VECTOR COMPUTER USED FOR CALCULATION OF 3D MAGNETOSTATIC FIELDS *

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Abstract

In this communication we describe the results of computations of 3D nonlinear magnetic field for a dipole magnet. Computations were carried out on vector computer CONVEX C120 by means of program MSFE3D [1].

I. FORMULATION OF THE PROBLEM

We consider the differential formulation of the magnetostatic problem for two scalar potentials [2], total – ψ and reduced – ϕ. Let Ω_F be a region with ferromagnetic material. We choose some region Ω, which contains Ω_F. Let Γ_0 be a boundary of the region Ω and $\Omega_A = \Omega \setminus \Omega_F$. Γ is a boundary between Ω_F and Ω_A. Then we have equations

$$div(\mu\nabla\psi) = 0, \qquad x\in\Omega_F; \tag{1}$$

$$div(\nabla\phi) = 0, \qquad x\in\Omega_A; \tag{2}$$

with the boundary conditions

$$\mu(\partial\psi/\partial n) = \partial\phi/\partial n - \mathbf{n}\cdot\mathbf{H}^S, \quad x\in\Gamma; \tag{3}$$

$$\psi = \phi + \phi^S, \qquad x\in\Gamma; \tag{4}$$

$$\phi\approx 0, \qquad x\in\Gamma_0. \tag{5}$$

Function $\mu = \mu(|\nabla\psi|)$ is given from the closed interval $1 \le \mu_* \le \mu \le \mu^*$ where μ_* and μ^* are known constants. Vector $\mathbf{H}^S$ is computed by the Biot-Savart's law

$$\mathbf{H}^S(x_0) = \frac{\mu_0}{4\pi}\int\limits_{\Omega_S} \mathbf{J}\times\nabla\frac{1}{|x-x_0|}d\Omega_S,$$

where Ω_S is a source region, $\mathbf{J}$ is a known vector of current density, μ_0 is the permeability of free space, $|x-x_0|$ is the distance between points x and x_0. Potential ϕ^S is defined from the Laplace equation in the region Ω_F

$$\Delta\phi^S(x) = 0, \quad x\in\Omega_F, \tag{6}$$

with the boundary conditions

$$\frac{\partial\phi^S}{\partial n} = -\mathbf{n}\cdot\mathbf{H}^S, \quad x\in\Gamma, \qquad \int\limits_\Gamma \mathbf{n}\cdot\mathbf{H}^S ds = 0. \tag{7}$$

It is known that the generalized solutions of the formulated boundary value problems exist. The generalized solution of nonlinear problem (1)- (5) is unique, and the generalized solution of Neumann problem (6) - (7) is defined with constant. Finite element approximations converge to the exact generalized solutions.

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II. USING THE FINITE ELEMENT METHOD

As a finite element $\bar\Omega_j$, we choose convex hexahedron. Let us divide Ω by the finite elements so that $\bar\Omega = \cup\bar\Omega_j$, $\Omega_i\cap\Omega_j = \emptyset$, when $i\neq j$, supposing also, that every face of element $\bar\Omega_j$ is either a subset of Γ_0, or a face of the other element, and the boundary Γ is formed by faces of the chosen elements.

We introduce in space with Cartesian coordinate system related to variable $\xi = (\xi_1, \xi_2, \xi_3)$, the linear shape functions N_m^e for cube $[-1,1]^3$

$$N_m^e(\xi) = (1/8)(1+\sigma_{m1}\xi_1)(1+\sigma_{m2}\xi_2)(1+\sigma_{m3}\xi_3),$$

$$m = 1,\ldots,8,$$

where the coefficients σ_{mk} are chosen as 1 or -1 so that N_m^e is equal to 1 in the vertex with number m and 0 in any other vertex. In view of that the cube $[-1,1]^3$ turns into the element $\bar\Omega_j$ under the use of the transformation [3]

$$x_k = \sum_{m=1}^{8} x_k^{j,m} N_m^e(\xi), \quad k = 1,2,3,$$

where $x_k^{j,m}$, $k = 1,2,3$ are coordinates of the element $\bar\Omega_j$ vertex with number m, the base functions $N_m(\xi(x)), m = 1,\ldots 8$ on element $\bar\Omega_j$ may be obtained. We shall find the solutions of the problems (1)-(5) and (6)-(7) in the form

$$\begin{aligned}
\psi(x) &= \sum_j \psi_j N_j(x), \quad x\in\bar\Omega_F;\\
\phi(x) &= \sum_j \phi_j N_j(x), \quad x\in\bar\Omega_A;\\
\phi^S(x) &= \sum_j \phi_j^S N_j(x), \quad x\in\bar\Omega_F,
\end{aligned} \tag{8}$$

where $\psi_j, \phi_j, j = 1,\ldots,M$ and $\phi_j^S, j = 1,\ldots,M_S$ are unknown potentials values in nodes of the space mesh, which is obtained as a result of partitioning the region $\bar\Omega$. Substituting the expressions for ψ and ϕ from (8) into generalized formulation of problem (1)-(5) and taking the boundary conditions into account, we perform the equation

$$\begin{aligned}
&\sum_{x^j\in\bar\Omega_F}\psi_j\int\limits_{\Omega_F}\mu\nabla N_i\cdot\nabla N_j d\Omega + \sum_{x^j\in\Gamma}\psi_j\int\limits_{\Omega_A}\nabla N_i\cdot\nabla N_j d\Omega +\\
&+\sum_{x^j\in\Omega_A}\phi_j\int\limits_{\Omega_A}\nabla N_i\cdot\nabla N_j d\Omega = -\int\limits_\Gamma N_i\mathbf{n}\cdot\mathbf{H}^S dS +\\
&+\sum_{x^j\in\Gamma}\phi_j^S\int\limits_{\Omega_A}\nabla N_i\cdot\nabla N_j d\Omega, \quad i = 1,\ldots,M.
\end{aligned} \tag{9}$$

Potential ϕ^S is found from the equation

$$\sum_{x^j \in \bar{\Omega}_F} \phi_j^S \int_{\Omega_F} \nabla N_i \cdot \nabla N_j \, d\Omega = -\int_{\Gamma} N_i \mathbf{n} \cdot \mathbf{H}^S \, dS, \qquad (10)$$

$$i = 1, ..., M_S.$$

Note that there is no need to solve this equation in whole Ω_F, because $\mathbf{H}^S = -\nabla\phi^S$ is orthogonal to gradients of the functions which are equal to 0 on the boundary of Ω_F [4]. I.e. it is enough to solve equation (10) in some situated near boundary volume of the region, completing the boundary conditions.

The integrals over regions Ω_F, Ω_A in (9) - (10) are computed by summation of the contributions from separate elements $\bar{\Omega}_j$. In such an element μ is a constant, its argument is computed in the centre. The centre is the point, every coordinate of which is the arithmetic mean of coordinates of hexahedron vertices.

III. SOLVING THE NONLINEAR SYSTEM

Let us write the discretized system of the nonlinear algebraic equations (9) with sparse matrix as

$$F(\mu)y = f, \quad \mu = \mu(y). \qquad (11)$$

Usually for solving such a system some linearization is used and then the finding of the solution of system (11) reduces to the sequential solving of the linear problems. General iterative scheme for solving the nonlinear equation (11) has the form

$$B_n \frac{y_{n+1} - y_n}{\tau_n} = -(F_n y_n - f), \quad n = 0, 1, ..., \qquad (12)$$

where y_0, F_0, τ_0 are given. Two kinds of this process are usually used:

1. $B_n = F_n$, $\tau_n \equiv 1$;
2. $B_n = F_n^{'}$, where $F_n^{'}$ is Jacobi matrix.

We use iterative scheme (12) for $B_n \equiv F_n$ and $\tau_n \in (0, 1]$. The parameter τ_n is chosen depending on behaviour of μ on every iteration. Note that the general theory of the iterative process (12) for self-adjoint, positive definite operator F is given in book [5].

Independently of B_n form, the linearized system of equations on every iteration should be solved

$$Az = b, \qquad (13)$$

where A– symmetric, positive definite sparse matrix. Usually for this purpose the incomplete Cholessky decomposition with conjugate gradient method is used [6]. The special algorithms developed by authors in [7] are used for solving the equation (13) on the vector computer CONVEX C120.

IV. EXAMPLE OF 3D MAGNETOSTATIC FIELD CALCULATION

As an example of 3D magnetostatic field calculation we present here the computed results for dipole magnet of the setup EXCHARM. More information about this computations was published in [4,8]. The setup EXCHARM is a forward-spectrometer for investigation of hadron production of charmed particles and the indication of the narrow resonances in neutron-nucleus interactions on the U-70 accelerator, at IHEP, Serpukhov, near Moscow. It is a further development of spectrometer BIS-2 [9]. The spectrometer includes the following basic elements: the SP-40 magnet, proportional chambers, a charged particle identification system, a Cherenkov shower detector, scintillation hodoscopes, an electronic system for event preselection and data acquisition.

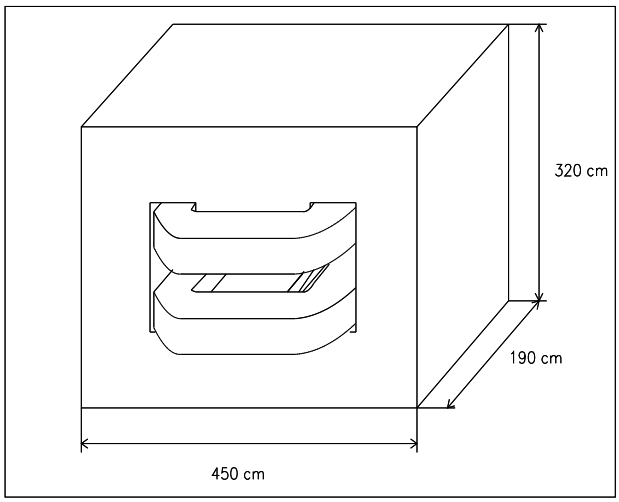

Figure 1. Schematic view of the SP-40 magnet.

The spectrometer magnet is a dipole with external dimensions $450 \times 320 \times 305\ cm^3$, with aperture $274 \times 48\ cm^2$. The length of the magnet is $190\ cm$ and the working magnitude of magnetic field is 0.75 .The schematic picture of the magnet is shown in fig.1.

All computed results are presented here in figures as the relations to the required magnetic field value B_0 in the magnet centre, where B_0 is equal to 0.75 T. The comparison of the computed (42800 nodes) and experimental curves for the relative main field component $B_y(z)/B_0$, has shown (fig.2) that inside the magnet the difference is not more than $\approx 10^{-3}$. In fig.3 the computed and experimental results are presented for the relative main component of the magnetic field for x = 0 cm., y = 19 cm.

In fig.4 the main field component at the end region of the magnet are given for y = 0 cm. The computations have shown that the absolute values of the field components B_x/B_0 and B_z/B_0 in this region are not more than 2,7 % and 4 % accordingly.

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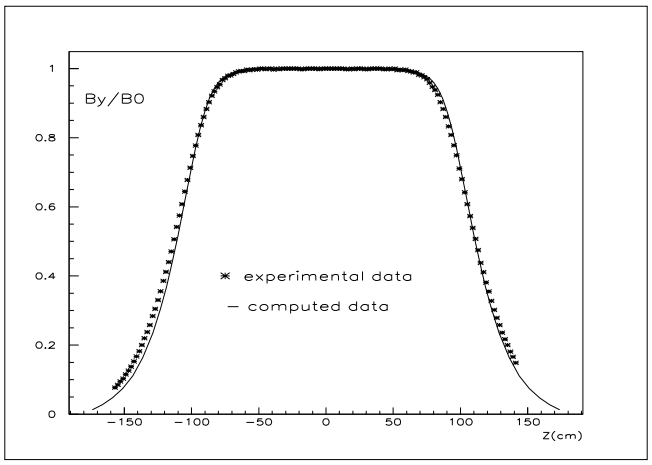

Figure 2. The comparison of computed and experimental data for relative field component B_y/B_0 for x=0 cm., y=1 cm.

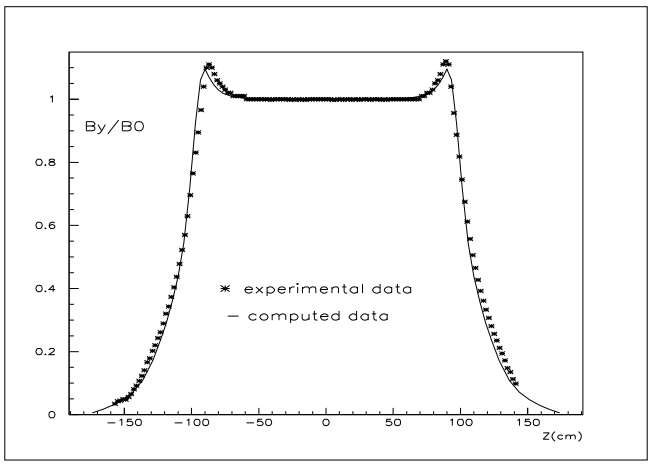

Figure 3. The comparison of computed and experimental data for relative field component B_y/B_0 for x=0 cm., y=19 cm.

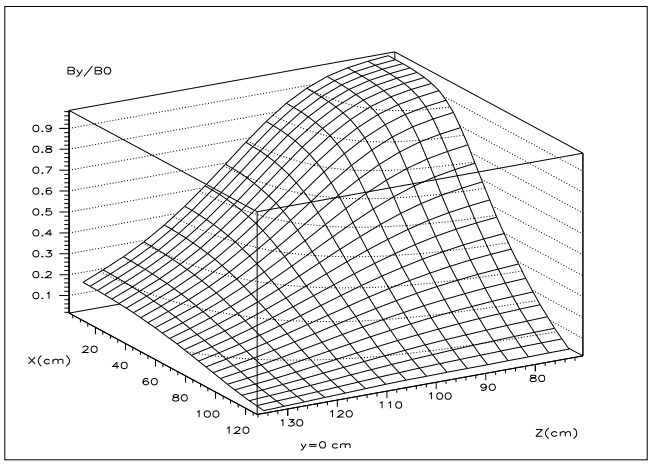

Figure 4. B_y/B_0 field component distribution at the end region of the magnet for y=0 cm.

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