Some Remarks on the Location of Higher Order Modes in Tapered Accelerating Structures with the use of a Coupled Oscillator Model

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ABSTRACT

This work is an attempt to overcome the difficulties which electrodynamical codes meet when calculating of RF characteristics of long accelerating structures consisting of cells of different geometry. The model of coupled oscillators (a kind of network model) seems to be an useful instrument in this case and may at least improve our understanding of higher order mode behavior in long tapered structures. The parameters of the model of multiple coupled oscillators (MCO) were determined on the basis of MAFIA calculations of the high order mode characteristics.

The frequency spectra, field distribution along the structure and normalized loss factor for the first band of HOMs in S-band accelerating section have been calculated. A comparison of the results of MCO calculations with the experimental data and the results of certain MAFIA calculations are presented.

1. INTRODUCTION

One of the most powerful and most general methods for numerical investigations of accelerating structures is the methods of moments. For example the modal field matching technique (or orthogonal expansion) and the finite element method can be derived from it. Both methods have to solve large algebraic systems - even in a case of a single resonator. The problem gets much more difficult for the calculation of accelerating structures consisting of a large number of coupled cells with different geometry. For example the calculation of the S-band Linear Collider (SBLC) structure with 180 different cells is out of the range of volume discretizing methods like FD- and FE-methods and needs a high com-putational effort for the modal field matching technique. To overcome this, it is reasonable to use resonant field itself as a basis function so that the discretization is much more ef-fective and only one or a few unknowns per cell have to be evaluated. This method expressed in the terms of a network model and based mostly on an analogy is well known and used successfully very often (see for example [1]). In this work we develop the approach in more general form to es-tablish more clear relations between electrodynamical values and model parameters. A particular goal of the work was to use the method for investigation of HOM in a long tapered S-band accelerating section and to get an information about their location (resonant frequencies of HOM, which ones of them and where are trapped inside the section etc.).

2. THE IDEA OF THE MULTIPLE COUPLED OSCILLATOR (MCO) TECHNIQUE

The electrodynamic eigenvalue problem is given by Maxwell's equation:

$$\frac{1}{\varepsilon}\nabla\times\frac{1}{\mu}\nabla\times\mathbf{E}=\omega^2\mathbf{E}\,. \qquad (1)$$

Let us divide the resonator into some subvolumes and suppose that the fields may be expressed in each subvolume by some eigenfunctions $\mathbf{E}_i$. This eigenfunctions need not to fulfill the same boundary conditions (on the surfaces of the subvolumes) and we can use eigenfunctions of different sets of subvolumes. The major purpose of this eigenfunctions is to serve as a basis functions so that the total field can be approximated by a linear combination

$$\mathbf{E}'=\sum x_j\mathbf{E}_j\,. \qquad (2)$$

By substituting this into the eigenvalue equation (1) we find the residuum function

$$\mathbf{r}=\sum x_j\frac{1}{\varepsilon}\nabla\times\frac{1}{\mu}\nabla\times\mathbf{E}_j-\sum x_j\mathbf{E}_j \qquad (3)$$

and can apply the Galerkin test method:

$$r_i=\int\mathbf{E}_i\,\mathbf{r}\,dV=\sum x_j\int\mathbf{E}_i\frac{1}{\varepsilon}\nabla\times\frac{1}{\mu}\nabla\times\mathbf{E}_j dV-\sum x_j\int\mathbf{E}_i\mathbf{E}_j\,dV \qquad (4)$$

Finally we claim that the tested residuum functions vanish and get a matrix eigenvalue problem:

$$\mathbf{A}\,\mathbf{x}=\omega^2\mathbf{B}\,\mathbf{x} \qquad (5)$$

with

$$(\mathbf{A})_{ij}=\int\mathbf{E}_i\frac{1}{\varepsilon}\nabla\times\frac{1}{\mu}\nabla\times\mathbf{E}_j dV \text{ and } (\mathbf{B})_{ij}=\int\mathbf{E}_i\mathbf{E}_j\,dV\,. \qquad (6)$$

The equation (5) can be interpreted in the terms of a network model, a mechanical system of pendulums and springs and so on. But a nature of the model is not that important, so we may constraint themselves by a following physical meaning of matrix elements - the ratios $((\mathbf{A})_{ii}/(\mathbf{B})_{ii})^{1/2}$ are the eigenfrequencies of some oscillators associated with given eigenfunctions (or subvolumes) and $(\mathbf{A})_{ij}$ and $(\mathbf{B})_{ij}$ $(i\neq j)$ are the couplings between them. Eigenvector $\mathbf{x}_n$ describes a field distribution along an accelerating section at resonant fre-quency ω_n.

One of the fundamental problems with this method is, that the eigenfunctions $\mathbf{E}_i$ have to be calculated numerically and there may occur some difficulties with singularities in the

coefficient integrals $(\mathbf{A})_{ij}$. Therefore the elements of matri-xes **A** and **B** are mostly calculated by indirect methods.

3. A CHAIN OF IDENTICAL CELLS.

To build a model for a tapered accelerating section we have at first to determine a structure of MCO matrixes and then to calculate MCO parameters for the cells of every geometry separately. A basis for the calculation is a frequency spectra (calculated or measured) of the modes under interest of a section consisting of the identical cells of given geometry. We are interested in the first dipole band (TM-like modes) which is considered to be the most dangerous for a beam sta-bility. But the second dipole band (TE-like modes) is very close to the first one and they combine a common spectra, so we have to take into account both of them. The same problem arisen at the time of investigation of DAW structure and a biperiodical chain of oscillators described by 5-diagonal symmetrical matrixes suggested in that work has been chosen as a model [2].

Let's associate the odd oscillators with TE-like modes and the even oscillators with TM-like modes. Following [3,4] we can derive a 2×2 matrix dispersion equation. Using this eq-uation, given frequency spectra and some relations origi-nating from the properties of HOMs we can calculate all MCO parameters for a cell of given geometry. In principle we get two solutions (two sets of MCO parameters) because the chain is biperiodical, but a choice of a right set is obvious and can be included in the procedure beforehand. A set of MCO parameters can be checked with the use of analytical solution of the matrix dispersion equation.

In Fig.1 the dispersion curves calculated by MCO for the cells N1 and N10 of a test S-band section [5] are given as an example together with MAFIA calculations.

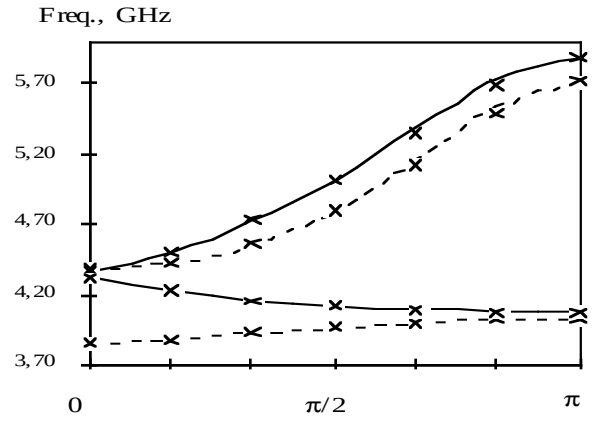

Fig.1. Dispersion curves for the cells N1 (dashed) and N10 (solid) of the test S-band structure calculated by MCO. Points - MAFIA calculations.

For magnetic walls inside end-irises E_z in centers of cells is described by $E_{0z}(\theta)\sin[(n-1/2)\theta]$. Even elements of vector $\mathbf{x}_\theta$ are $a(\theta)\sin[(i-1)\theta/2]$. (Here θ - phase advance per cell, n - number of cell, i - even). Fig.2 demonstrates $E_{0z}(\theta)$ and $a(\theta)$ for lower branch of dispersion curve of cell N1 calculated by MAFIA (2 mm apart the structure axis) and MCO. Vectors are normalized as $\mathbf{xx} = 1$, E_{0z} is normalized by stored energy, both amplitudes at $\theta = \pi$ are chosen equal to 1.

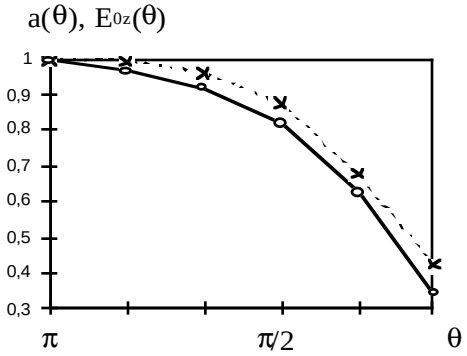

Fig.2. Amplitudes $a(\theta)$ and $E_{0z}(\theta)$ for lower branch of dispersion curve of cell N1. MAFIA - dashed, MCO - solid.

So, more or less accurate relation between absolute values of $E_{0z}(\theta)$ and $a(\theta)$ can be calculated.

4. A CHAIN OF TAPERED CELLS.

A real S-band section for Linac-II at DESY consisting of 148 cells has been chosen as an example of tapered acce-lerating section by two reasons: it is close to SBLC section by its parameters and the measurements of it were available.

To form the matrixes for a such long section a following procedure has been used: the MCO parameters for the sepa-rate cells (NN 1,26,50,76,100,126 and 148) have been cal-culated, the parameters of intermediate cells have been found by approximation and then all oscillators have been com-bined into full matrixes.

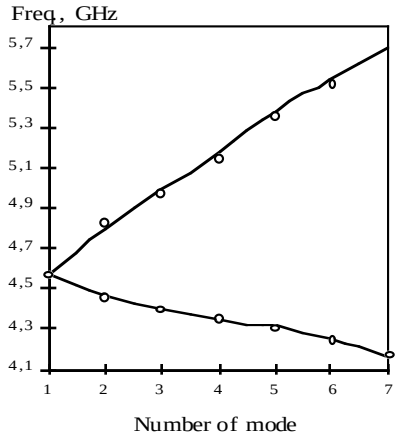

Fig.3. "Dispersion curve" of 6-cell strongly tapered section. Line - MCO, points - MAFIA.

To check the model a short strongly tapered section consisting of cups mentioned above was calculated by MAFIA and the results were compared to MCO calculations. Fig.3 shows a "dispersion curve" of the short strongly tapered structure.

5. RESULTS OF CALCULATIONS AND EXPERIMENTS

The MCO model of 148-cell Linac-II section was made and the resonant frequencies and field distributions for all modes were calculated. The results shown that the main part of modes, including dangerous ones with π-like distribution pieces, are not trapped inside section and can be excited through the first cell or even input coupler. This had been used when the spectra measurements and bead-pull field distribution measurements were performed in Linac-II section. Fig.4 shows calculated "dispersion curve" of the section and experimental resonant frequencies. Fig.5 shows a field distribution (calculated and measured) along structure of one of the modes.

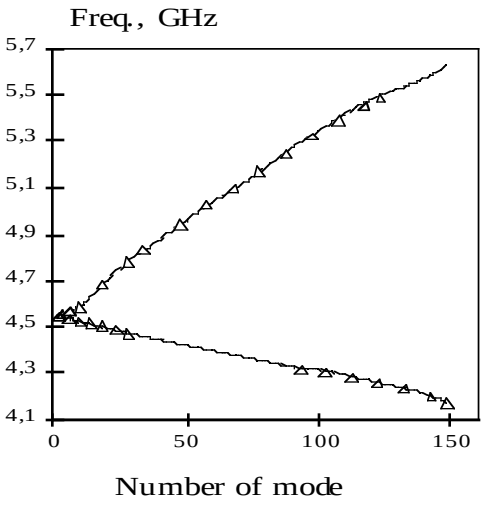

Fig.4. "Dispersion curve" of full 148-cell Linac-II S-band section. Lines - MCO calculations, points - measurements.

Calculations of loss factor parameter was done for the first band of Linac-II section on a basis of MCO data. Absolute values of E_z wasn't calculated, so a curve in Fig.6 should be considered just as normalized and qualitative.

6. CONCLUSION.

The comparison of the MCO calculation results to the experimental data and the test MAFIA calculations shows that this model can be useful and sufficiently accurate instrument for investigation of long tapered structures and in other ca-ses, which require high computational efforts by other met-hods. Also it should be mentioned that MCO technique gives a possibility to investigate accelerating structure properties in analytical form.

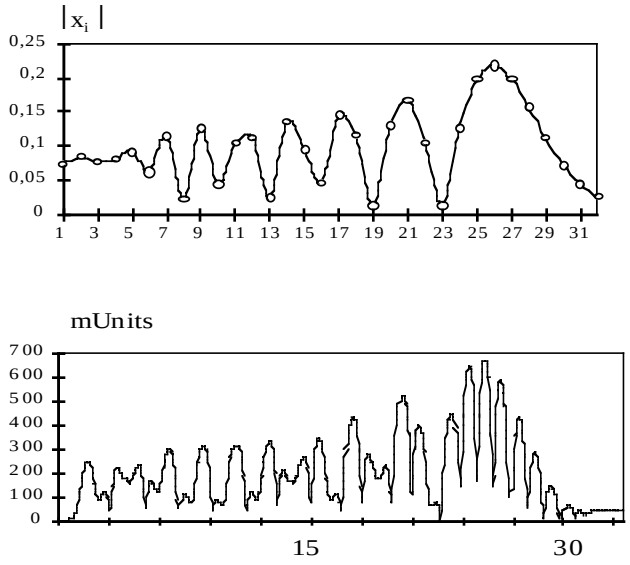

Fig.5. Distribution of E_z against number of cell of mode 138 of lower branch. Lower drawing - experimental data.

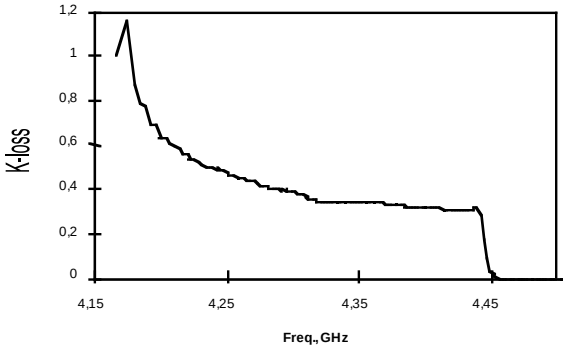

Fig.6. Normalized loss-factor for lower band of Linac-II S-band section.

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