A Proof of Principle of A Storage Ring with Fifth-Order Achromatic Bending Arcs

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Abstract

The design of a storage ring consisting of two identical 180-degree bending arcs and two short straight sections is presented. Each of the bending arcs is a four-cell fifth-order achromat designed according to a recently developed theory about arbitrary-order achromats. Instead of repetition of cells, which is widely used in achromat design based on normal form theory, we utilize cells which are obtained from the the original ones through mirror imaging about the x-y plane, which corresponds to a reversion. In our design, the second cell is the reversion of the first one. The third and fourth cells are identical to the first and second ones, respectively. Long term stability is studied through high-order tracking using code COSY INFINITY [1].

1 Introduction

In the past few years, various third-order achromatic systems containing at least seven repetitive identical cells have been found using normal form theory [2] [3] [4]. The number of bending magnets needed ranges from 7 to 300. Each solution requires a specific number of cells depending on the choice of the tunes of a cell.

By introducing mirror symmetry into the consideration, we developed a new theory which requires only four cells and as few as one bend per cell to obtain achromats of, in principle, arbitrary orders [5] [6]. The use of mirror symmetry enables us to choose from four kinds of cells, namely the forward cell (F), the cell in which the order of elements is reversed (R), the cell in which the direction of bend is switched (S), and the cell where reversion and switching is combined (C). According to the theory, the minimum number of conditions required for a four-cell fifth-order achromat with an arbitrary forward cell are five for the first order, four for the second order, fifteen for the third and the fourth orders and thirty nine for the fifth order. The optimal four-cell systems which require only the minimum number of conditions are listed in Table 1, together with the first-order requirements.

One of the possible applications of high-order achromats is achromatic bending sections of accelerators. In this report, a storage ring with two fifth-order achromatic bending arcs is presented. The detail of the design is discussed in Section 2. In Section 3, the repetitive stability is studied through tracking. Conclusions are given in Section 4.

Systems	Linear Conditions
F R S C	$(a\|\delta) = 0$, $(x\|a) = (a\|x) = 0$
F R F R	$(a\|\delta) = 0$, $(x\|x) = (a\|a) = 0$
F C S R	$(x\|\delta) = 0$, $(x\|a) = (a\|x) = 0$
F C F C	$(x\|\delta) = 0$, $(x\|x) = (a\|a) = 0$

Table 1: The optimal four-cell systems Additional linear conditions are $(y|y) = (b|b) = 0$ or $(y|b) = (b|y) = 0$ for each one.

2 Design of the Achromat

2.1 *First- and Second-Order Design*

In order to design an achromatic bending arc, no switched (S) or switched-reverse (C) sections can be used. Thus, the only choice is FRFR. The first-order layout should avoid large changes in the beta functions in order to minimize nonlinear aberrations; furthermore, there should be room for the insertion of correction multipoles. Another consideration is that, if possible, the number of first-order conditions should be further reduced through symmetry arrangements inside a cell.

The result of these thoughts is shown in Figure 1, where the 180-degree bending arcs are achromatic. The forward cell itself also consists of two parts, where one is the reversion of the other. This guarantees that at the end of it, $(x|x) = (a|a)$ and $(y|y) = (b|b)$. The building block of the arc is a FODO cell consisting of a defocusing quad, a 5.625° bend, and a focusing quad. All four FODO cells within one part of a cell are identical except the last one, which has an extra quadrupole for dispersion correction. So there are three knobs for the first-order design which can zero $(x|x)$, $(a|a)$, $(y|y)$, $(b|b)$, $(x|\delta)$ and $(a|\delta)$ at the same time. Figure 2 shows that the beam moves around the arc in a quite uniform manner avoiding large ray excursions and beta functions.

According to the arbitrary-order theory, four independent sextupoles are required to obtain a second-order achromat. However, because of the fact that to the first order, the cell R

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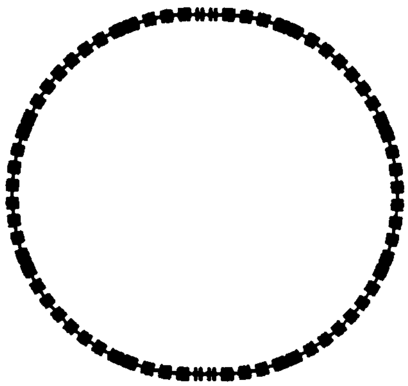

Figure 1: The layout of a storage ring with fifth-order achromatic bending arcs; the circumference is 1451.06 m; the tunes are $T_x = 0.03654, T_y = 0.03721$.

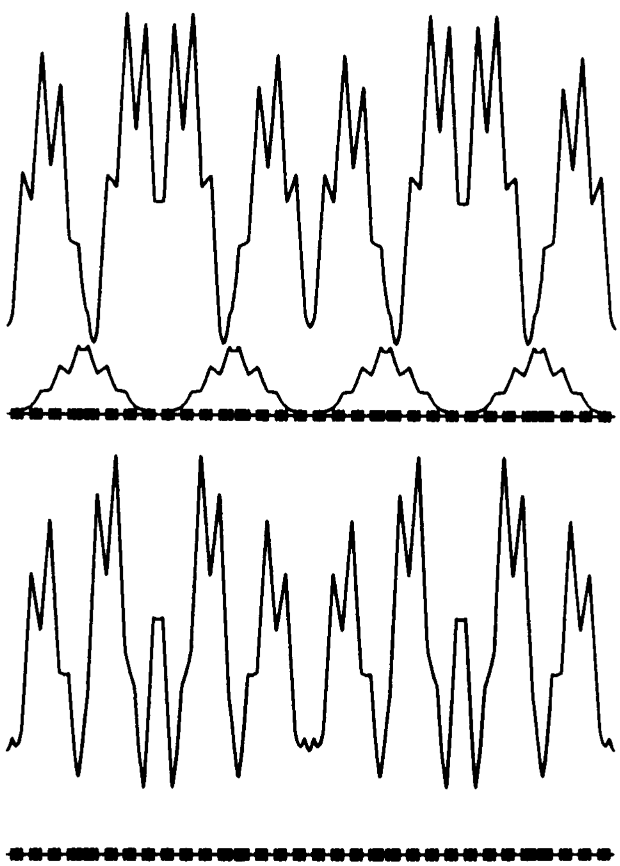

Figure 2: The beam envelope and the dispersive ray of the horizontal (top) and vertical (bottom) motion; emittance: 10π mm mrad (horizontal and vertical); $\Delta E/E$: 0.1%.

is identical to the cell F, a simplification is possible based on Brown's theory of second-order achromats ([7] [8]). In this theory, it is shown that a second-order achromat can be achieved by placing two pairs of sextupoles in dispersive regions and cancelling one chromatic aberration in each transverse plane. Therefore, a second-order achromat can be achieved on the arc using two sextupoles per cell. In our case, it turns out to be advantageous to split the sextupoles into symmetrically excited pairs to ensure that up to the second order the second cell (R) still is the reversion of the first.

2.2 Higher-Order Design

After the investment in a careful first-order layout, the third-, fourth- and fifth-order corrections actually turn out to be conceptually straightforward, even though they are computationally more demanding. In the whole process of nonlinear optimization, only two aspects seem to be worth considering. First, the required multipole strengths are quite sensitive to the average distance among multiples of the same order. So, in order to keep their strength limited, it is important to dimension the total size of the ring and the dispersive region sufficiently large, as done in the previous section, and distribute multipoles of the same order roughly uniformly.

Secondly, all the decapoles have to be placed in regions with sufficient dispersion because all the fourth-order aberrations remaining after third-order achromaticity is achieved are of chromatic type. Thus it is advantageous to use a substantial dispersive region.

The combination of these considerations results in reasonably weak multipole strengths for third-, fourth- and fifth-order corrections. Table 2 shows that a fifth-order achromat is achieved.

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 1.00     0.000E+00 0.000E+00 0.000E+00 0.0000E+00 100000
0.000E+00  1.00     0.000E+00 0.000E+00 0.0000E+00 010000
0.000E+00 0.000E+00  1.00     0.000E+00 0.0000E+00 001000
0.000E+00 0.000E+00 0.000E+00  1.00     0.0000E+00 000100
0.000E+00 0.000E+00 0.000E+00 0.000E+00  1.000     000010
0.000E+00 0.000E+00 0.000E+00 0.000E+00  33.63     000001
0.000E+00 0.000E+00 0.000E+00 0.000E+00 -38.31     000002
0.000E+00 0.000E+00 0.000E+00 0.000E+00 -11044     000003
0.000E+00 0.000E+00 0.000E+00 0.000E+00 -2.124E+05 000004
0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.9870E+09 000005
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Table 2: The fifth-order map of the arc (Zero means smaller than 2E-01.)

3 Repetitive Stability

With the arc at hand, a storage ring is designed, which contains two identical achromatic arcs and short straight sections. Each of the straight sections consist of two FODO cells with weak quads (less than 0.1 kG/cm as opposed to 1 kG/cm in the arc), which means that the quads only produce weak nonlinearities.

To study the repetitive stability of the ring, a 7th-order one-turn map is generated by COSY INFINITY and used for tracking. To be specific, we analyze the 10,000-turn dynamic behavior for both horizontal and vertical motions through the inspection of phase space plots. As an example, Figure 3 depicts the horizontal motion of on-energy particles up to 10,000 turns.

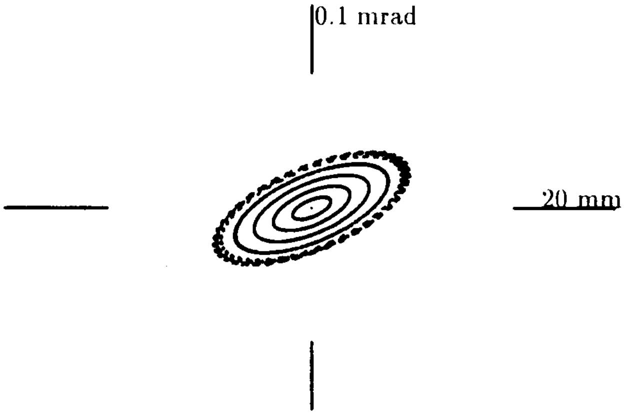

Figure 3: The 10,000-turn tracking of the x-a motion of on-energy particles

4 Conclusion

It has been shown that it is possible to design fifth-order achromatic bending arcs. Careful first-order considerations allow the use of relatively weak correction elements, and thus also weak nonlinearities beyond the orders that can be corrected. A storage ring containing the achromatic arcs is presented, and the repetitive stability is studied.

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