The Computation of the Dynamic Inductance of Magnet Systems and Force Distribution in Ferromagnetic Region on the Basis of 3–D Numerical Simulation of Magnetic Field

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Static and dynamic inductances are ones of the main technical parameters of magnet systems at the designing stage. Ponderomotive force distribution is required for mechanical stress calculations.

The static inductance is used for evaluations of the stored energy, magnetic flux linkage in coils at instantaneous currents. The dynamic inductance allows to define the interrelation between the instantaneous flux linkage and currents determining a transient process in coils.

In the given paper a technique for determination of the dynamic inductance for magnet systems on the assumption of no eddy currents in ferromagnetic elements of a construction is proposed. This technique necessitates the evaluation of magnetic energy at two rather close values of current in a coil on the magnetization curve, i.e. static parameters of a magnet system are applied for the determination of the dynamic inductance. As, at present, magnet systems are more frequently designed on the basis of a magnetic field distribution analysis, obtained as a result of numerical simulation, the calculation of magnet energy involves no difficulties.

Algorithmic aspects of numerical simulation of specific and surface ponderomotive force loads for practical needs for designing electrophysical devices are given.

I. DETERMINATION OF THE DYNAMIC INDUCTANCE

The total magnetic energy W can be determined as follows [1], [2]

$$\begin{aligned} W &= \frac{1}{2}\int(\vec{B}\cdot\vec{H}) \quad dV = \frac{1}{2}\int(\vec{A}\cdot\vec{j}) \quad dV = \\ &= \frac{1}{2}\sum_k^N \int(\vec{A}\cdot\vec{j_k}) \quad ds_k\, dl_k \\ &= \frac{1}{2}\sum_k^N I_k \int(\vec{A}\cdot d\vec{l_k}) j_k/I_k \quad ds_k = \\ &= \frac{1}{2}\sum_k^N I_k \int([\nabla\times\vec{A}]\cdot d\vec{S_k^{(j)}}) j_k/I_k \quad ds_k = \\ &= \frac{1}{2}\sum_k^N I_k \int(\vec{B}\cdot d\vec{S_k^{(j)}}) j_k/I_k \quad ds_k = \\ &= \frac{1}{2}\sum_k^N I_k \int \check{\psi}_k j_k/I_k \quad ds_k = \frac{1}{2}\sum_k^N \Psi_k I_k, \end{aligned}$$

where $\vec{B}$,$\vec{H}$ - are the magnetic induction and strength vectors;

$\vec{A}$ - is the vector potential ($\vec{B} = rot\vec{A}$);

$\vec{j}$, I_k, Ψ_k - are the density vector, total current and flux linkage of the K-th coil;

$S_k^{(j)}$ - is the transverse cross-section of the K-th coil;

l_k - is the loop of an elementary current filament with S_k cross-section.

In the general case the dynamic inductance of a current coil is known [4] to determine the velocity of the magnetic flux linkage with this coil

$$\frac{d\,\Psi_i}{d\,t} = L_{din_{ik}} \cdot \frac{d\,I_k}{d\,t}$$

Let us define:

$$\frac{\partial\, W}{\partial\, I_k} = \frac{1}{2}\sum_i^N \left(\frac{\partial\,\Psi_i}{\partial\, I_k}\cdot I_i + \Psi_k\right) = \frac{1}{2}\sum_i^N I_i\left(L_{din_{ik}} + L_{st_{ik}}\right),$$

where $L_{st_{ik}}$ - is the static inductance.

Without limitation of the commonness let us consider the case of one coil ($N = 1$). Then

$$\frac{d\,W}{I\,dI} = \frac{1}{2}\left(L_{din} + L_{st}\right)$$

The final expression for L_{din} is the following:

$$L_{din} \quad = \quad \frac{2d\,W}{Id\,I} - L_{st}, \tag{1}$$

where

$$L_{st} = \frac{\Psi}{I} = \frac{2\,W}{I^2}$$

The determination of L_{din} according to (1) necessitates the two-fold computation of the problem for a magnet system to define the energy increment $\Delta\,W$. However, due to a small current increment $\Delta\,I$ in a coil the results of the previous numerical simulations are rather well initial approximation for subsequent computations. The efficiency of a similar procedure is substantially increased, if determination of $L_{din} = L_{din}(I)$ dependency is needed.

In the case, when $W_1 = W(I_1)$ and $W_2 = W(I_1 + \Delta\,I)$ are known, L_{din} can be defined using the central difference of the form

$$L_{din}(I_1 + 0.5\Delta I) \;=\; 4\frac{W_2 - W_1}{(k^2-1)\,I_1^2} - \frac{W_2 + k^2 W_1}{k^2 I_1^2}, \tag{2}$$

where

$$k = 1 + \frac{\Delta\,I}{I_1}$$

For the case of N coils, matrix of the dynamic inductances is calculated in the similar way to (2).

II. PONDEROMOTIVE FORCE SIMULATION

The problem of ponderomotive force determination has been discussed in [1], [6], [3], [7].

In using the finite element method for spatial magnetic field simulation it is assumed that magnetic permeability μ to be constant in each finite element. Such an approach permits a required accuracy of calculations of magnetic induction components and magnetic intensity ones, as well as field, energy, inductance and so on. In this case it is naturally to use a linear dependence between magnetic inductance B and magnetic intensity H for calculations of the ponderomotive force. Thus, detailed distribution of "the equivalent density "of ponderomotive force in ferromagnetic [1] can be constructed by using the "Maxwell Stress", $B^2 \,/\, 2\mu_0$. In the given model all of the finite element sides are "strong break surfaces" [6] in electromagnetic field. For real geometry of magnet systems such surfaces are interfaces, on witch the surface density of ponderomotive force has physical sense [6]

Defining the outward normal from media "1" to media "2" one can obtained the expression for the ponderomotive force density, acting upon the interface

$$\vec{f} = \frac{1}{\mu_o}(B_{2n}\vec{H}_2 - B_{1n}\vec{H}_1) - \frac{1}{2\mu_o}(B_2 H_2 - B_1 H_1)\vec{n} \qquad (3)$$

Thus, an algorithm of ponderomotive force calculations permits to find the specific ponderomotive force density, as well as the surface one, acting upon the interfaces of electromagnetic value break. Also it is possible to find the resultant force applied to the whole body, as well as to the part of the body taking into account small construction gaps . Such an approach has been used and appropriate software FERROPON (Finite Element, FERRomagnet continua, PONderomotive force distribution) has been developed. Though this software is a part of the KOMPOT program package [5], it can be easily used separately for ponderomotive force calculations for available distribution of magnetic field.

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