij} - V(i,j;\mathbf{a})]^2}{\sigma_{ij}^2},$$

where $\sigma_{ij}^2$ is the data error of the i-th electrode at the j-th measurement. For simplicity, the data errors are assumed the same as $\sigma_{ij}$=$\sigma_0$.

The curvature matrix, $[\alpha]$, defined by

$$\alpha_{k\ell} \equiv \frac{1}{2}\frac{\partial^2\chi^2(\mathbf{a})}{\partial a_k \partial a_\ell}$$

is very important not only for performing the minimization but also for knowing the variance and the covariance of the fitting parameters. The covariant matrix, $[\mathbf{C}]$, is just equal to the inverse of the curvature matrix at the minimum point. The diagonal element gives the variance of the estimated parameter in a way that

$$\sigma^2(a_k) = C_{kk} = \alpha_{kk}^{-1},$$

and the off-diagonal element $C_{k\ell}$ shows the correlation between $a_k$ and $a_\ell$.

## IV. SIMULATION RESULTS

Imagine a circular pipe having four electrodes as shown in Fig.2. This pick-up model has such a pleasant symmetry that all of the response functions can be expressed with only one function, $F_1(x, y)$.

$$F_2(x,y) = F_1(-x,y), \qquad F_3(x,y) = F_1(-x,-y),$$
$$F_4(x,y) = F_1(x,-y).$$

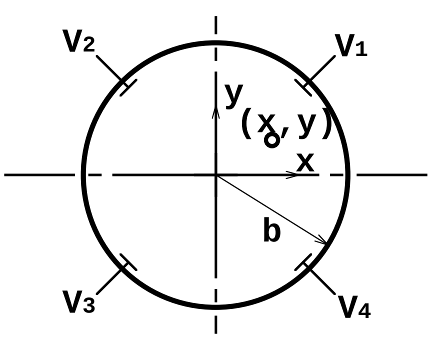


Figure. 2. A circular pick-up unit model.

If the electrode dimension is chosen sufficiently small, the response function can be given by the wall current distribution at the electrode location. With a pipe radius $b$, the response function to the 3-rd order is

$$\begin{aligned} F_1(x,y) &= 1 + \frac{\sqrt{2}}{b}(x+y) + \frac{4}{b^2}xy \\ &\quad + \frac{\sqrt{2}}{b^3}(-x^3 + 3xy^2 - y^3 + 3x^2y). \end{aligned}$$

In simulations, the pipe radius is 50 mm. The first step is to give reasonable values to the gains, and to the charge and displacements at each measurement. The relative gains, $g_2$, $g_3$ and $g_4$, are typically not far from 1. The unreal measurement is done at 5 or 9 displaced positions, as shown in Fig.3. The charge, $q_j$, is chosen around 1 with a proper scaling. The second step is to calculate output data from the model monitor with the assumed response functions. Finally estimation of gains and beam parameters at each measurement is carried out, from the simulated data, with the chi-square method. At the same time the variance of the estimated parameter is obtained.

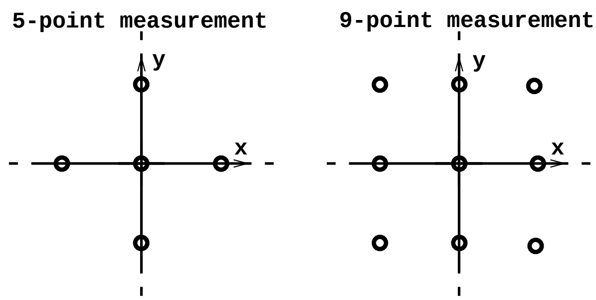


Figure. 3. Orbit displacements at the model monitor.

The simulation results are summarized in Table.I, which shows values given to relative gains and displacements, and the variance of each parameter. With an expected error of $\sigma_0 < 10^{-3}$, which is estimated from the TRISTAN monitor system, the geometrical center can be found within 100 $\mu$m in Case 1 and Case 2. Further improvement can be seen in Case 4 with the nine-point measurement.

Table I
Preset values and simulation results for the variance of the estimated parameters.

| | | | | | |
|---|---|---|---|---|---|
| Case 1 : m=5 | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| preset | 1 | 1 | 1 | 1 | ±5,0 mm |
| $\sigma/\sigma_0$ | 5.5 | 7.7 | 5.5 | 3.9 | 98 |
| Case 2 : m=5 | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| preset | 0.9 | 0.95 | 1.1 | 1 | ±5,0 mm |
| $\sigma/\sigma_0$ | 5.0 | 7.5 | 6.1 | 3.9 | 100 |
| Case 3 : m=5 | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| preset | 1 | 1 | 1 | 1 | ±1,0 mm |
| $\sigma/\sigma_0$ | 25 | 35 | 25 | 18 | 440 |
| Case 4 : m=9 | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| preset | 1 | 1 | 1 | 1 | ±5,0 mm |
| $\sigma/\sigma_0$ | 3.3 | 4.7 | 3.3 | 2.6 | 63 |

## V. BEAM TEST

The present method was tested with a stripline monitor in TRISTAN. Measurement was done with a single beam, an electron beam or a positron beam. The configuration of four striplines and the monitor chamber is the same with that of the numerical model. The stripline monitor is 150 mm long, and its inner radius is 42 mm, which is the only necessary parameter for the position estimation.

Each stripline electrode has two output ports for ensuring the signal directivity. The directivities of the four striplines may be different, and also dependent on the beam direction. In this experiment, not only the upstream port signals but also the downstream signals are analyzed.

This test provides an ideal setting to demonstrate the usefulness of the present method. The relative gains are different on the upstream and downstream sides. Due to the variation of the directivities, the range of the relative gains on the downstream side is wider than that of the upstream. Estimation of the beam position can be done with either the upstream or downstream data independently, and the results are compared with each other.

The position measurement was done for 9 different orbits at the monitor. The signal detection was made with a narrow-band detector, sampling a 500MHz frequency component, with the help of a coaxial switch choosing one from 8 output signals.

Test results are summarized in Table II, and shown in Figures 4 and 5. The directivity, the ratio of downstream/upstream, was 17∼20 %. The measurement error was not analyzed, but may be the same on the upstream and downstream sides because the signal was detected with the same detector gain. With a typical relative error of $10^{-3}$ on the upstream side, the measurement error would be $\sim 3\times10^{-4}$, which is consistent with the difference between the positions estimated independently from the data on either side. The covariant matrix also shows a strong positive correlation among the estimated positions. This fact can be seen in the figures as an offset between the two sets of estimated positions.

The beam test was so successful that the present method will be surely helpful for finding the geometrical monitor center in future accelerators having the optics extremely sensitive to the orbit, for example, in the KEKB.

## References


[1] W. Press, B. Flannery, S. Teukolsky and W. Vetterling, Numerical Recipes (Cambridge University Press).


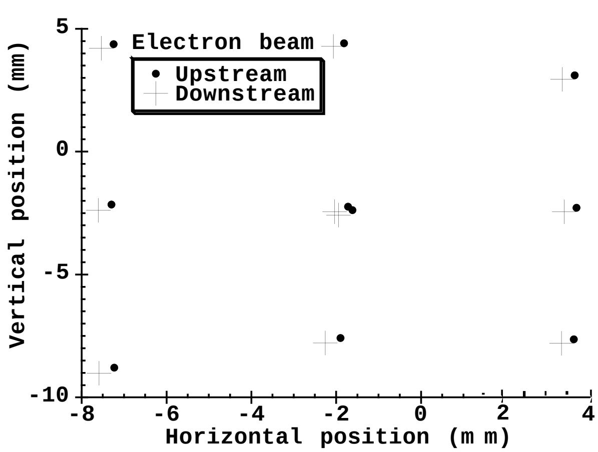


Figure. 4. Estimated positions with an electron beam.

Table II

Estimation for gains and the variance of the fitting parameters in the beam test.

| | | | | | |
|---|---|---|---|---|---|
| Electron beam | | | | | |
| Upstream | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| fit | 1.045 | 1.031 | 1.037 | ∼0.35 | Fig.4 |
| $\sigma/\sigma_0$ | 10 | 14 | 9.2 | 3.0 | 160 |
| Downstream | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| fit | 1.121 | 0.987 | 1.049 | ∼0.065 | Fig.4 |
| $\sigma/\sigma_0$ | 57 | 73 | 49 | 3.0 | 880 |
| Positron beam | | | | | |
| Upstream | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| fit | 1.027 | 1.009 | 1.039 | ∼0.25 | Fig.5 |
| $\sigma/\sigma_0$ | 12 | 17 | 12 | 2.4 | 200 |
| Downstream | | | | | |
| | $g_2$ | $g_3$ | $g_4$ | $q_j$ | $x_j, y_j$ |
| fit | 1.068 | 1.043 | 1.032 | ∼0.045 | Fig.5 |
| $\sigma/\sigma_0$ | 70 | 98 | 66 | 2.4 | 1100 |

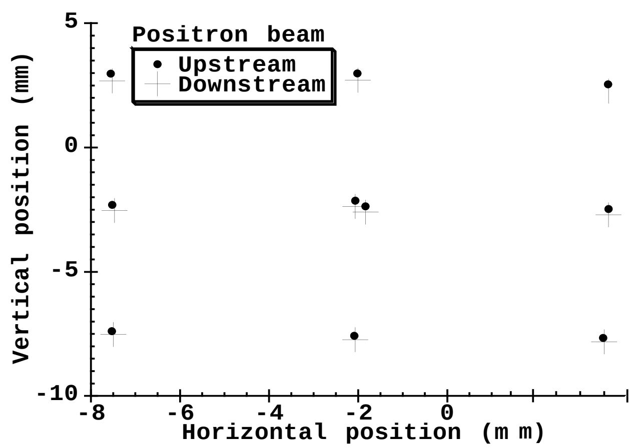


Figure. 5. Estimated positions with a positron beam.